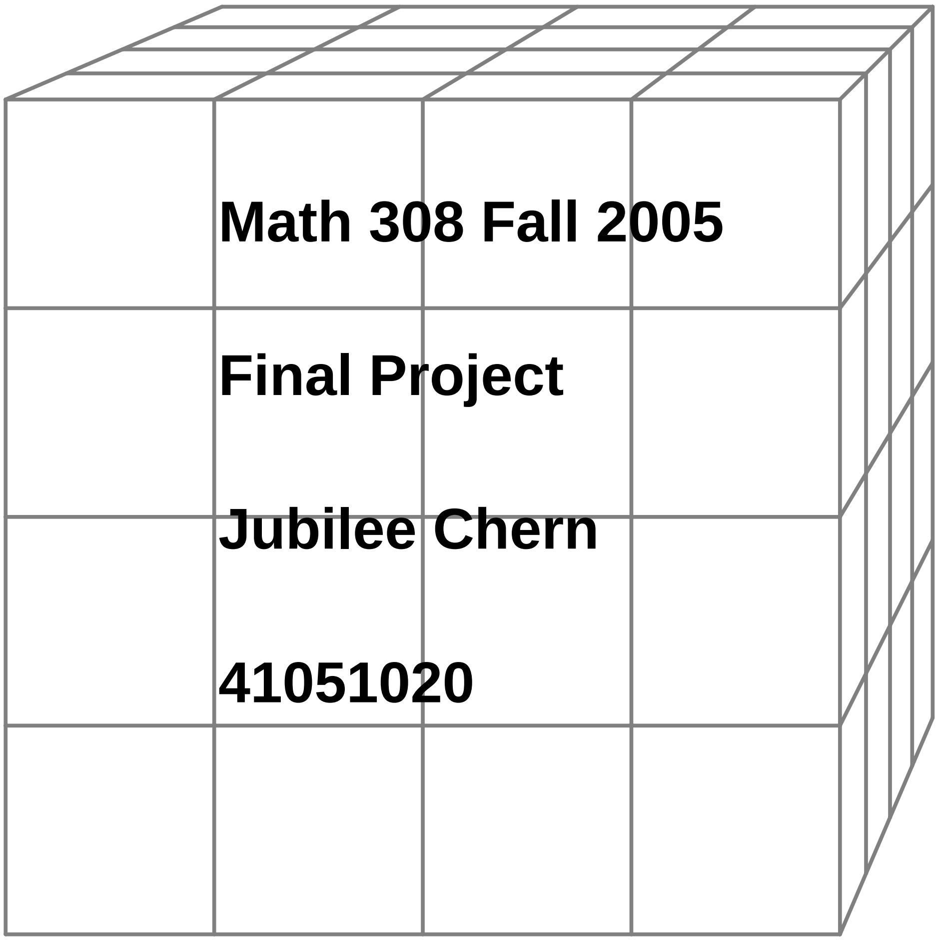
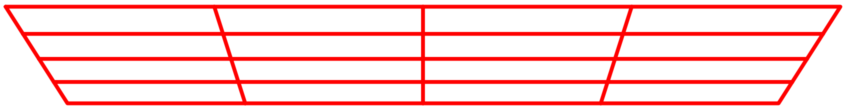


Math 308 Fall 2005

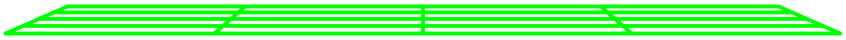
Final Project

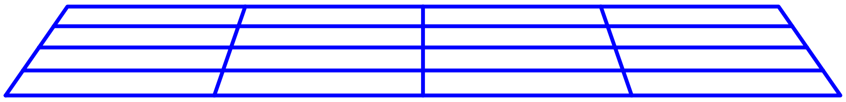
Jubilee Chern

41051020

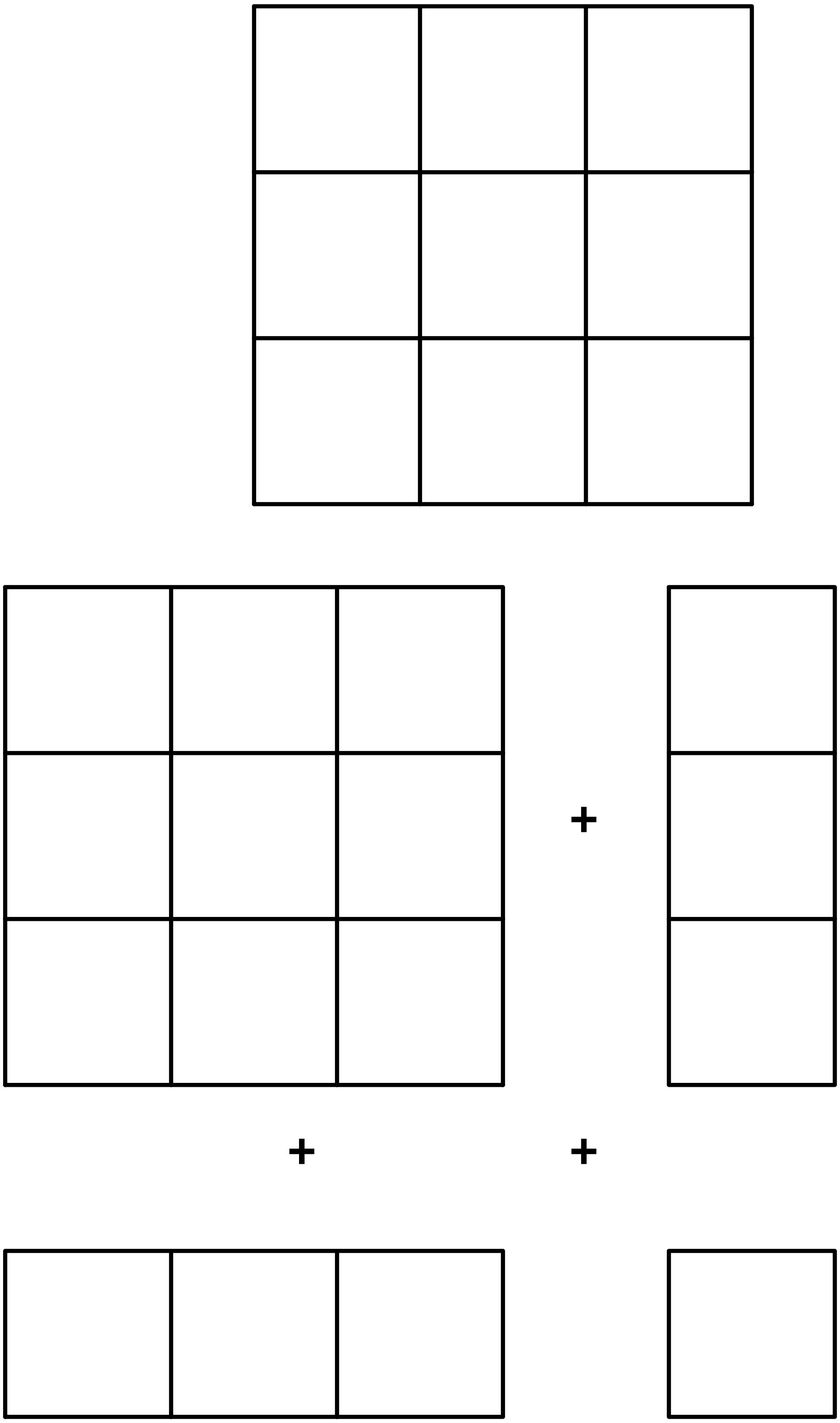


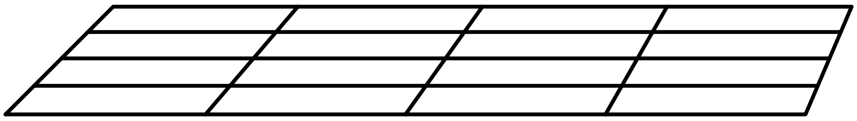
Qubic Unfoiled

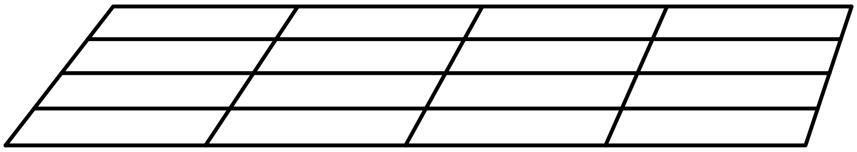


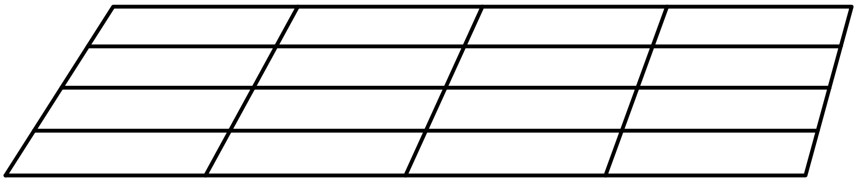


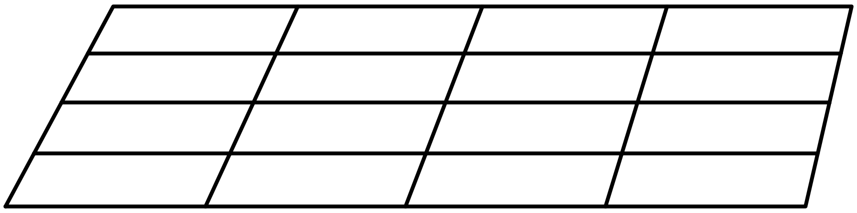
Qubic is a three dimensional (4x4x4) extension of the original 3x3 tic-tac-toe.



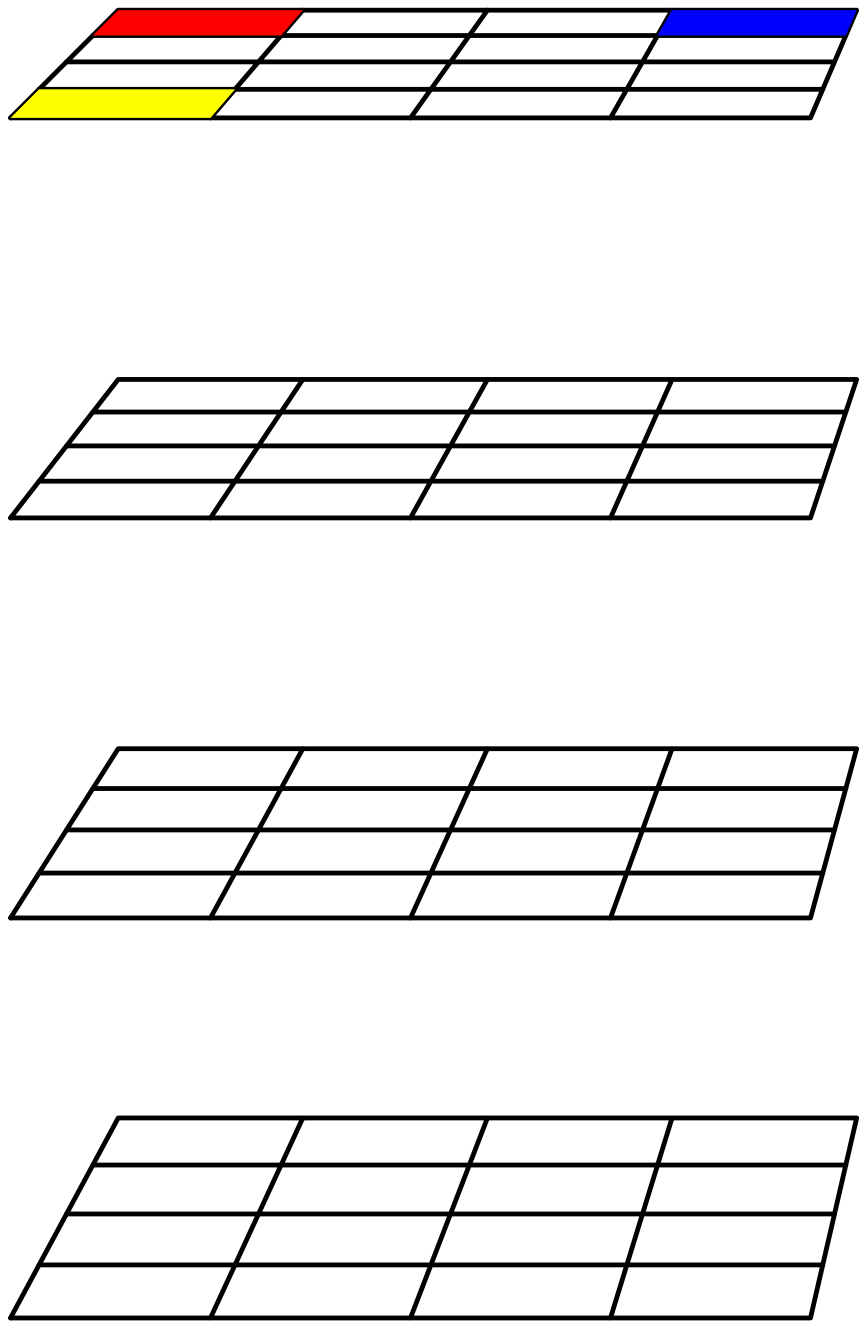








Just as the 3x3 version, 3D tic tac toe is a two people game with perfect information - both parties know all of the moves. The objective is to obtain four squares in a row in either 1D, 2D, or 3D.



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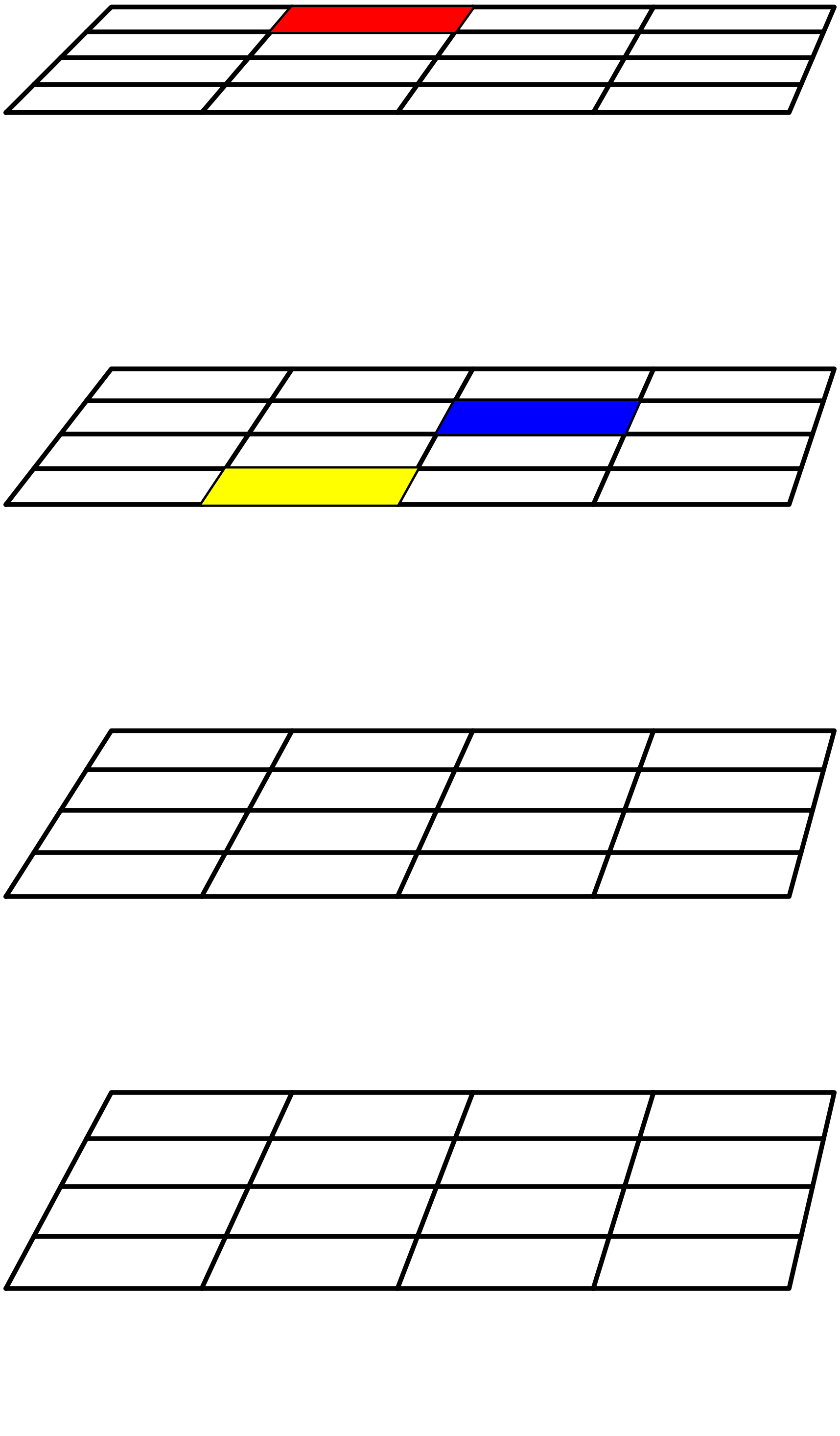

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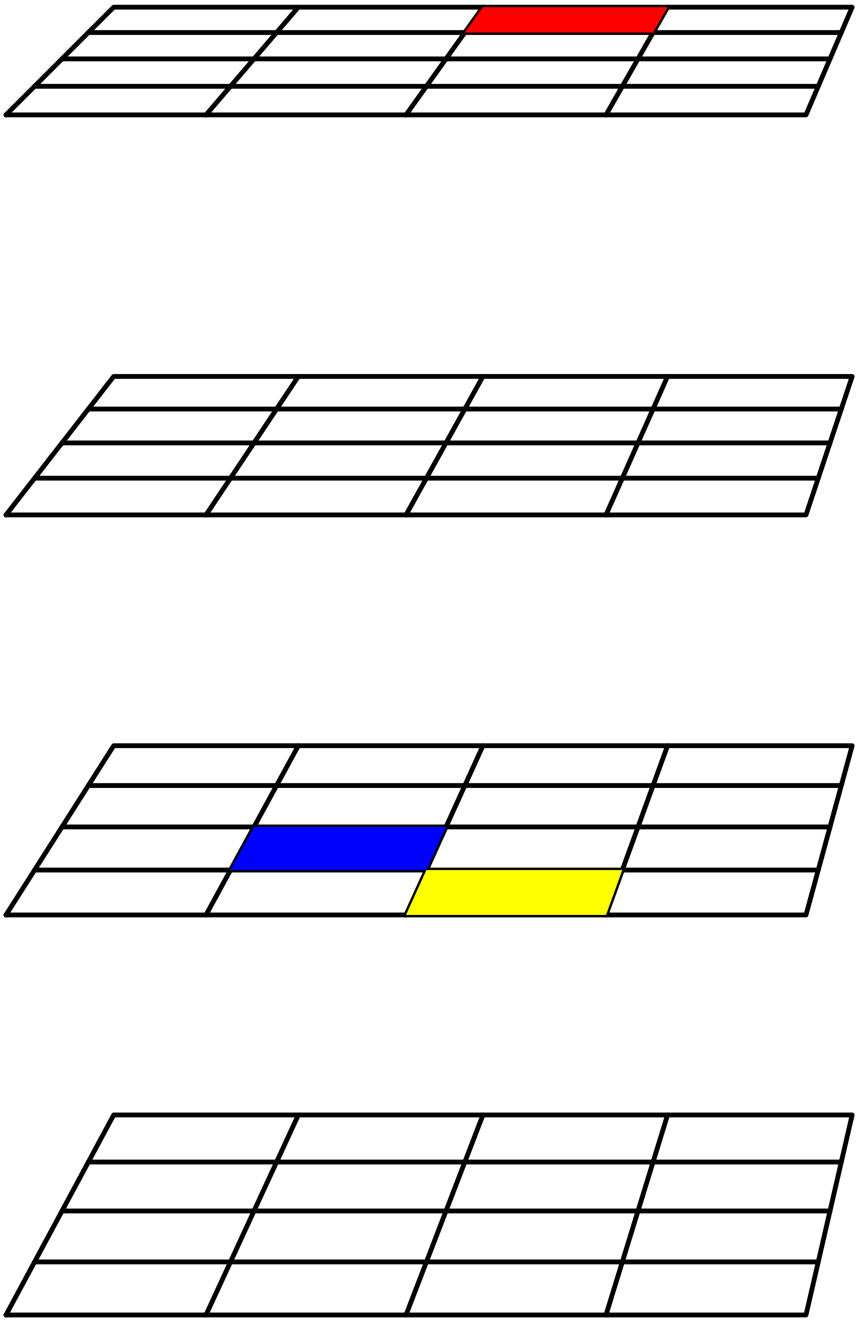

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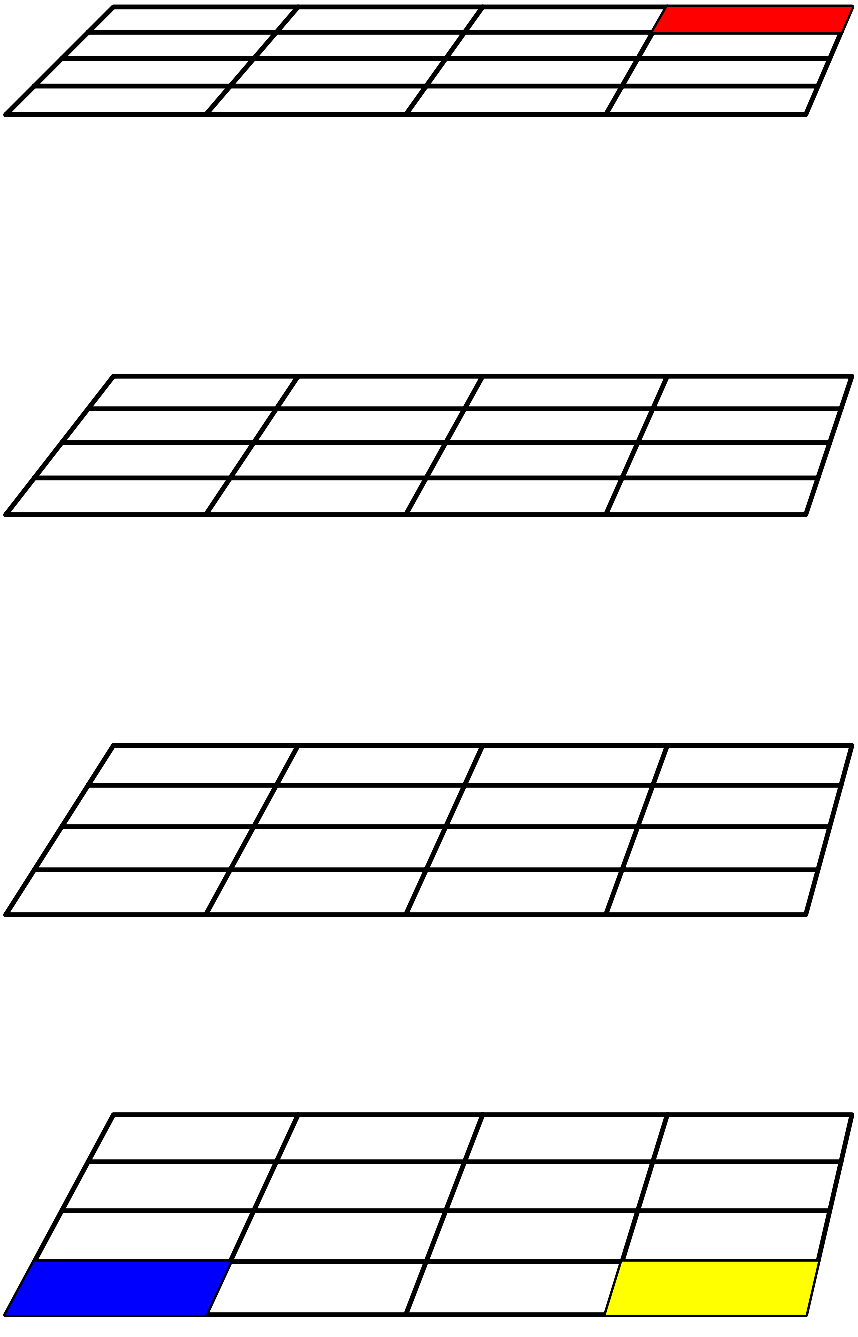
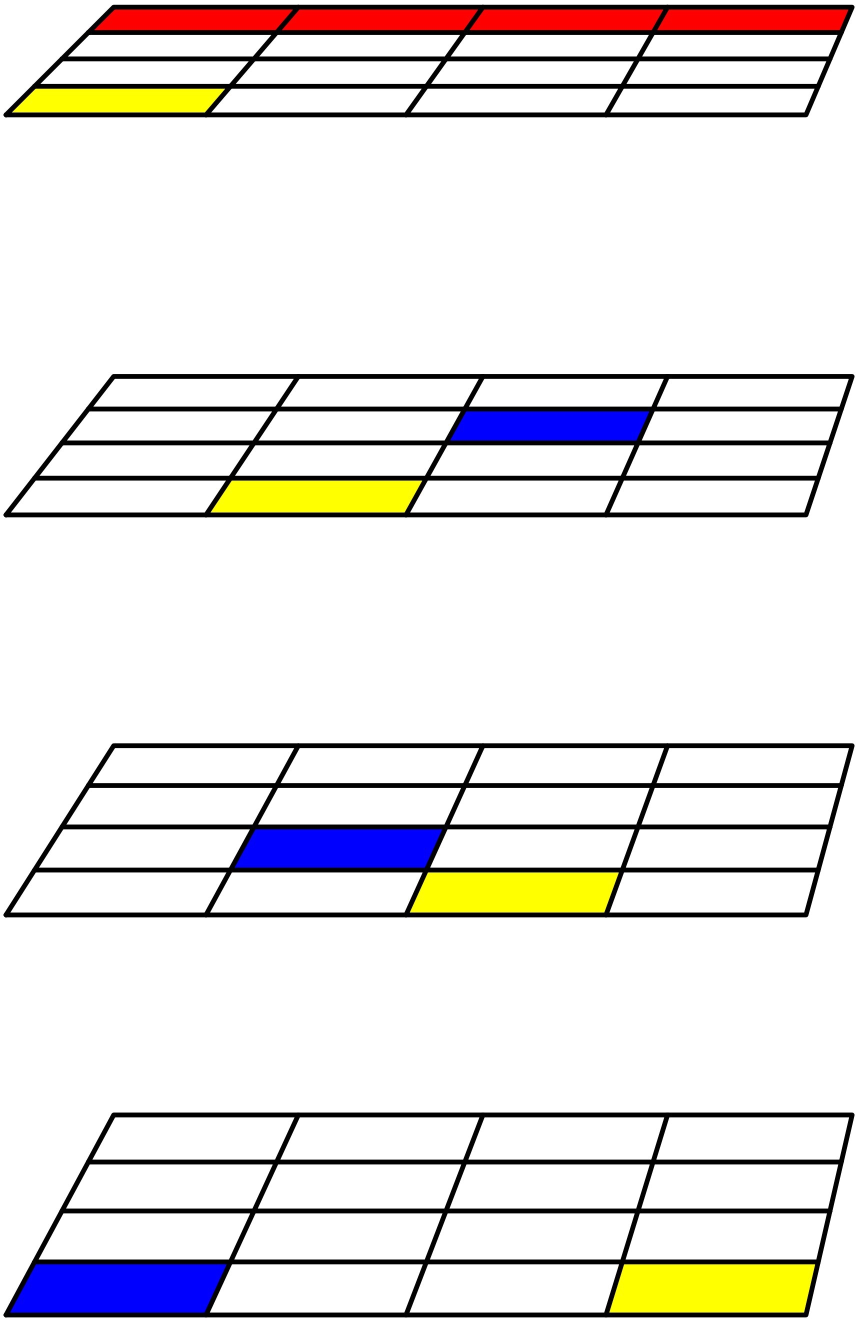


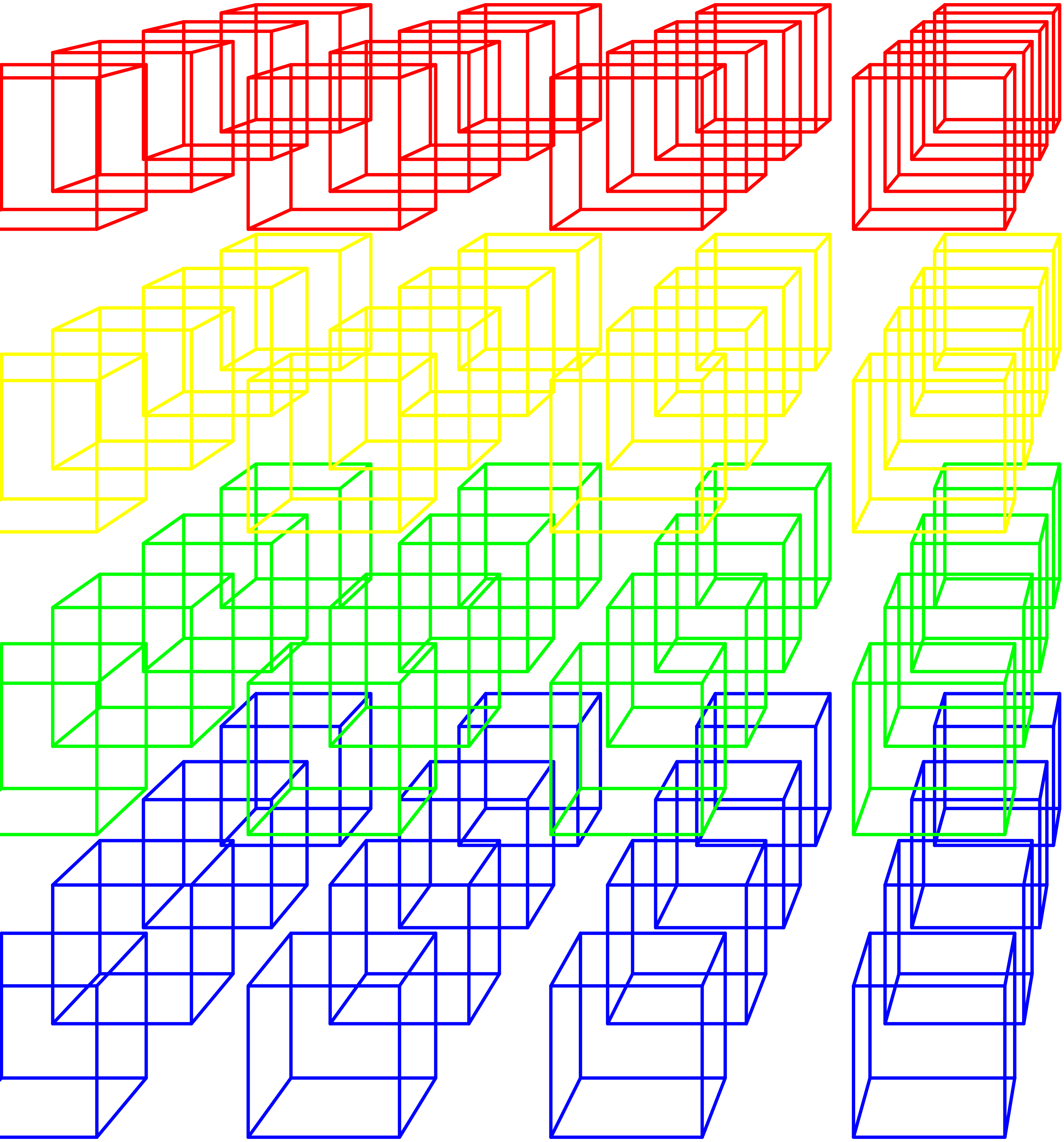
Red represents 4 in a row in 1D

Yellow represents in 2D

Blue represents in 3D



A Qubic gameboard is made of 64 points - each point is represented by a 1x1x1 cube.



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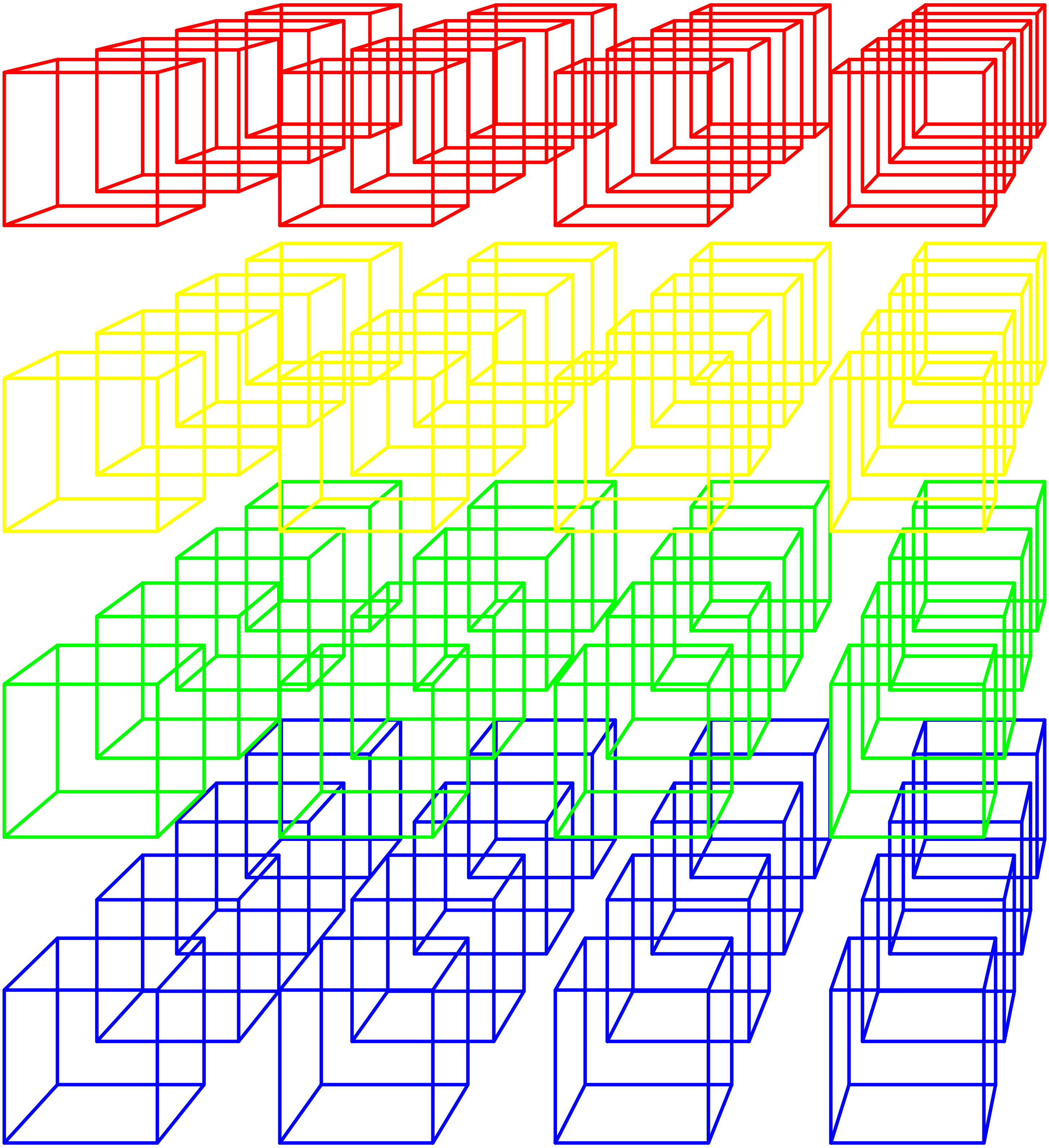

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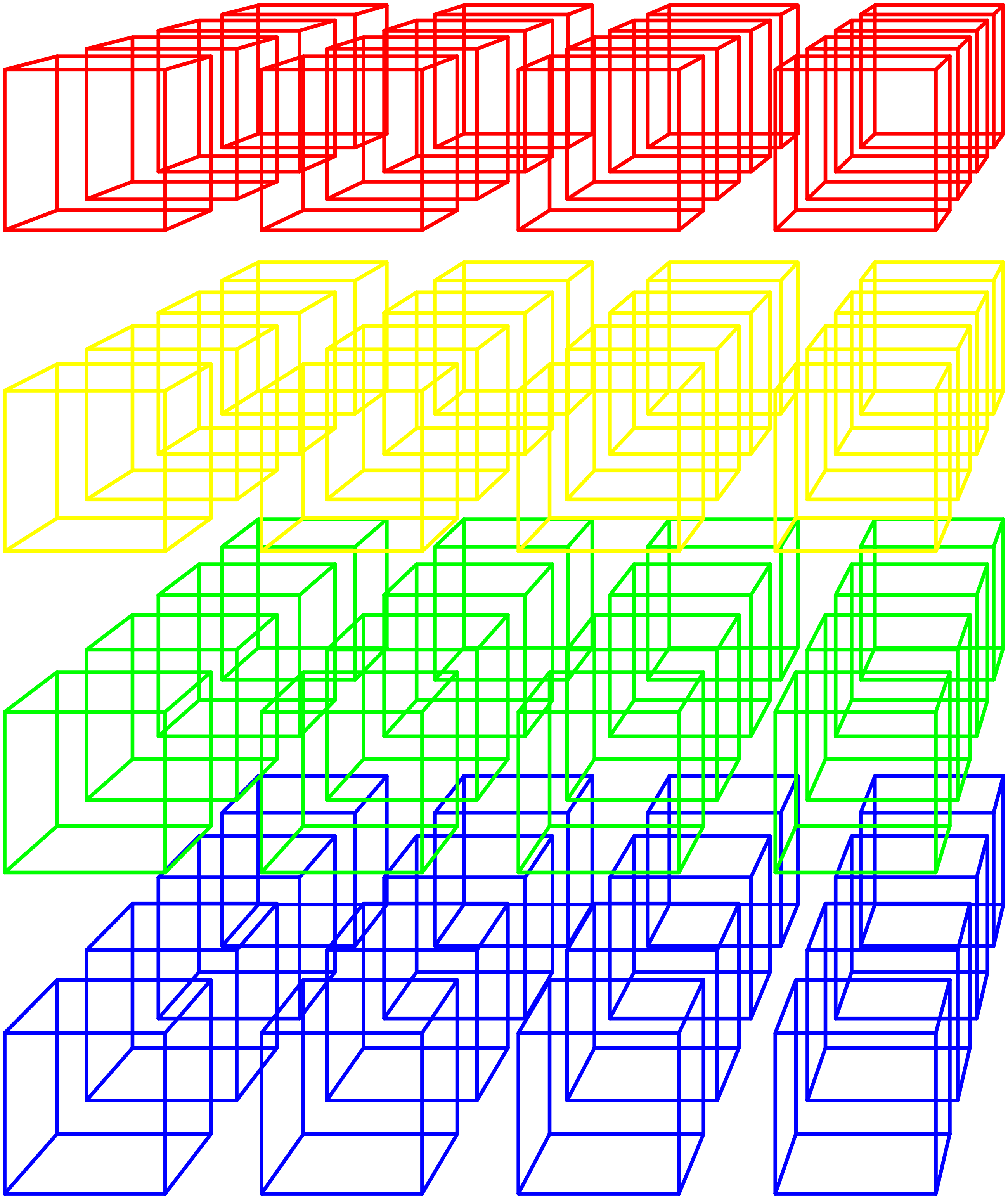

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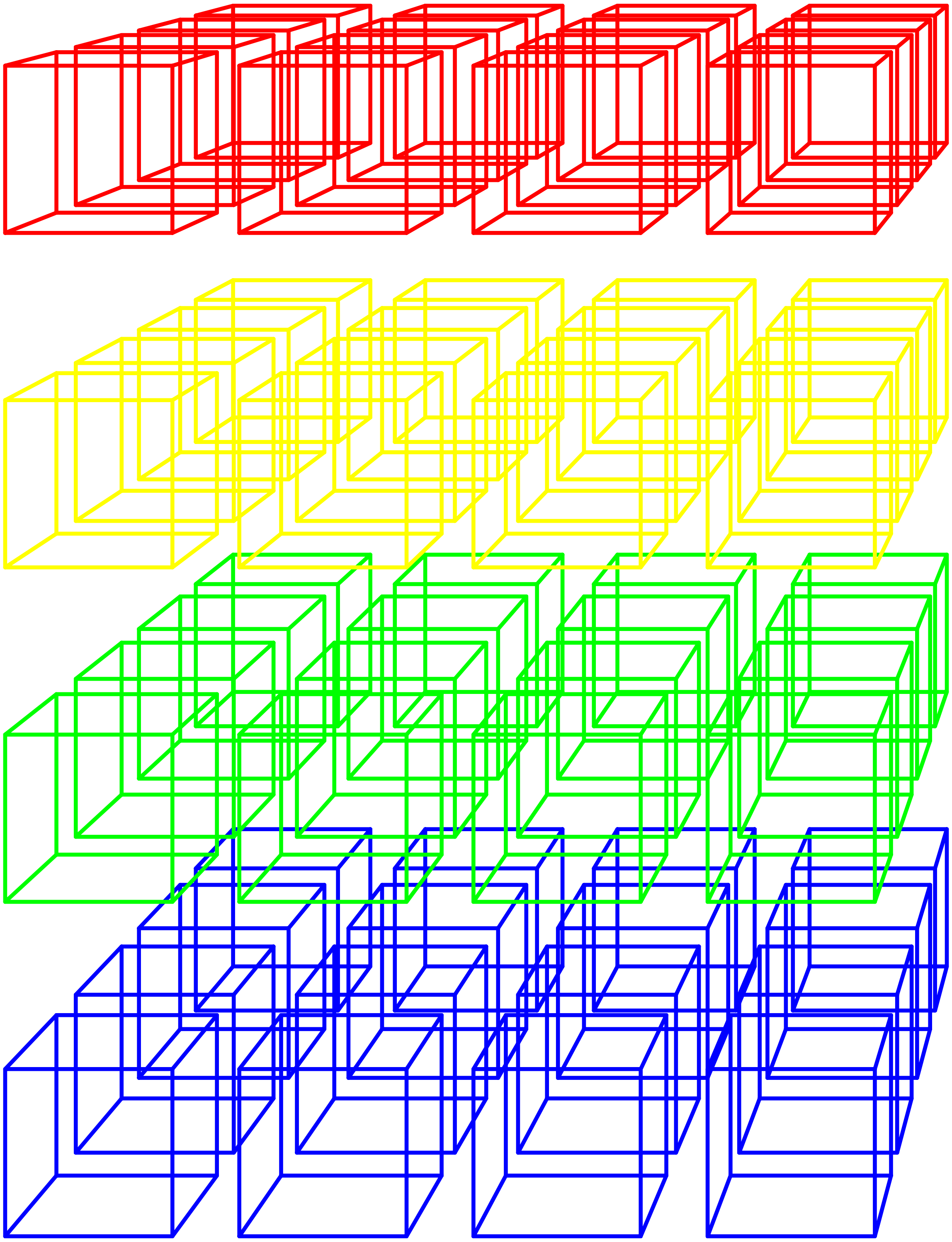

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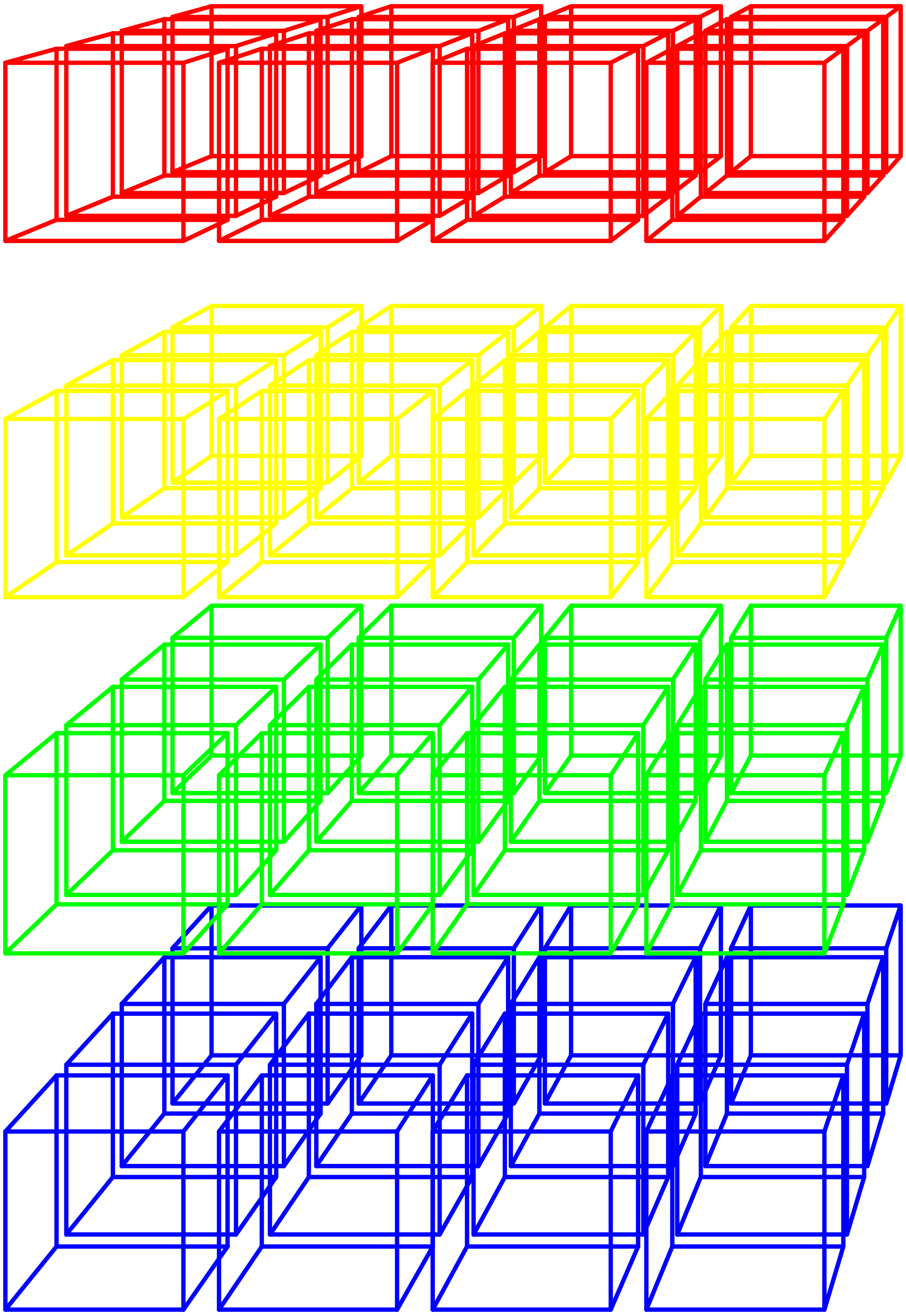

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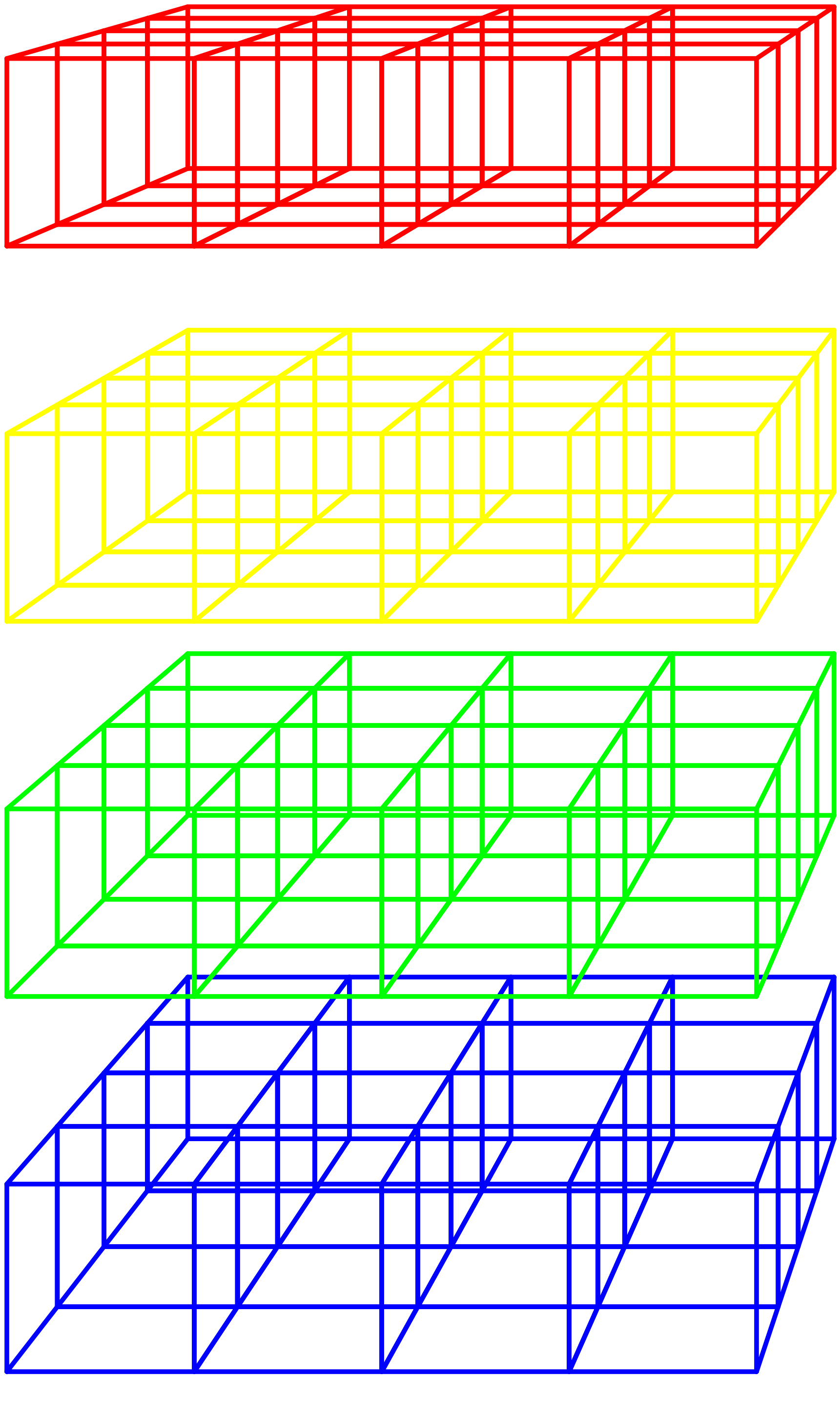
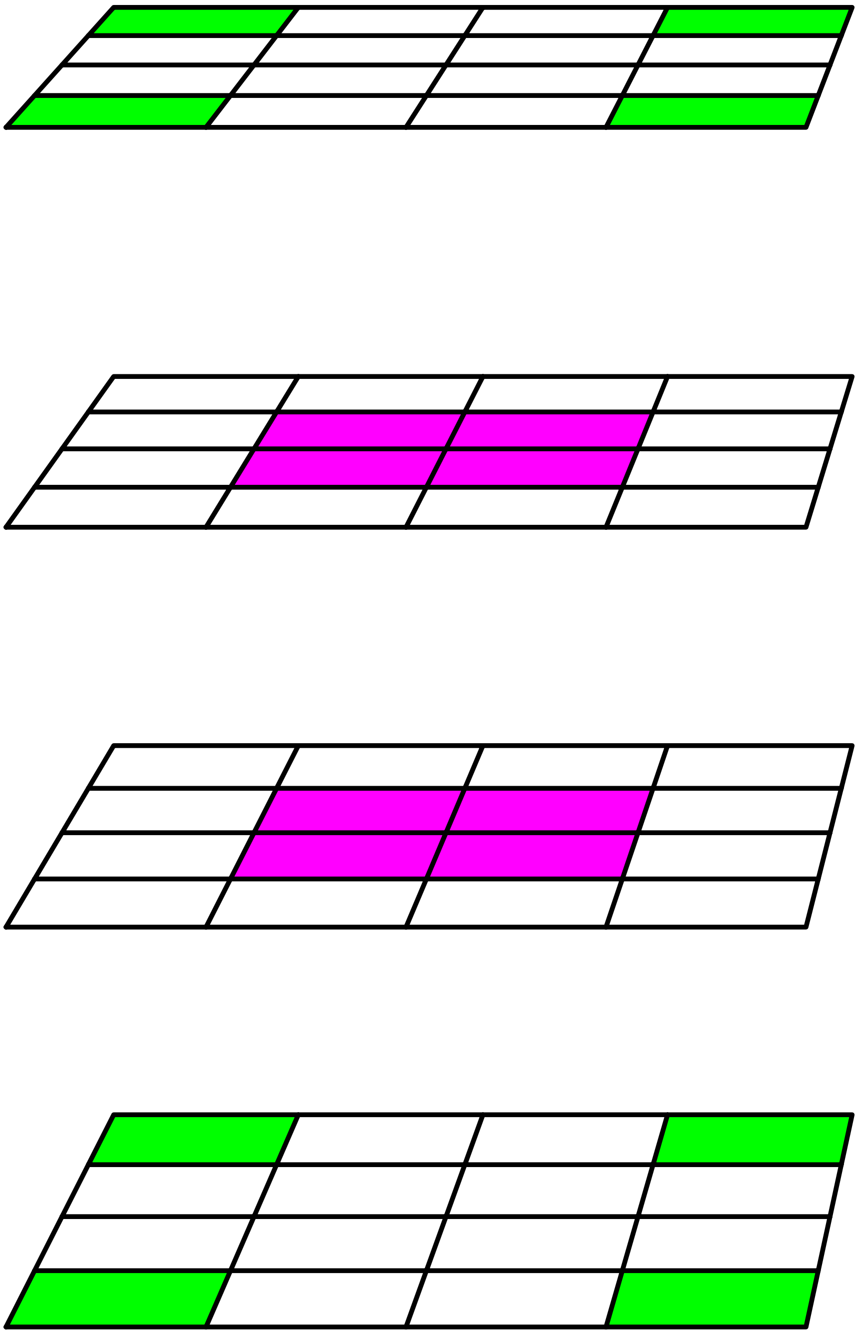


Out of 64 points, 8 corner cubes (green) and 8 centre cubes (purple) are stronger cubes compared to the rest of the 48 cubes. These cubes allow most number of crossings which ensure a greater chance of winning.



Another interesting property of a Qubic is its 'Automorphisms'*. Due to the 4x4x4 cube's symmetry, it has an amazing 192 automorphisms. 48 of which are ordinary rotations or reflections; the remaining 144 are combinations of these 48.
Patashnik (1980)

By rotation: each of the six sides can be brought on top in four different ways, resulting in a total of 24 automorphisms. There are three more symmetries each doubling the number of automorphisms. One of which is reflection in a plane through the centre of the cube.
Allis Schoo (1992)

Other two can be described in the following ways:

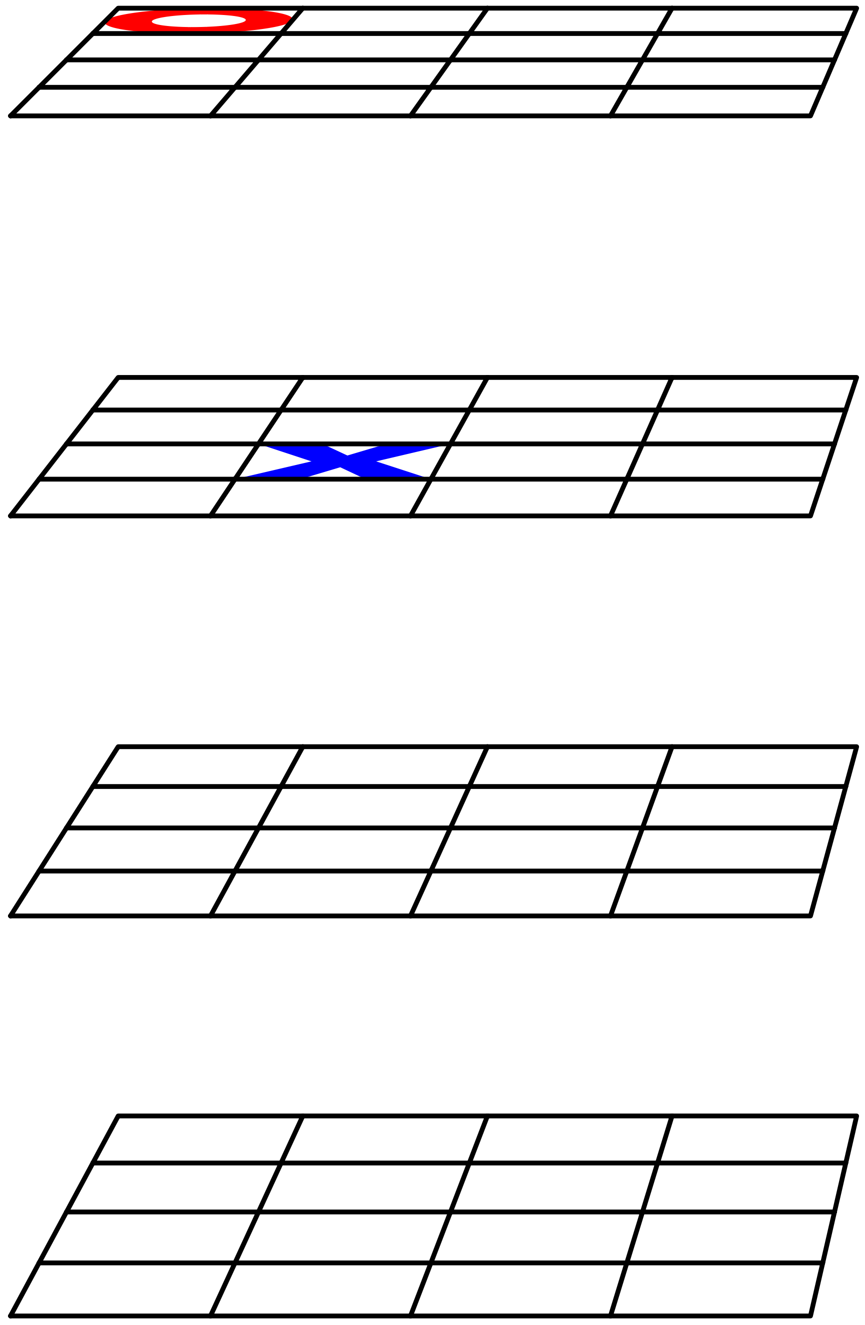
1. Turning the cube inside out, exchanging in all three dimensions, i.e. the inner planes with the outer planes.
2. Internal exchange, exchanging the inner planes in all three directions, while leaving the outer planes untouched.

Allis Schoo (1992)

*Automorphism: a one-to-one mapping of the set of points onto itself that preserves the set of lines.
Allis Schoo (1992)

I will now illustrate the different automorphisms:

First by rotation:



Each of such rotation is applied to each of the six faces; thus 24 automorphisms.

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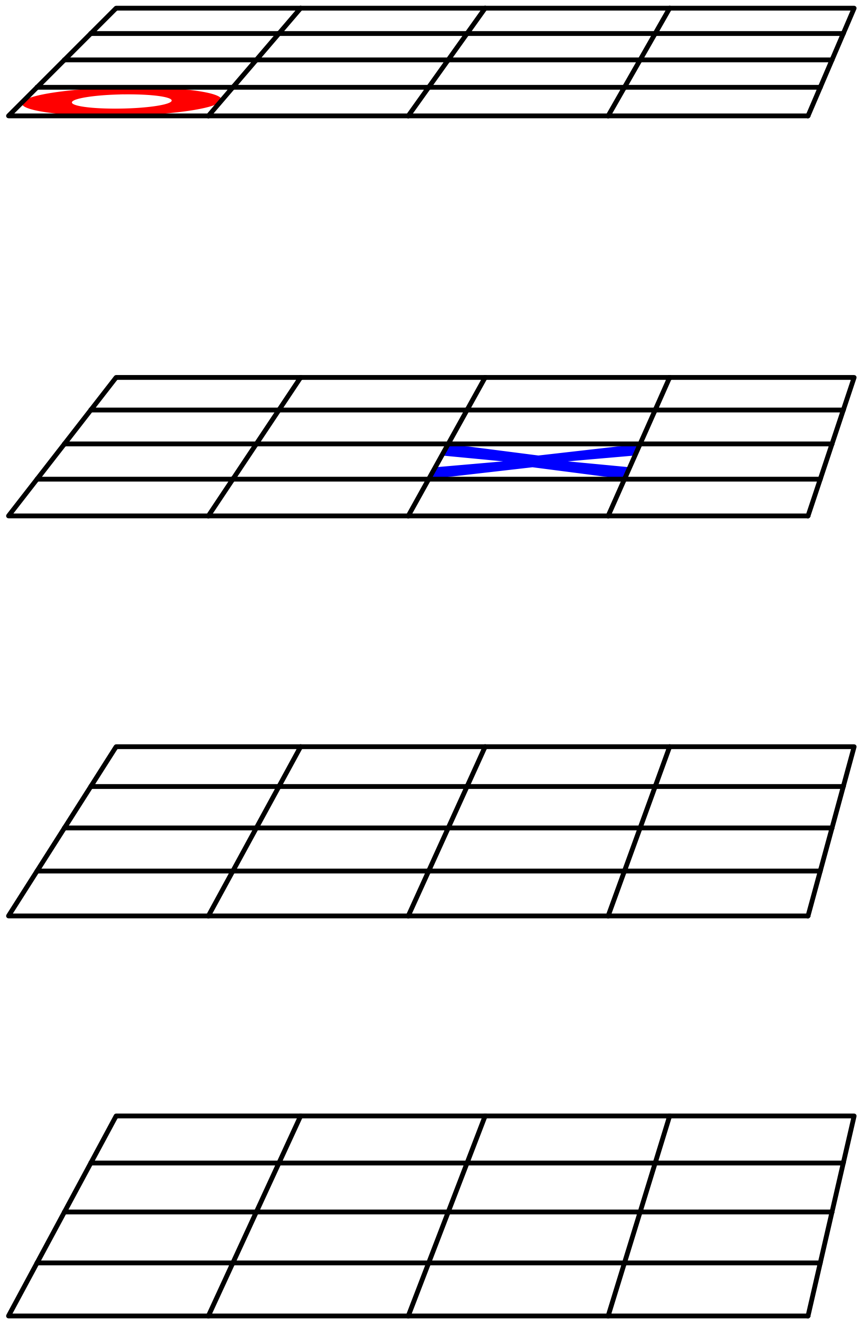

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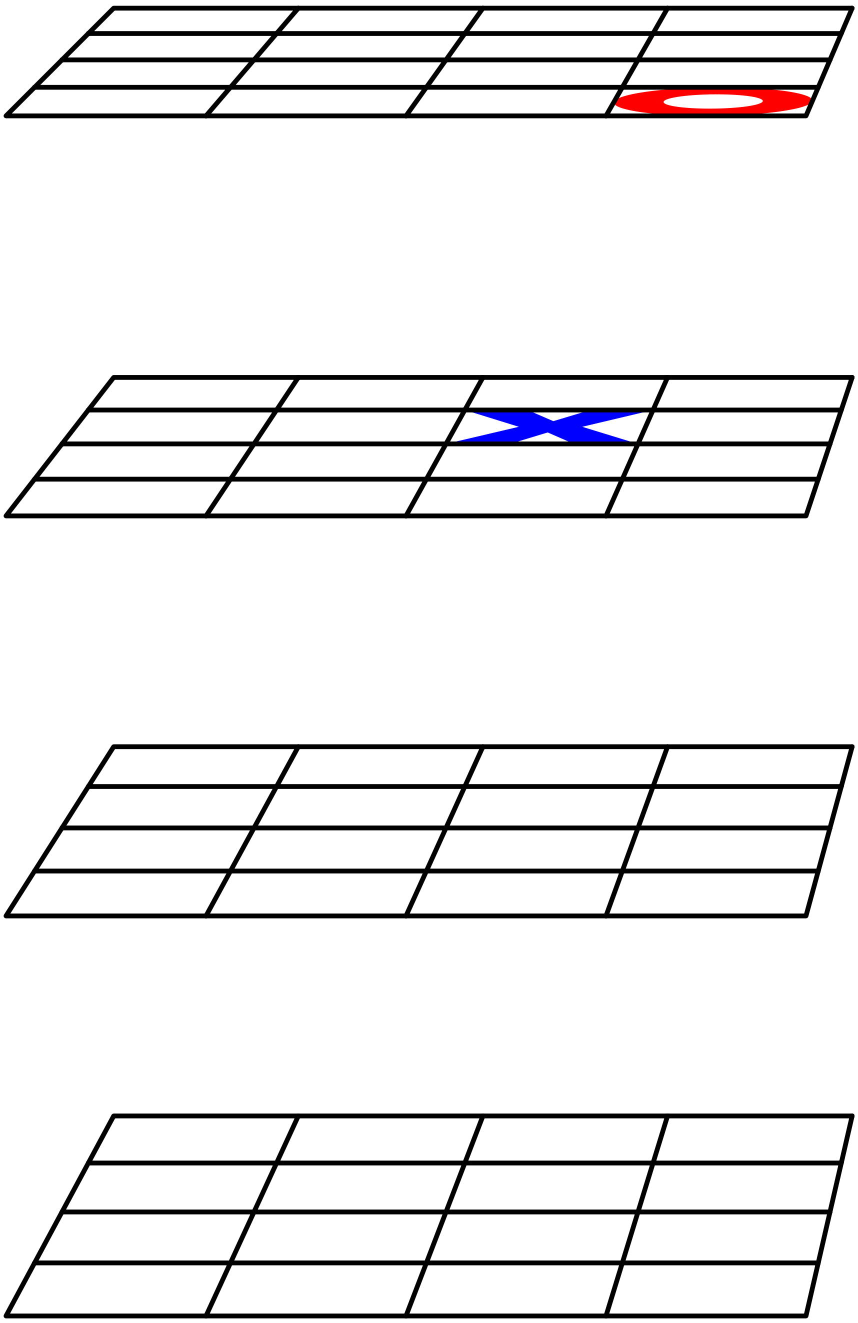

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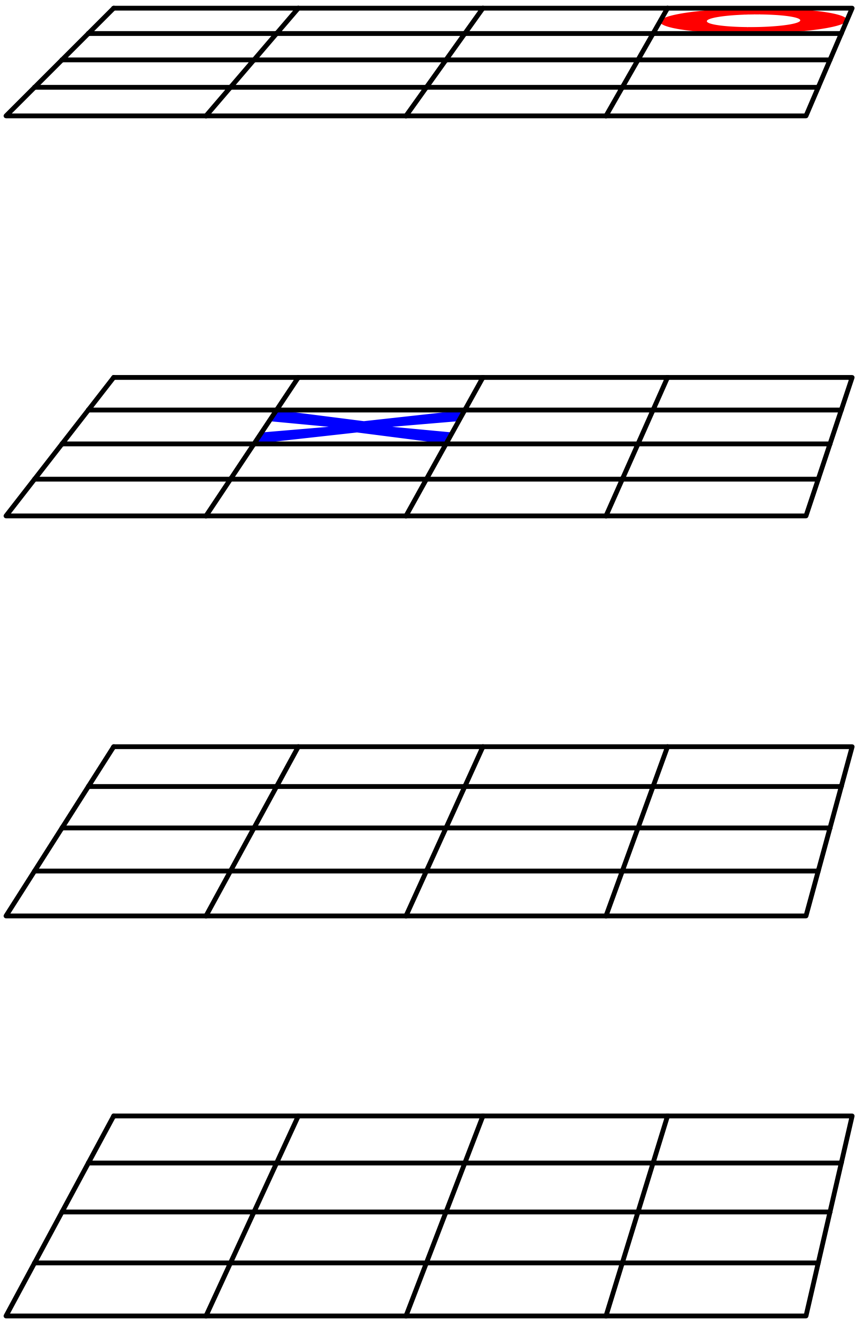
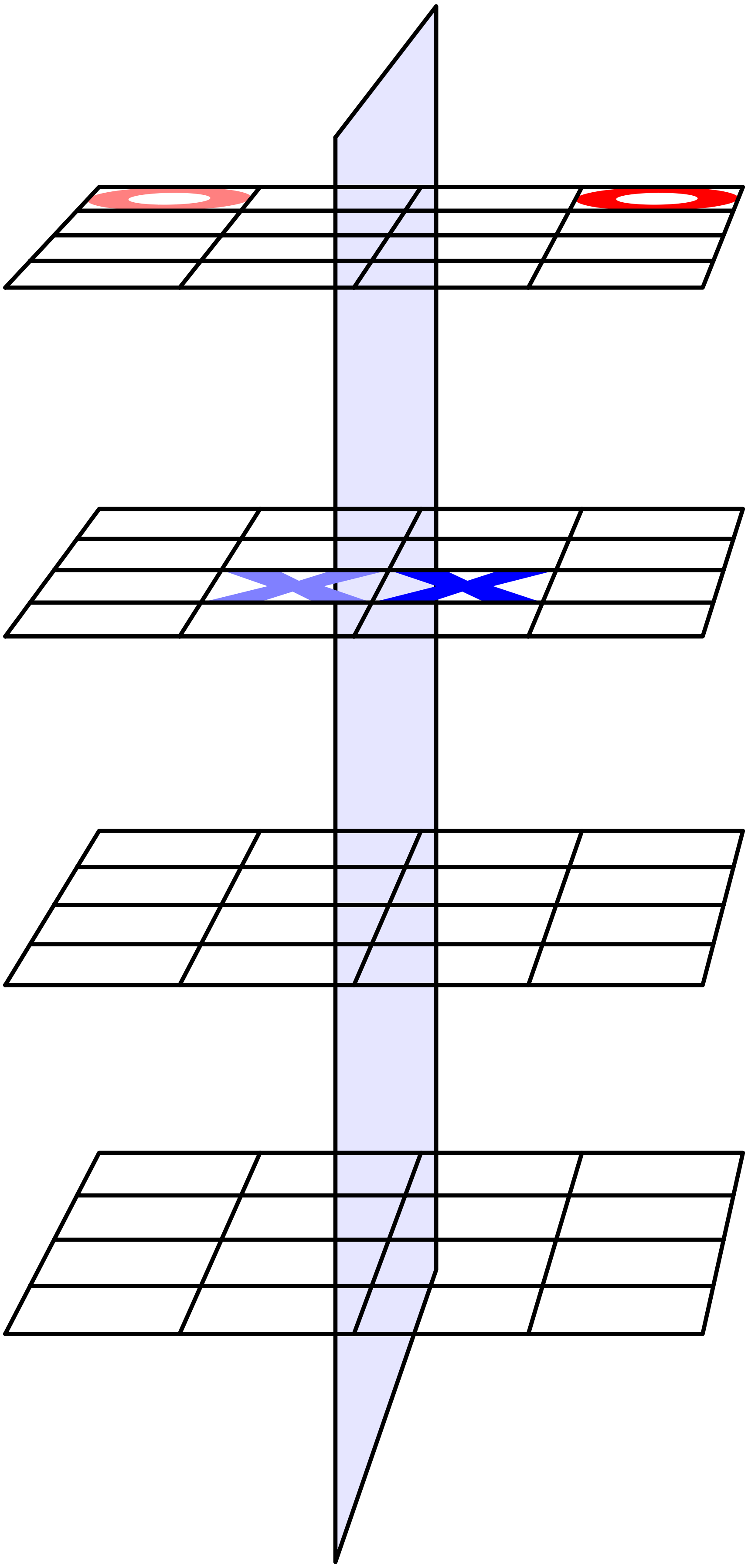


Each of such rotation is applied to each of the six faces; thus 24 automorphisms.

Second by Reflection:

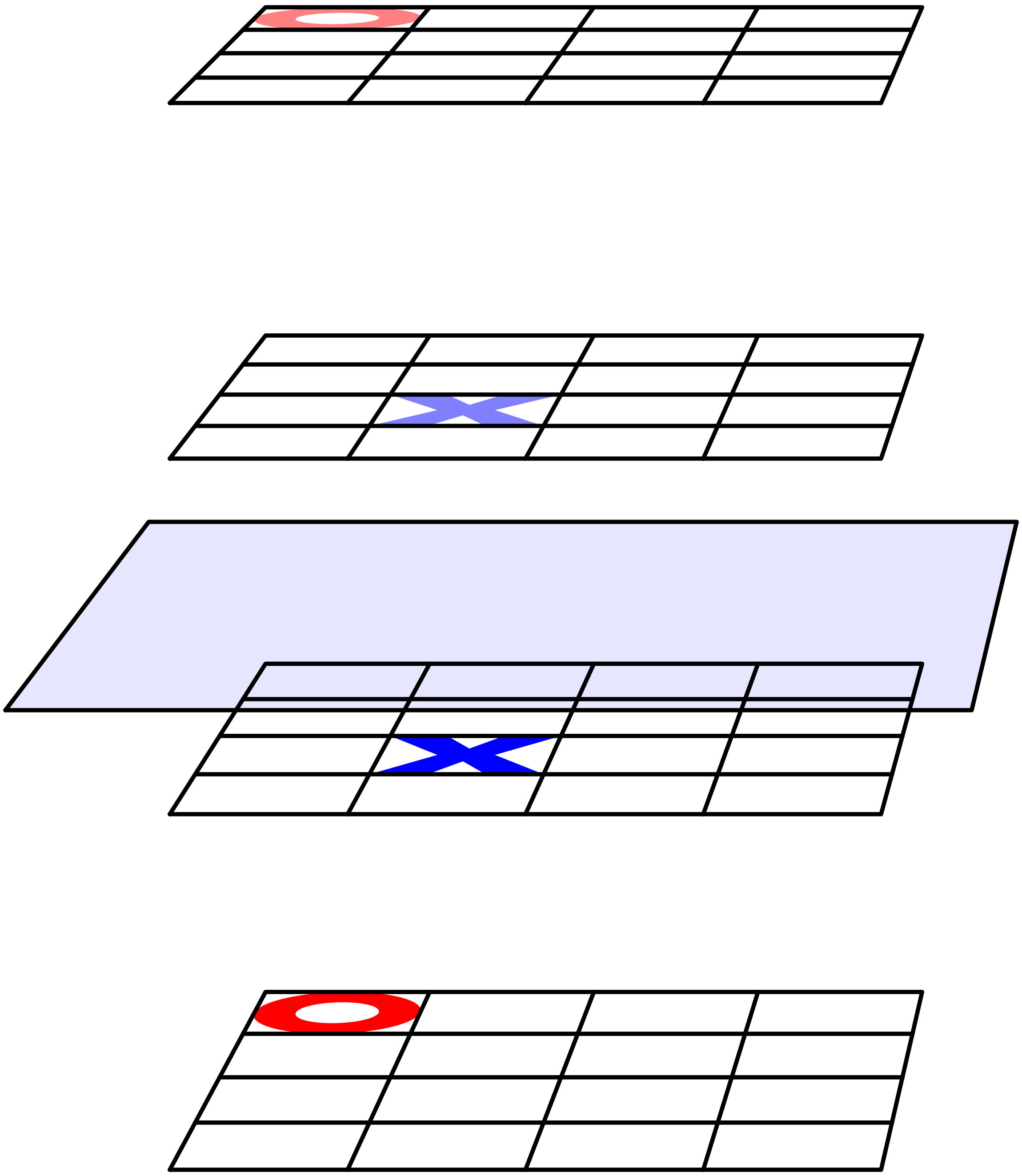
First type: About different planes

Reflection about the x = 0 plane

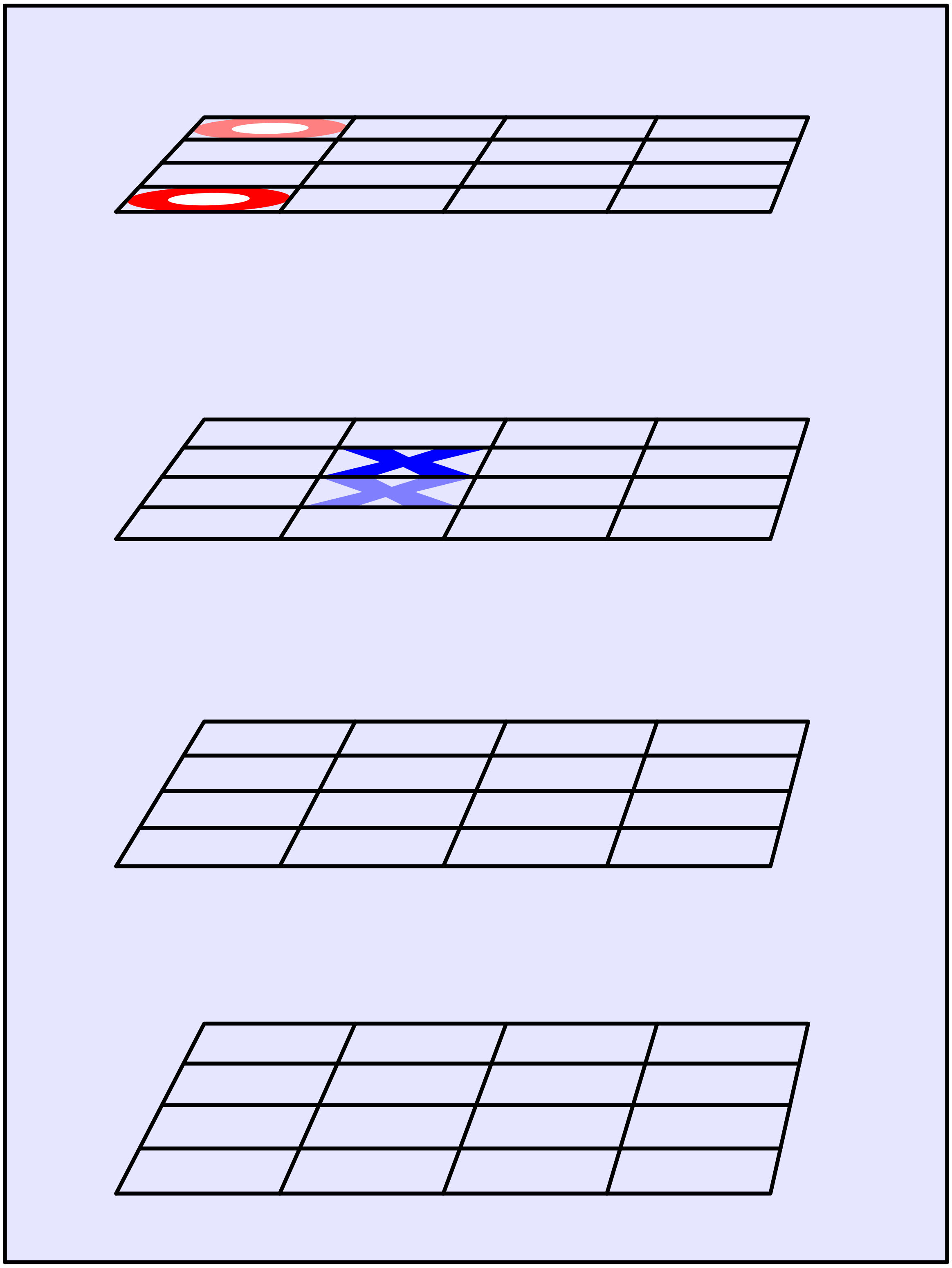


First type: About different planes

Reflection about the y = 0 plane



First type: About different planes
Reflection about the z = 0 plane



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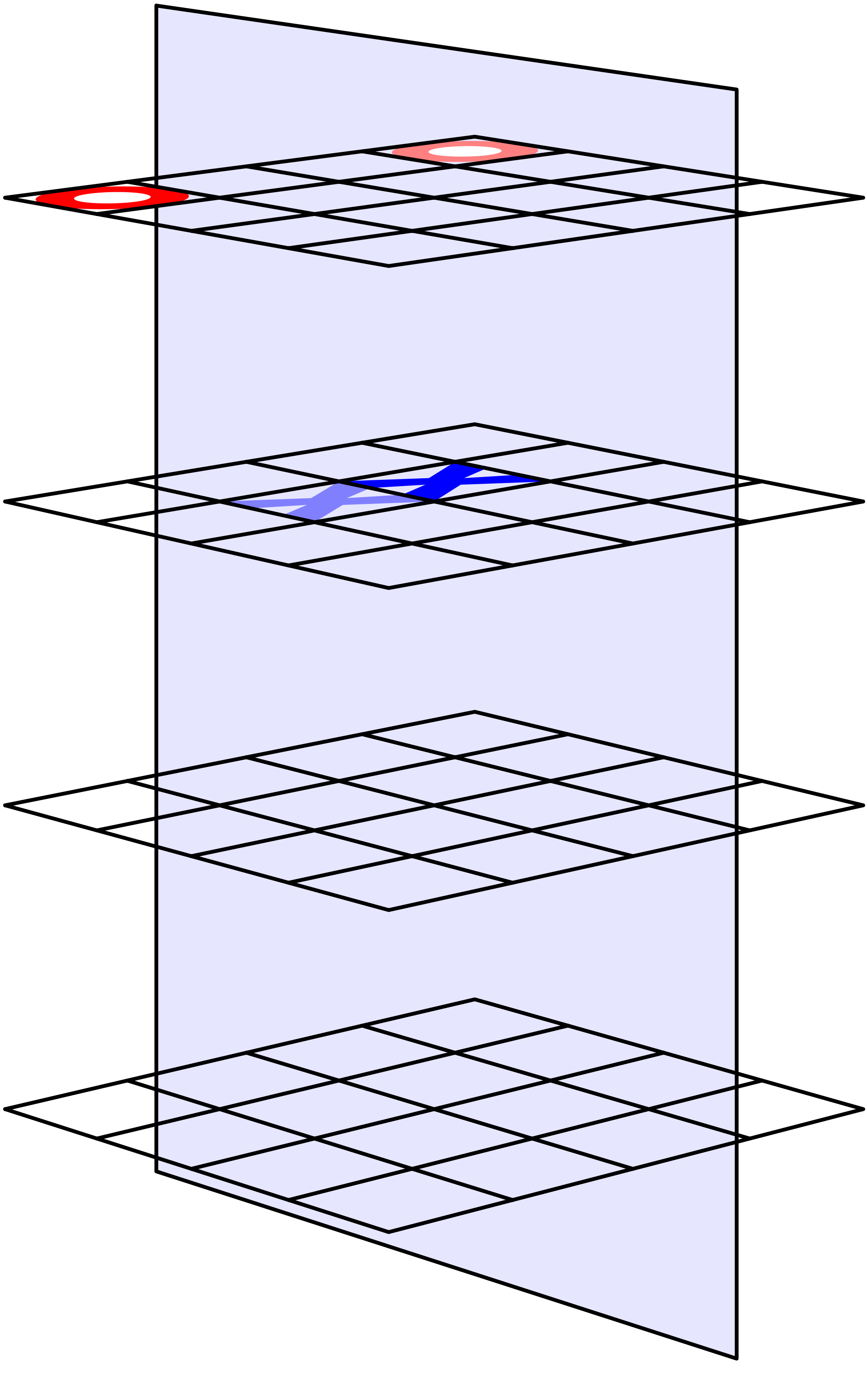

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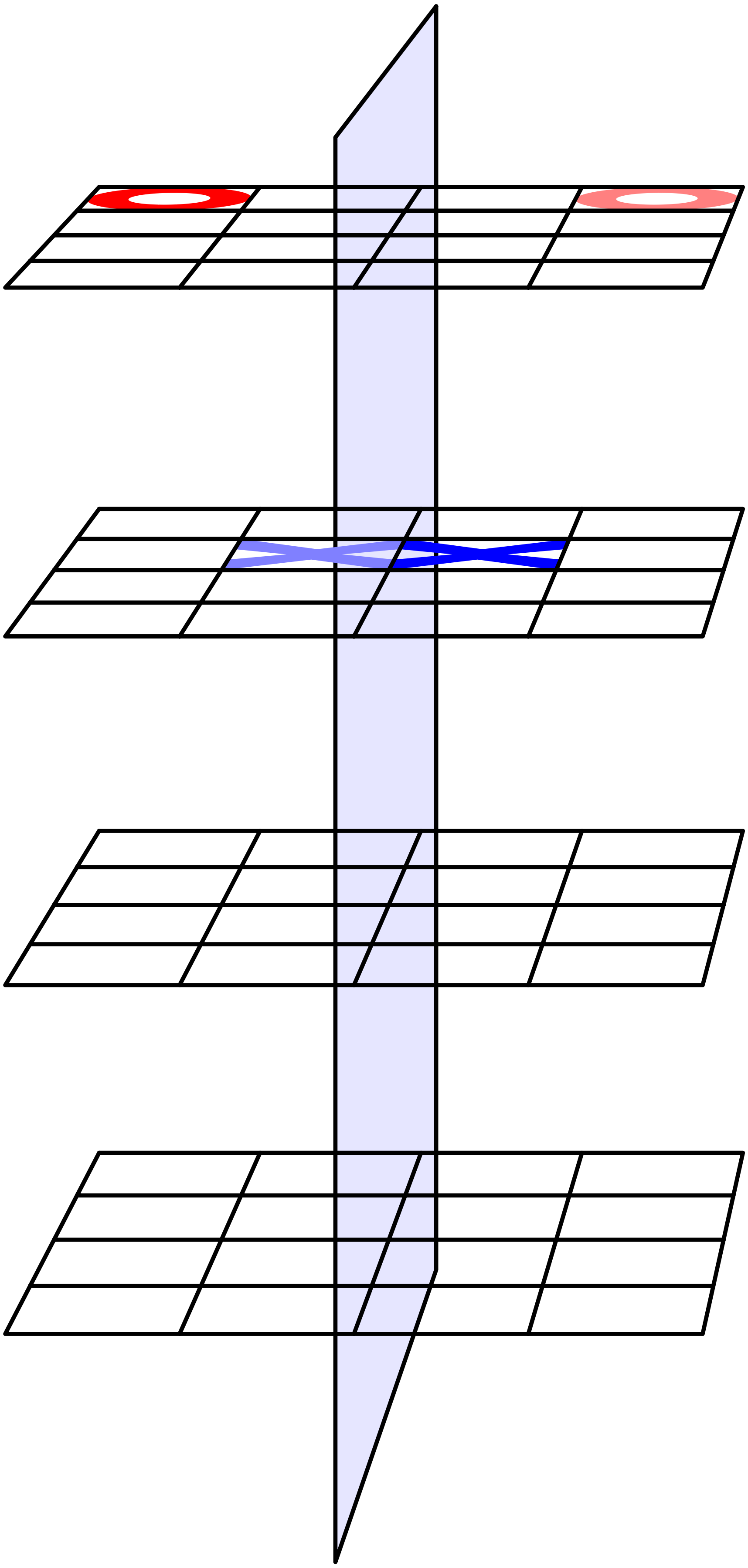

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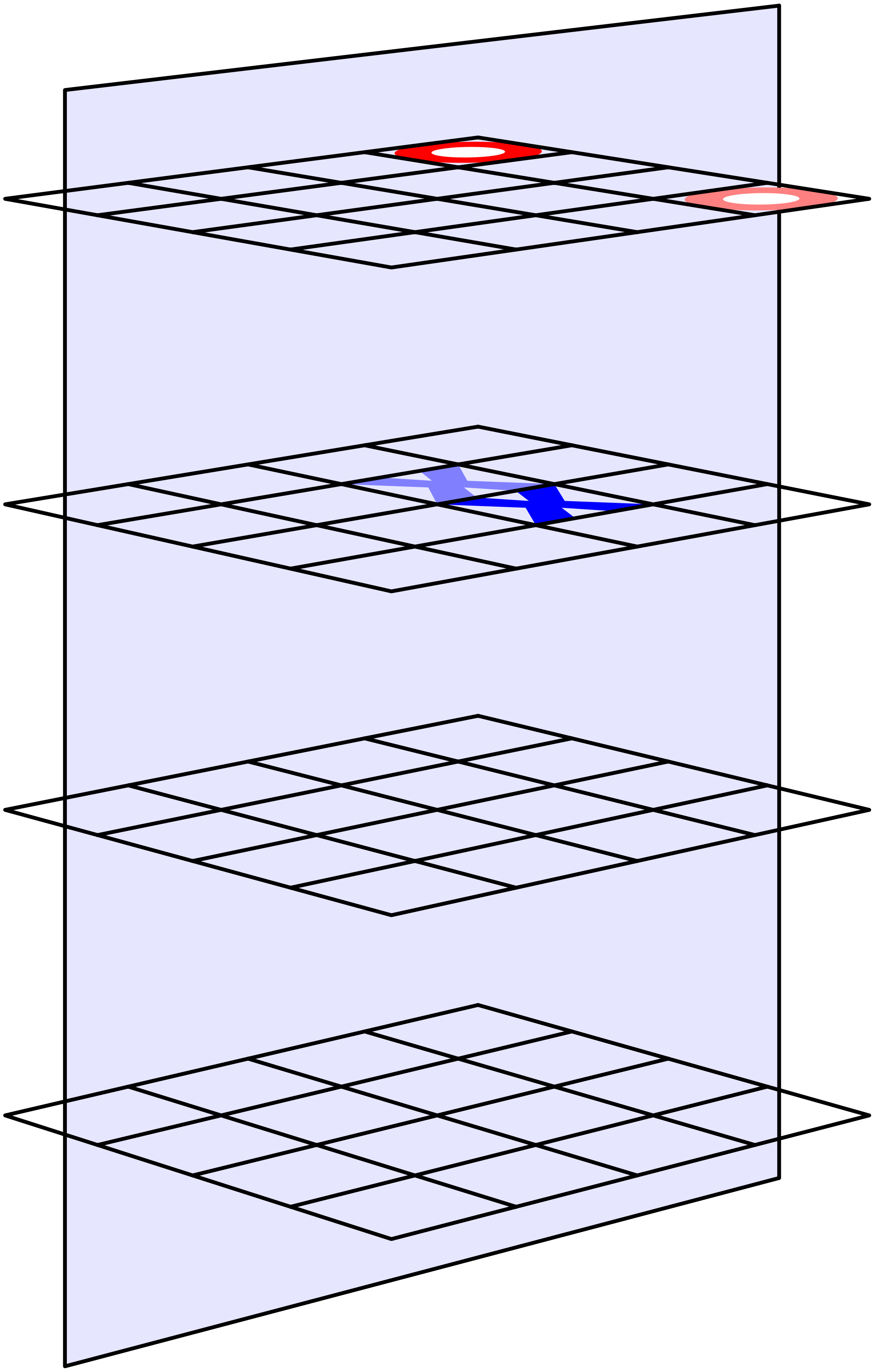

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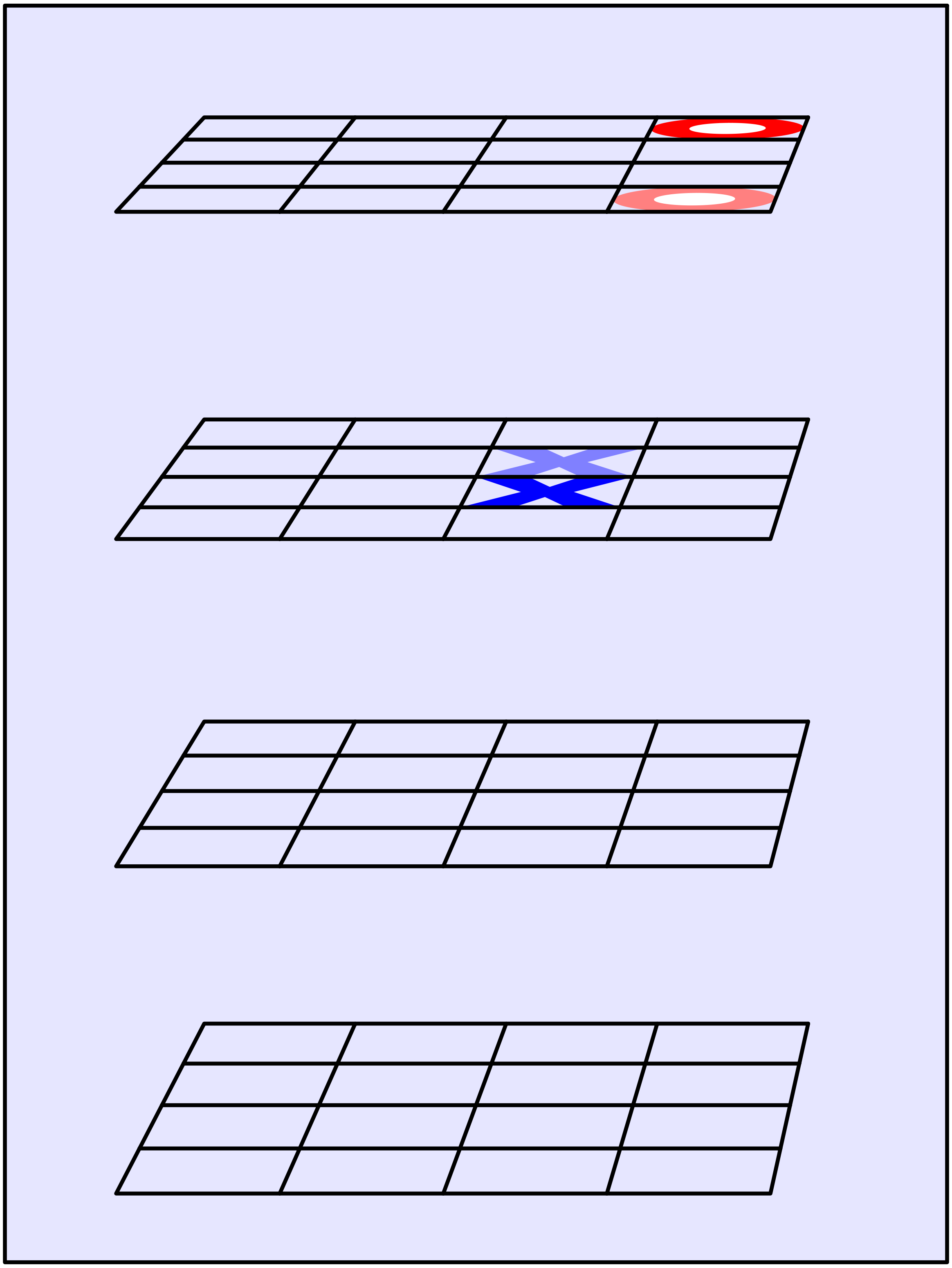

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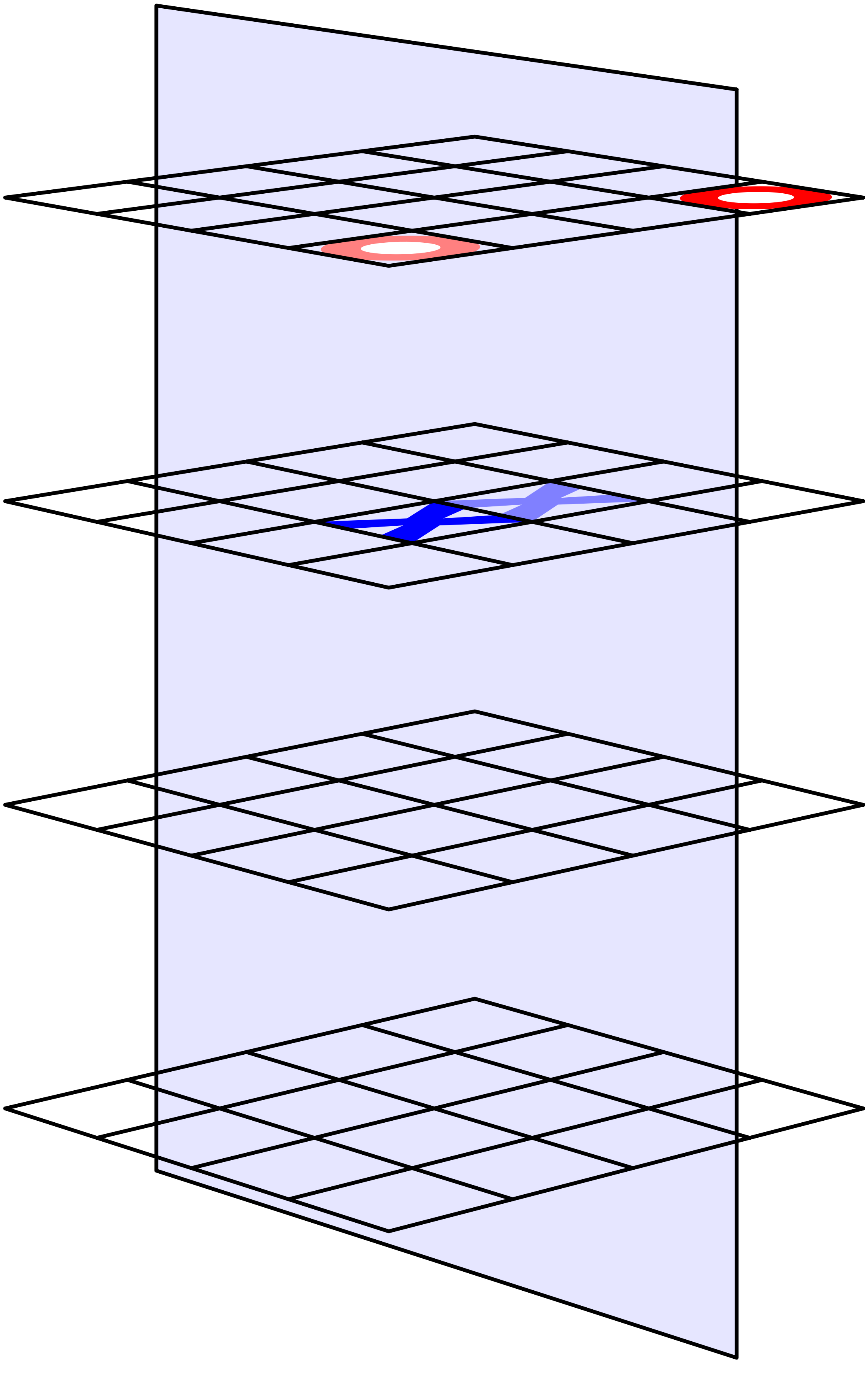

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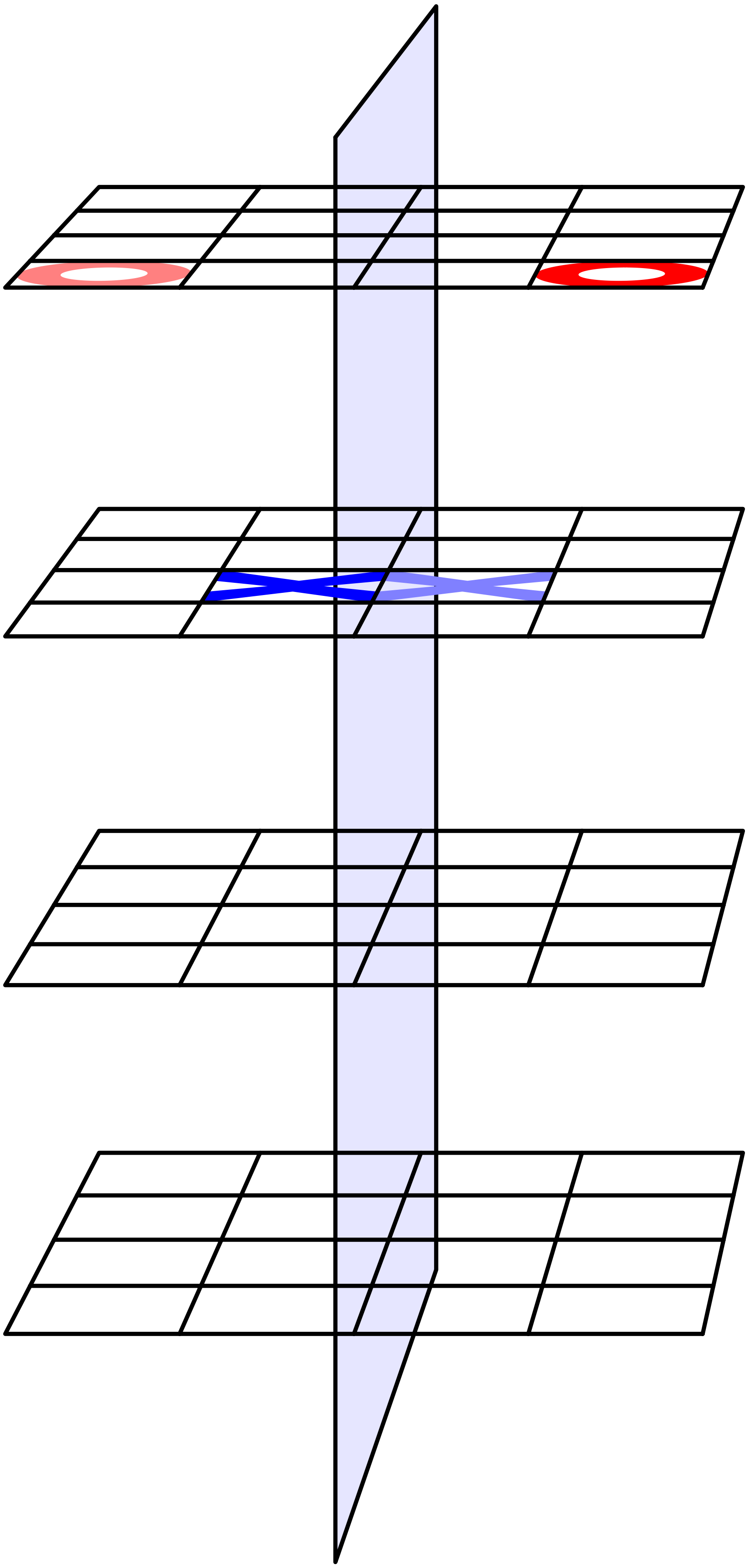

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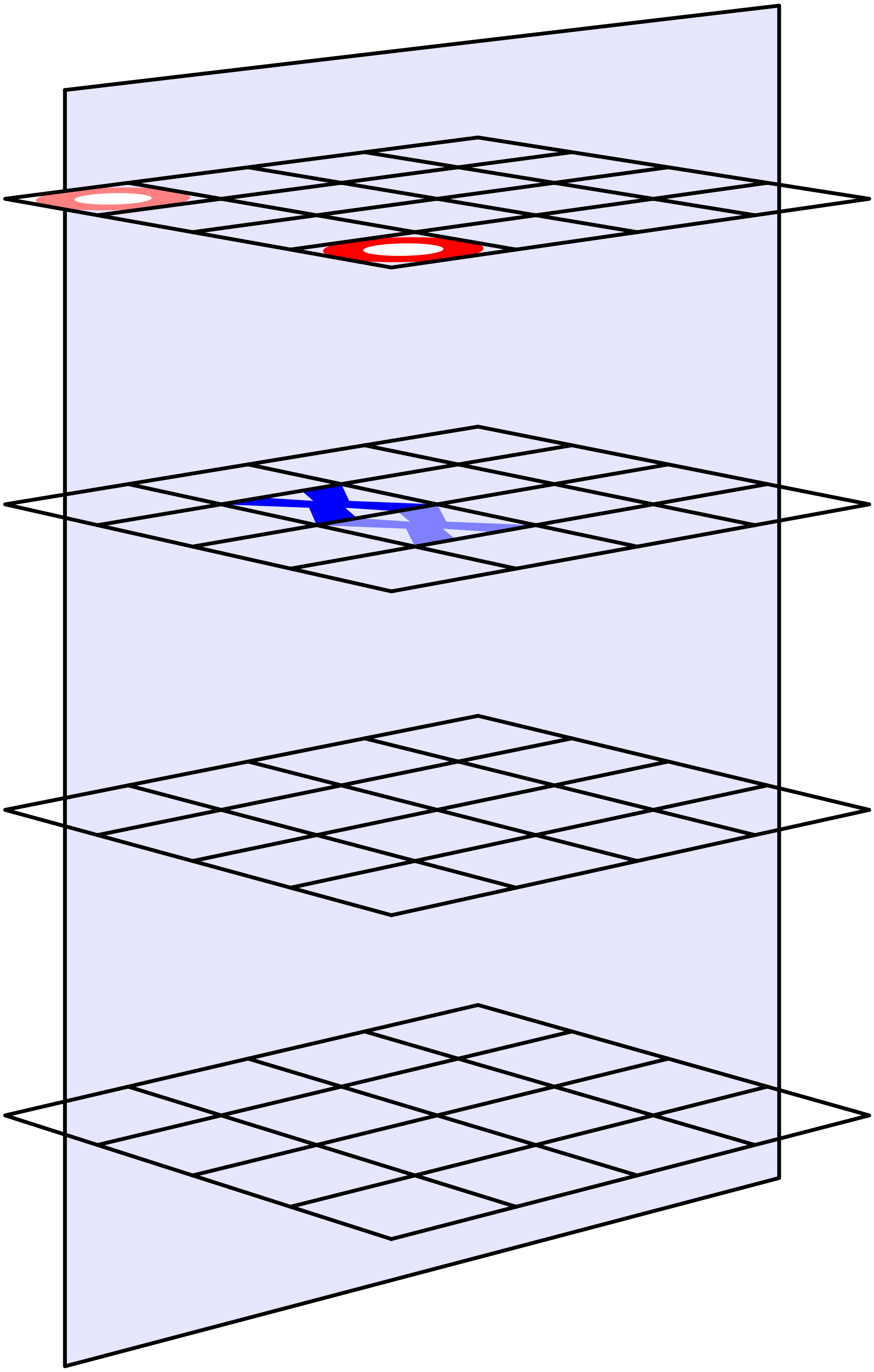

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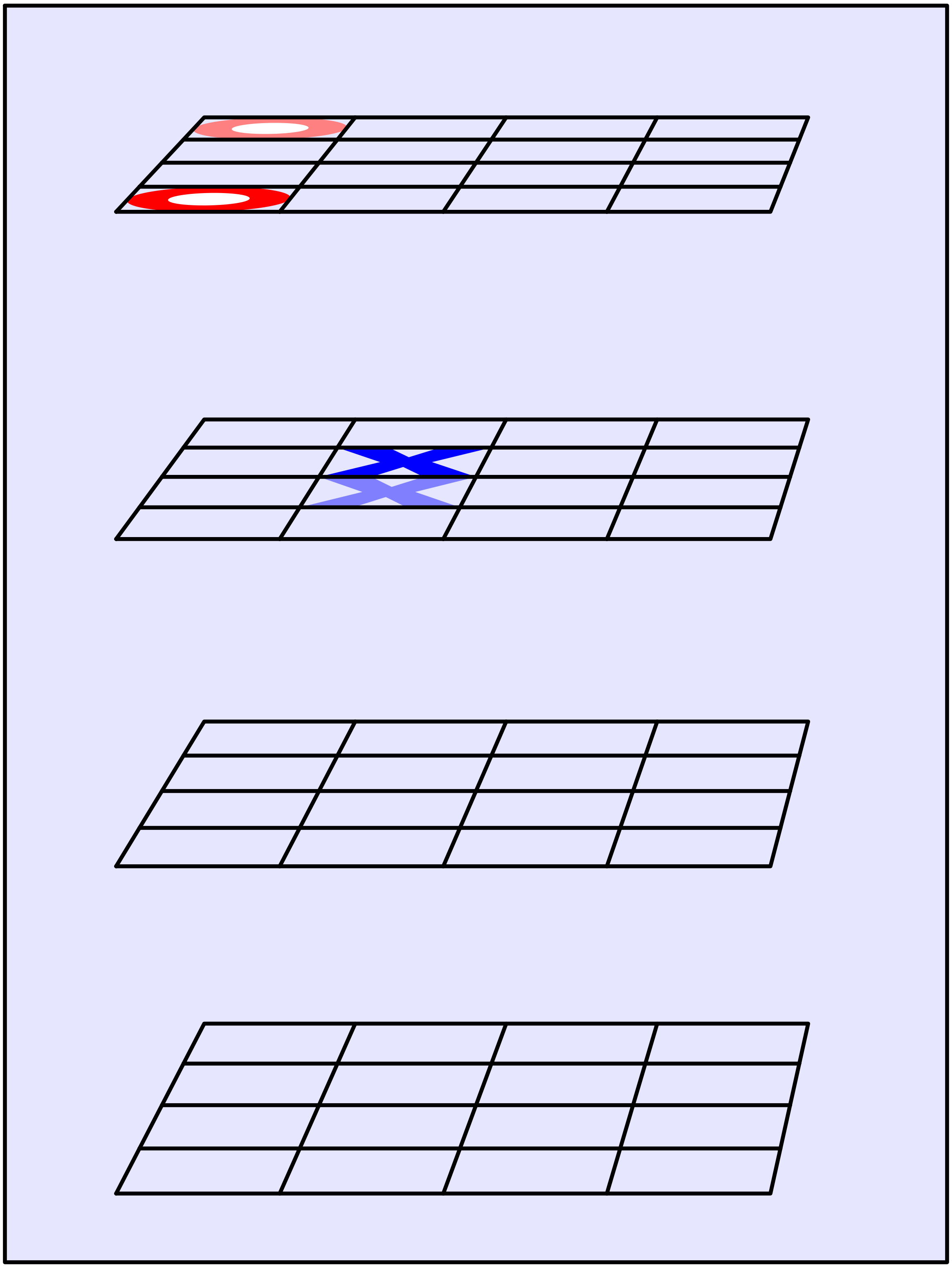
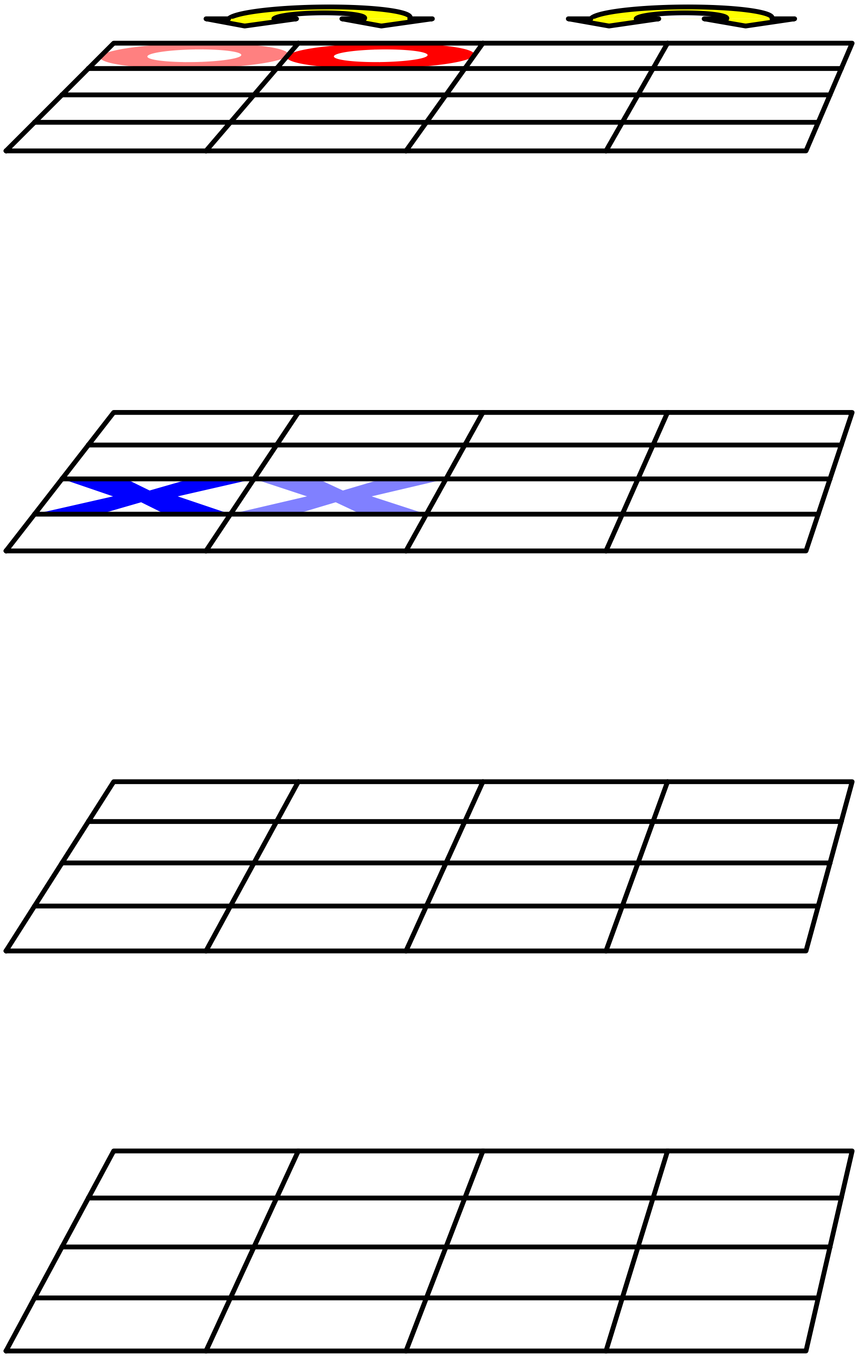


Reflection:

Second type: Turning the cube inside-out

Switching the inner and the outer planes in the z direction

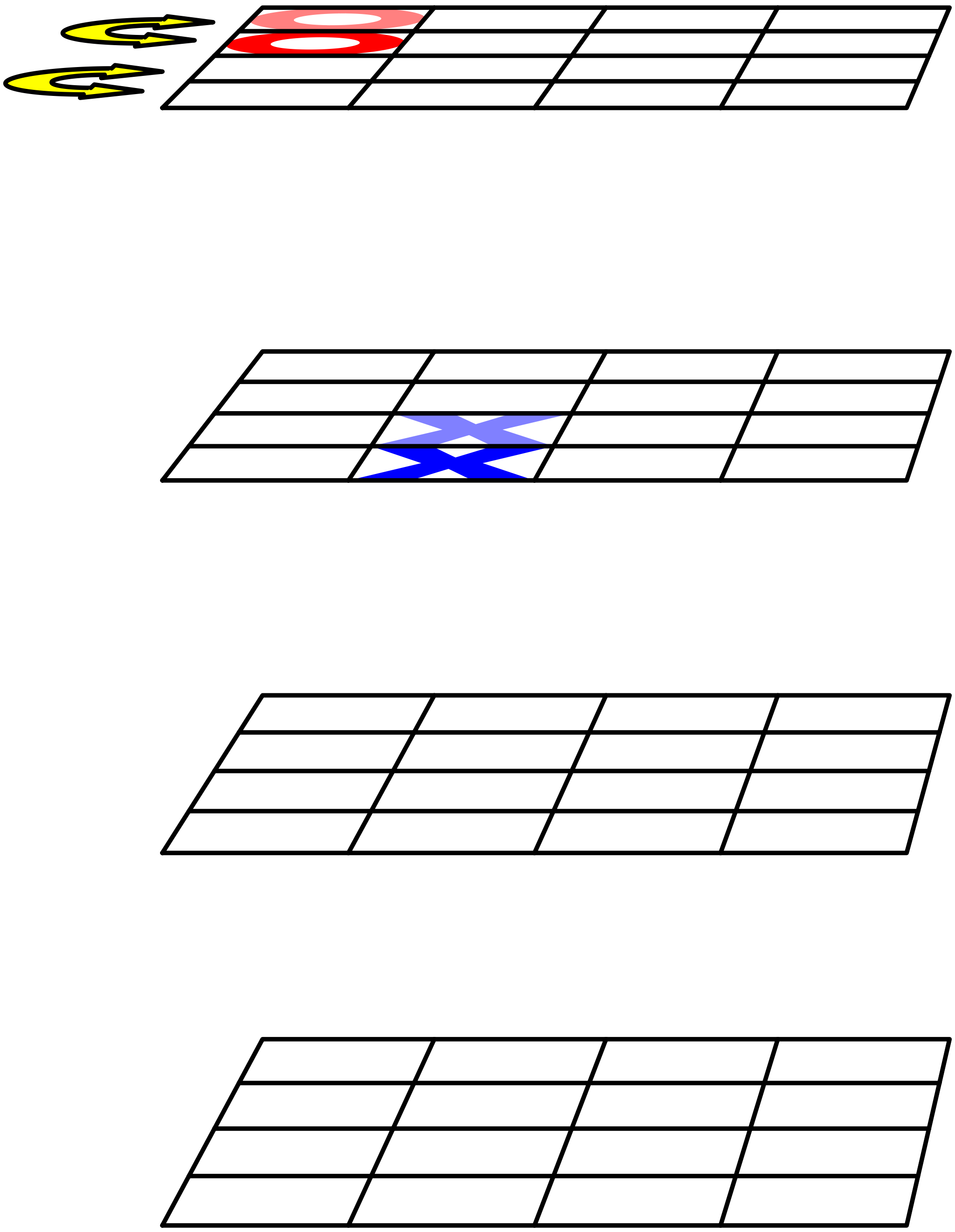


Reflection:

Second type: Turning the cube inside-out

Switching the inner and the outer planes

in the x direction

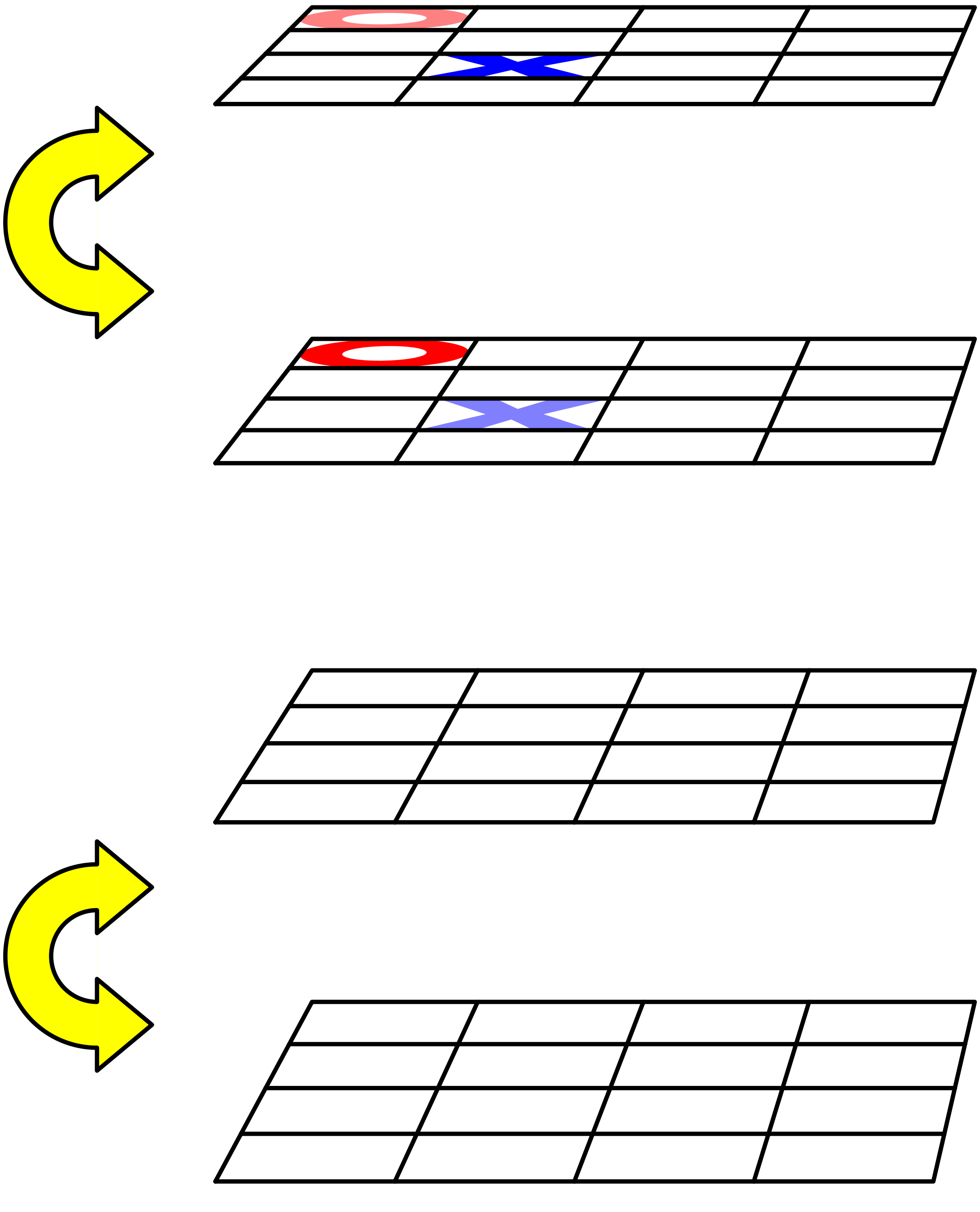


Reflection:

Second type: Turning the cube inside-out

Switching the inner and the outer planes

in the y direction

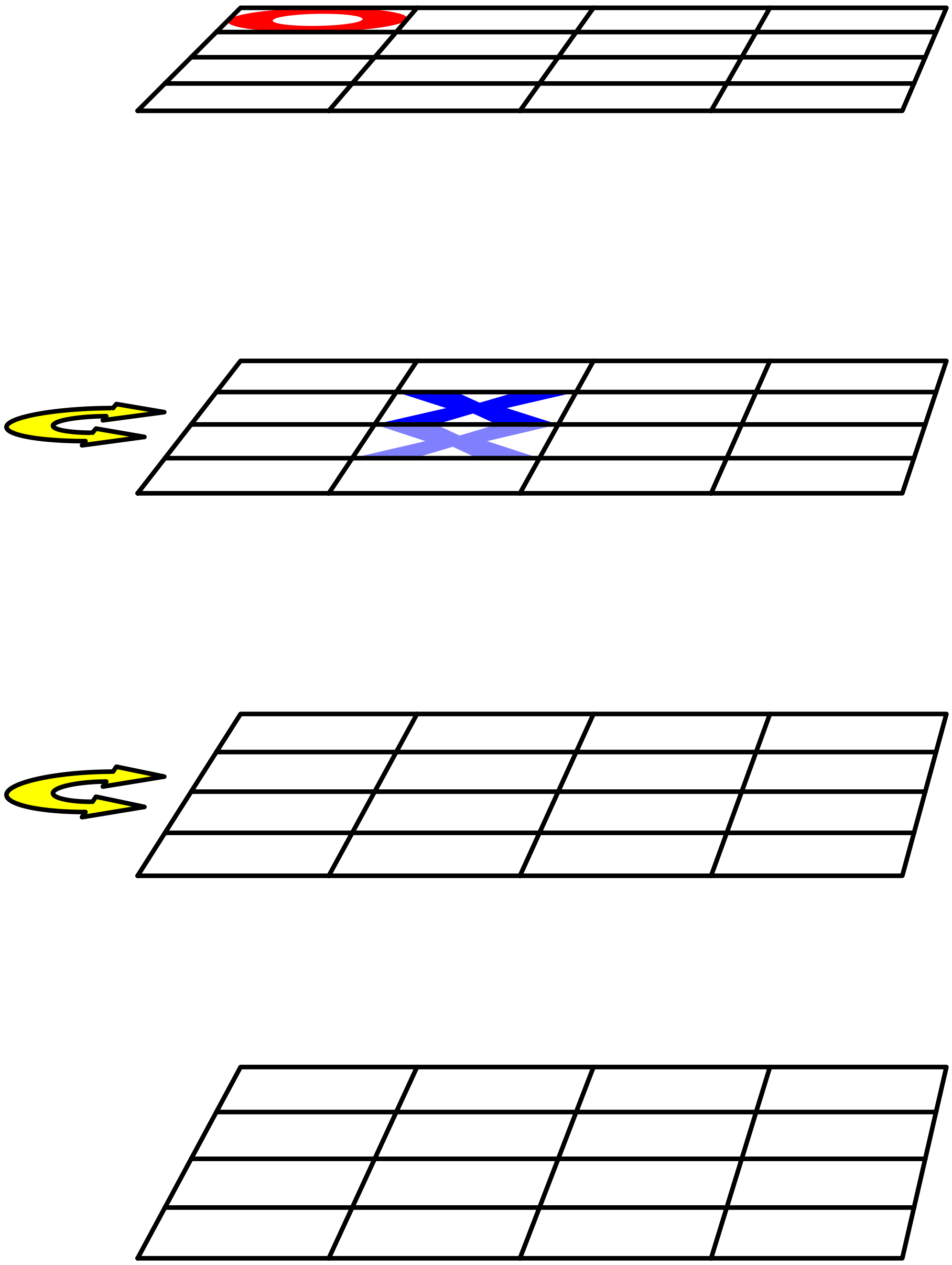


Reflection:

Third type: Exchanging inner planes

Switching the inner two x planes

and leaving the outer ones unaltered

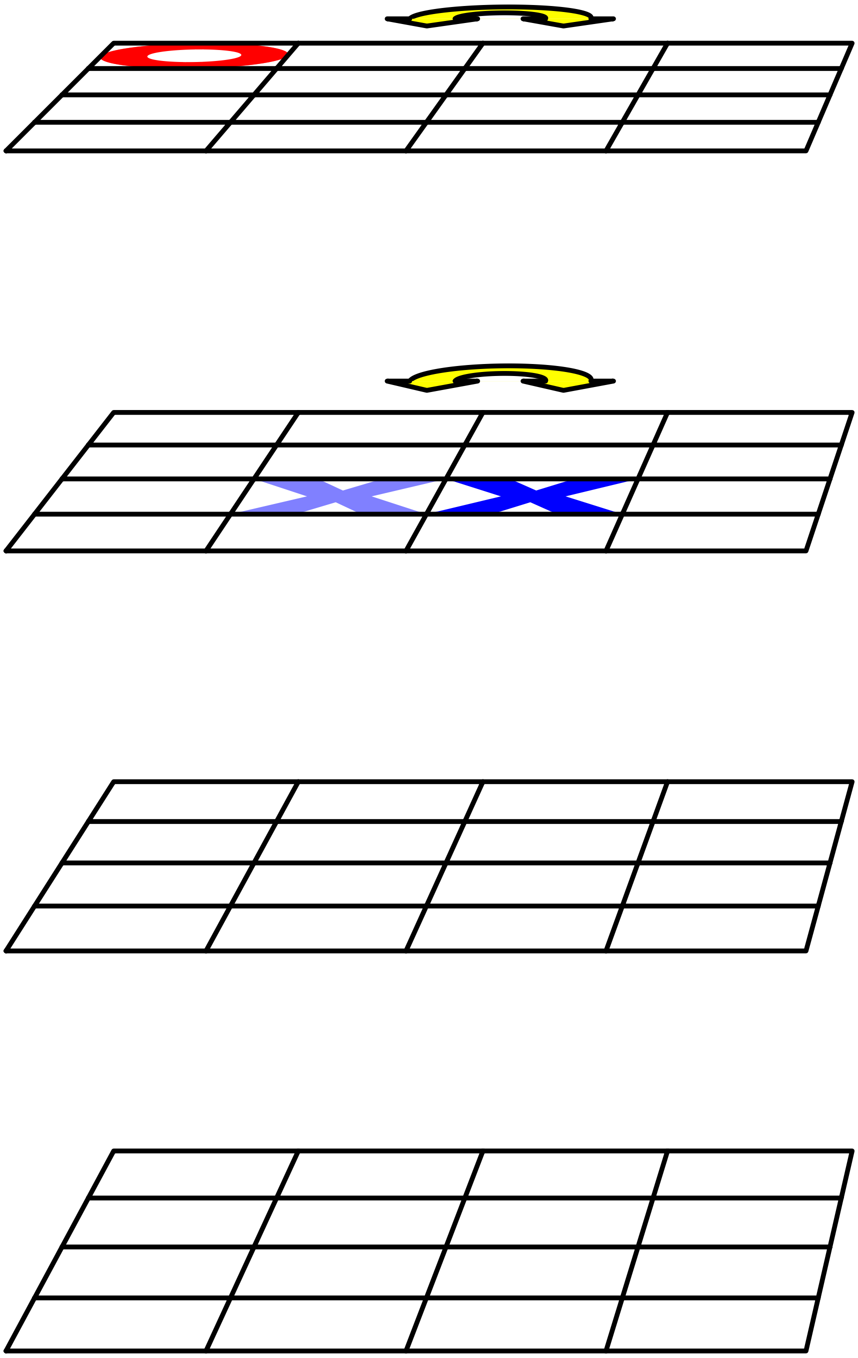


Reflection:

Third type: Exchanging inner planes

Switching the inner two z planes

and leaving the outer ones unaltered



Reflection:
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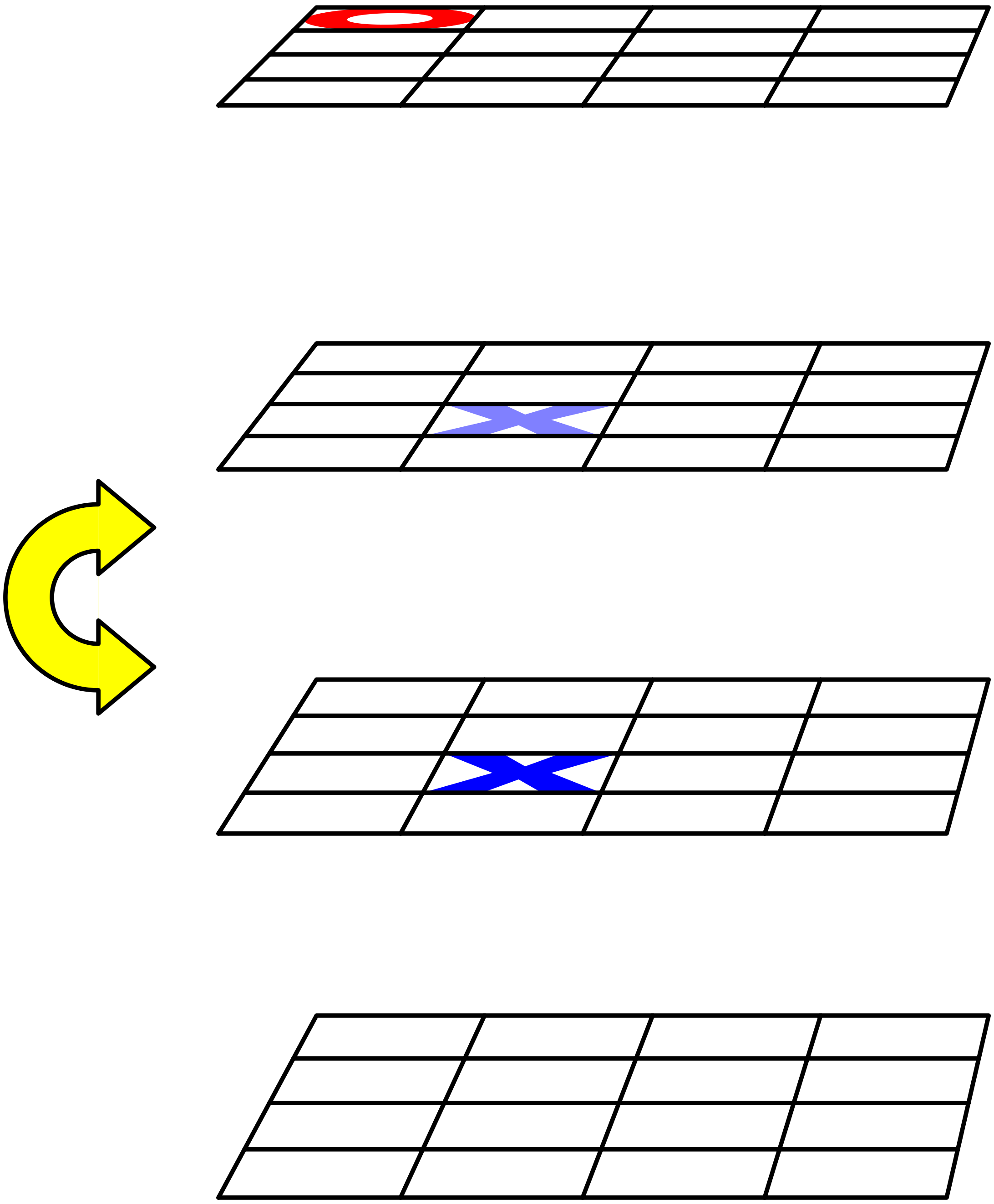


Each of the reflections can be combined with one another as well as with the rotations Here is a chart showing the combinations of the Automorphisms:

Reflections:

	Type 1	Type 2	Type 3
1.	present	present	present
2.	present	absent	absent
3.	present	present	absent
4.	present	absent	present
5.	absent	present	absent
6.	absent	present	present
7.	absent	absent	present
8.	absent	absent	absent

Each of the 8 combinations of the reflections can be further combined with 24 rotations. Resulting in a total of 24 x 8 = 192 automorphisms!

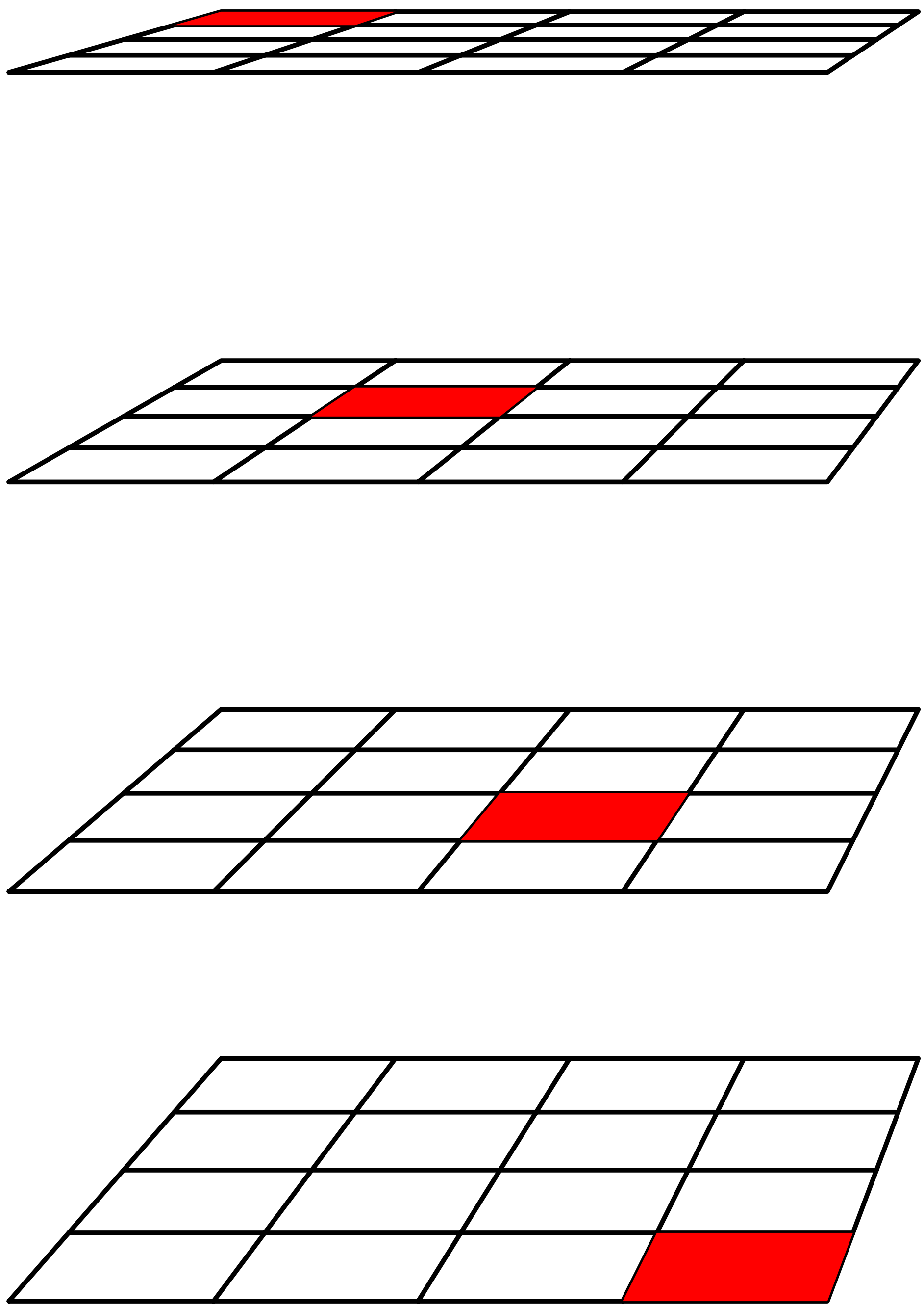
With some new insights into Qubic, now is a good opportunity to explore the strategies of winning*.

Recall the strong and weak positions I pointed out earliear. In the following, I will discuss about them in more detail.

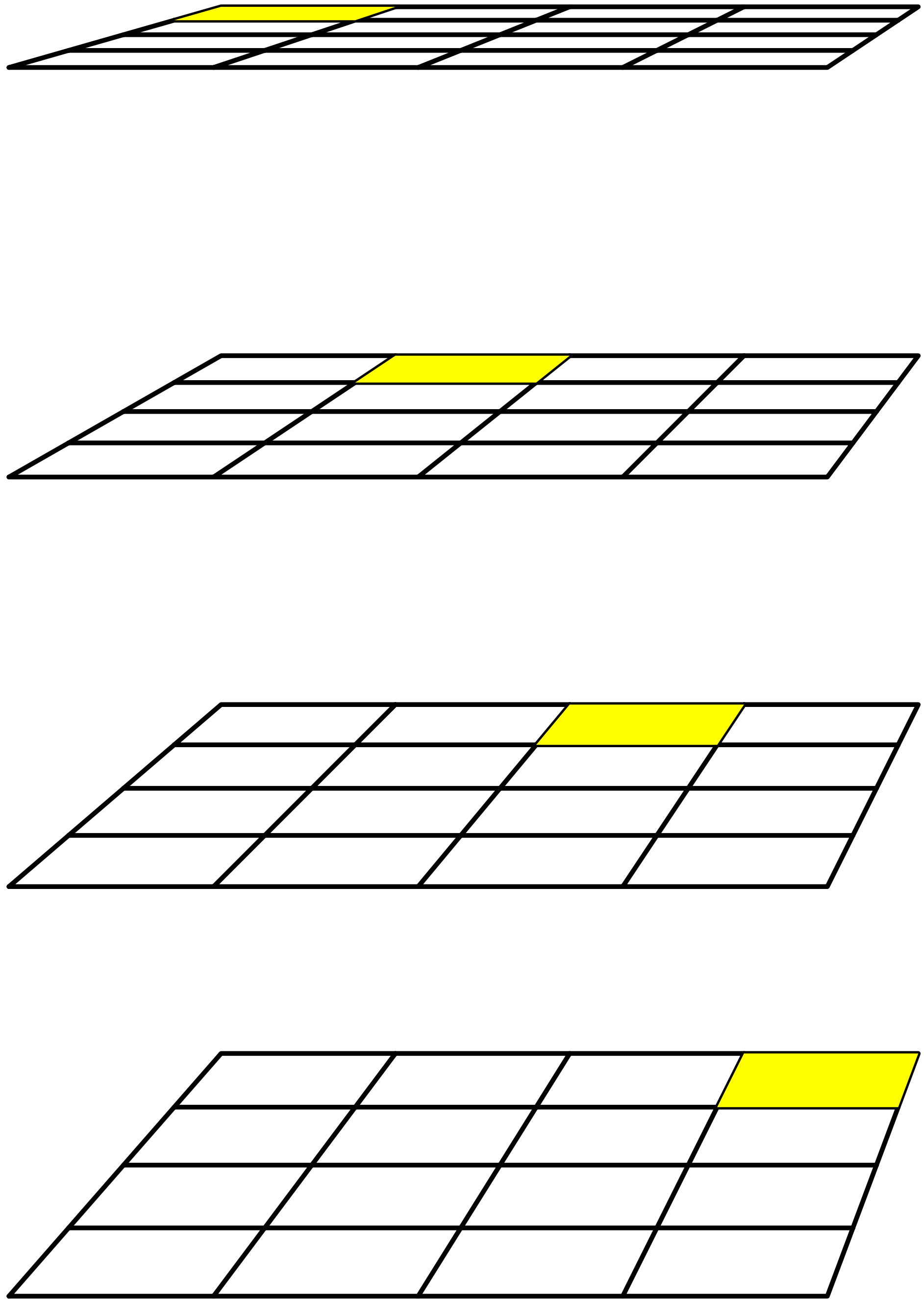
***Knowing these strategies does not gaurantee winning.**

Each strong (also called rich) point lies on 7 lines:

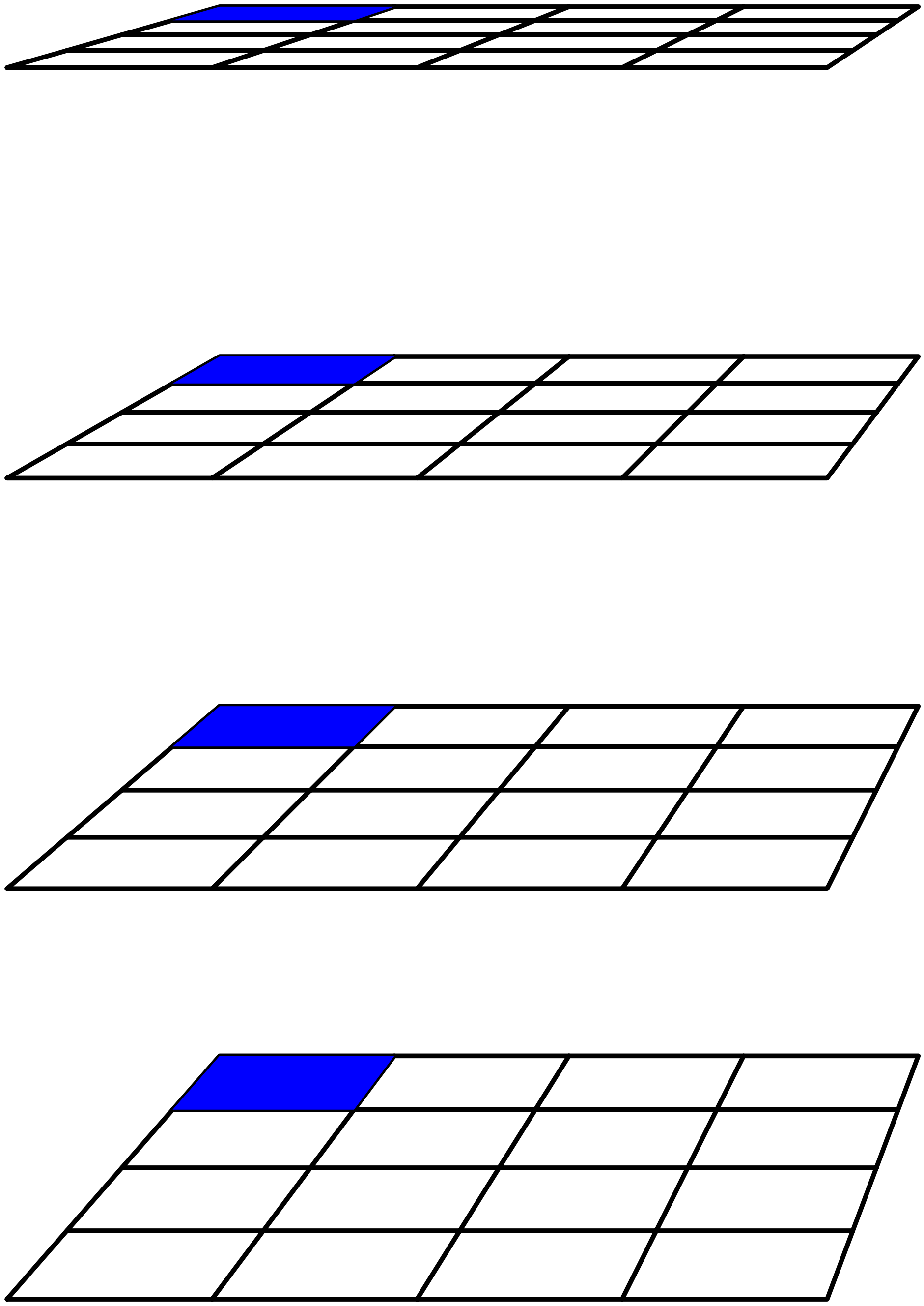
1 line consists of all 4 rich points - as illustrated



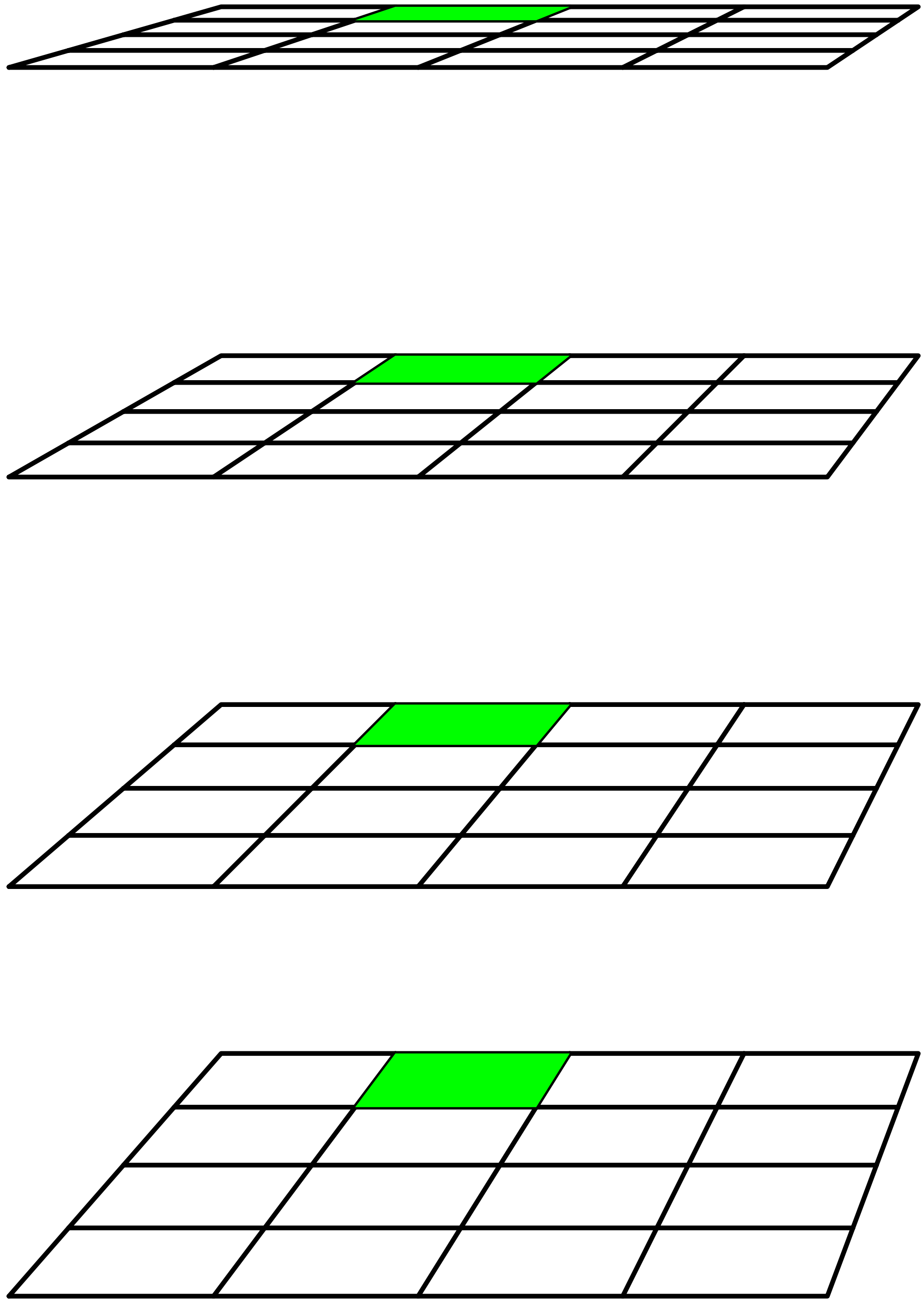
Each strong (also called rich) point lies on 7 lines:
3 lines consist of two rich and two poor points
and pass through two opposite corners



Each strong (also called rich) point lies on 7 lines: 3 lines consist of two rich and two poor points and are parallel to the axes.



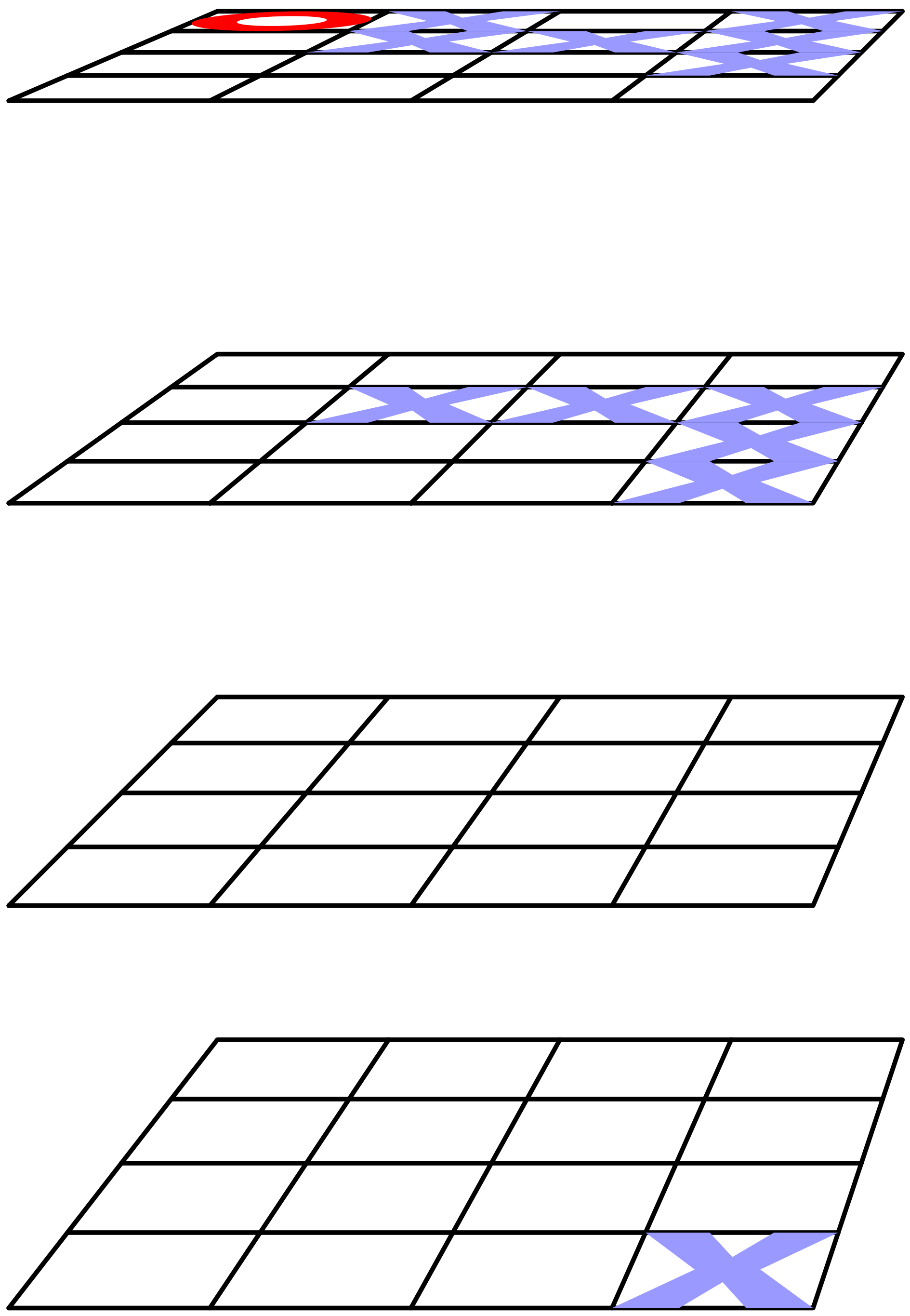
Each weak (also called poor) point lies on 4 lines:
2 lines of which contain no rich points
2 lines contain two rich and two poor as shown earlier.



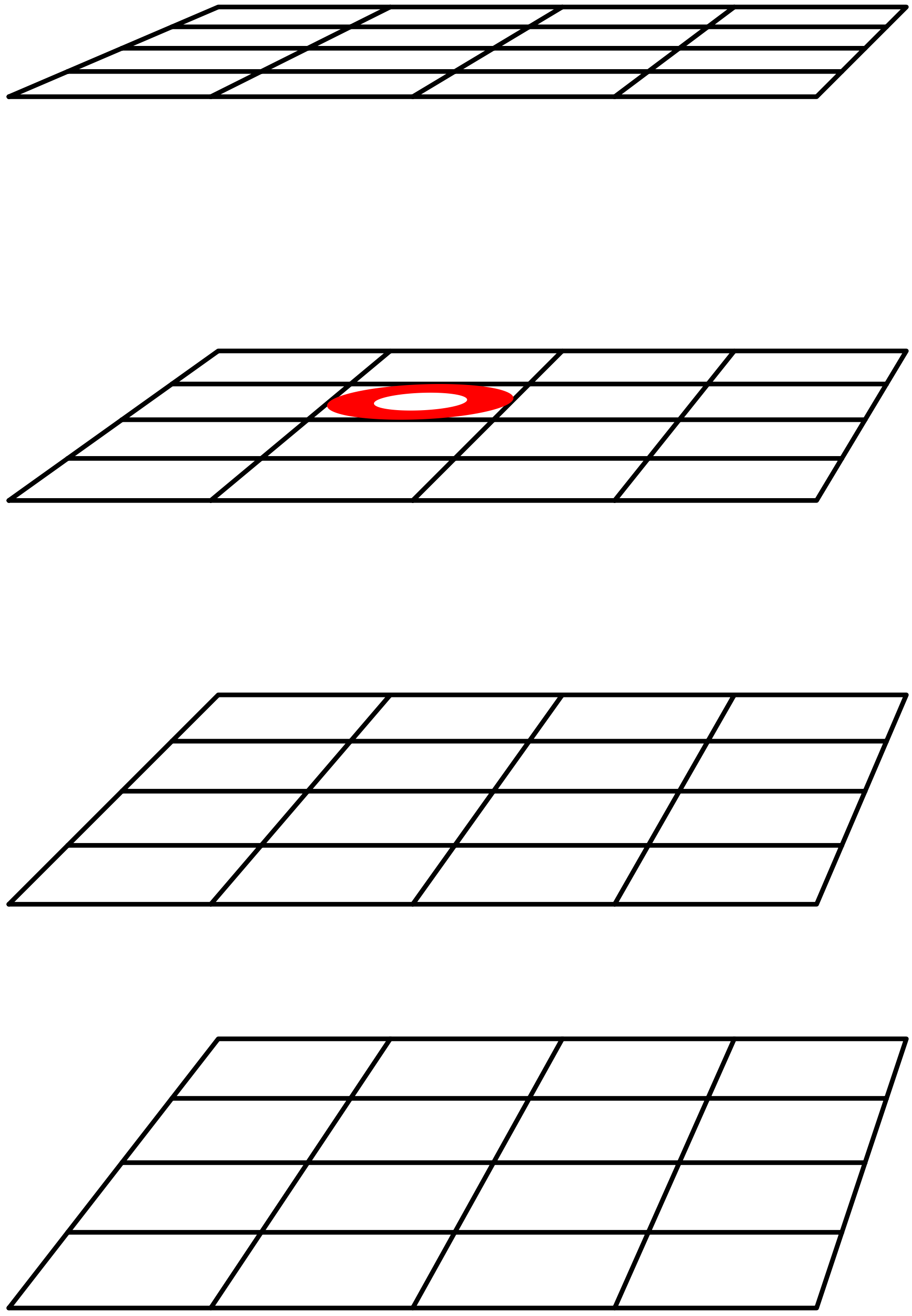
As a result of Qubic's automorphisms, there are only two distinct opening moves, any rich point or any poor point.

As soon as one of the player forces a winning series, where he or she forces the other player to continually play the defensive role, a winner is soon determined. When one player is able to obtain all or most of the rich points and create 3-in-a-rows or threats, a fork (two 3-in-a-rows) ensures the win. It is believed that the first player has a sure win. However, this is only proved by computer generated solutions. Generalization on proof-number search can be applied to such conclusion that the first player has an advantage, but no specific research has been done on Qubic.

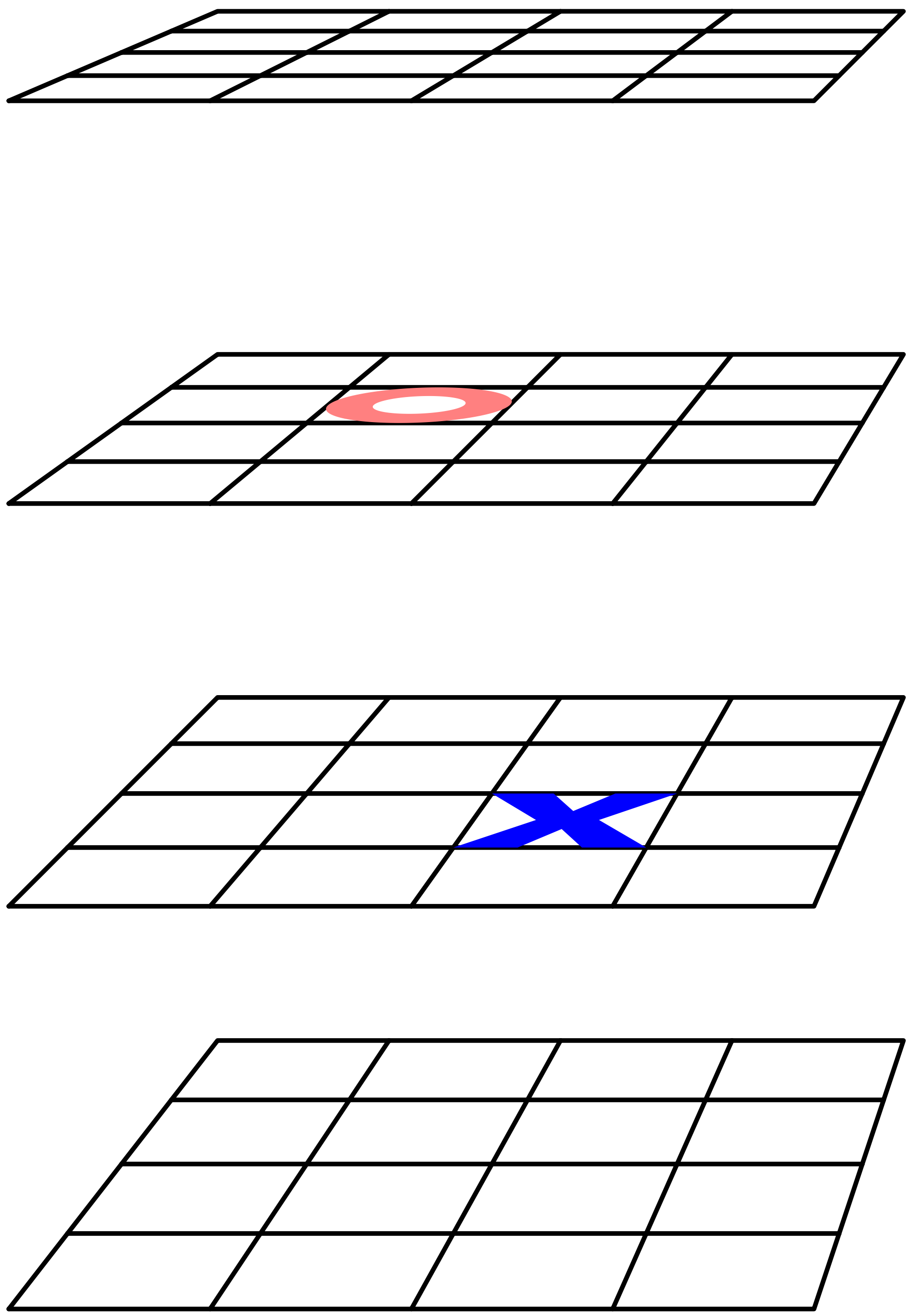
Here are examples of some opening plays:
If ’O’ takes a rich point in the upper corner,
due to symmetries, there are 12 distinct responses.
If one tries to put an ’X’ in any other position, a
’repeat’ will appear due to the symmetries.



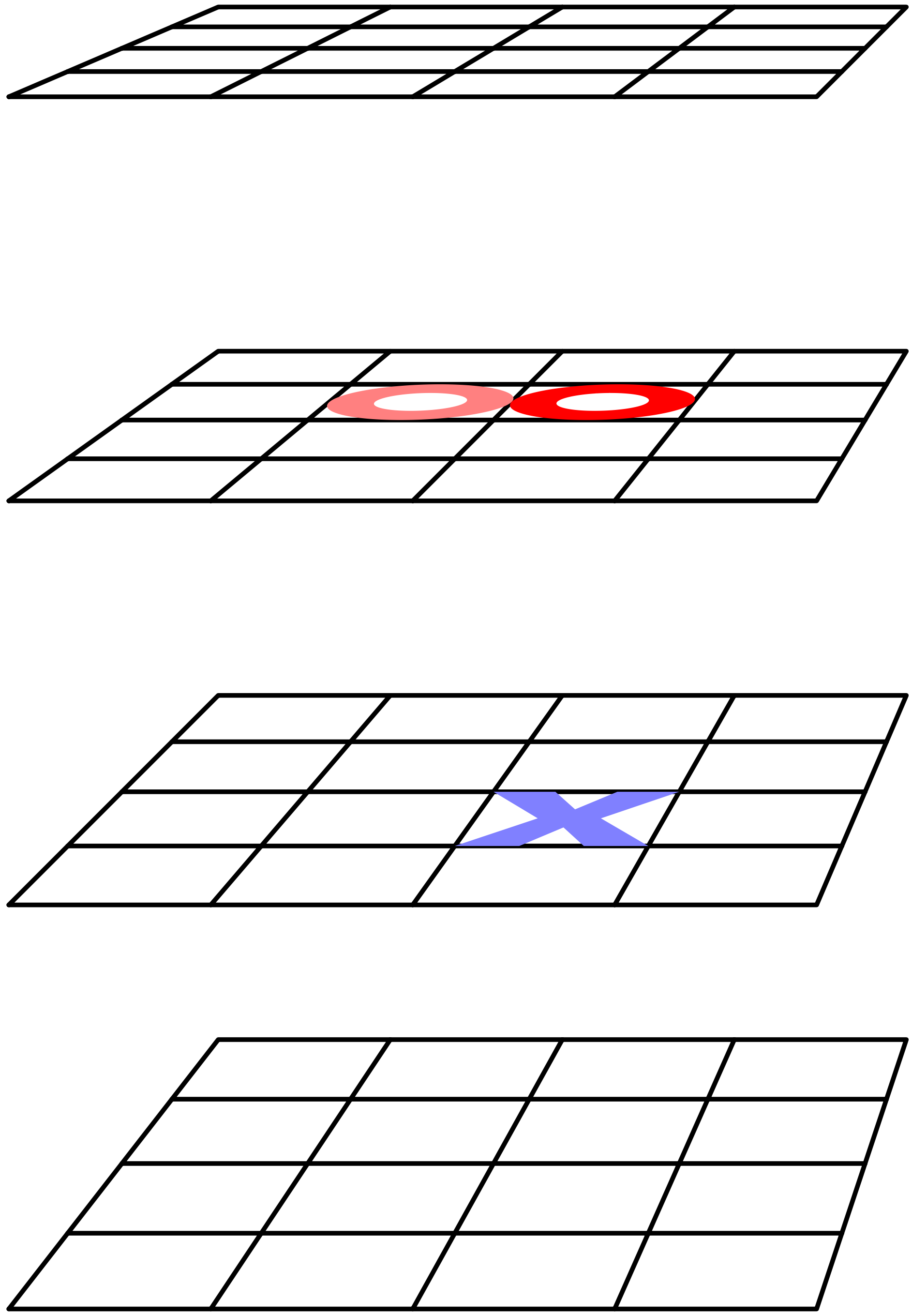
Here is an example of a threat-search game where the first player tries to force the second player to play the poor game spaces. The first player places the ’O’ in a rich space.



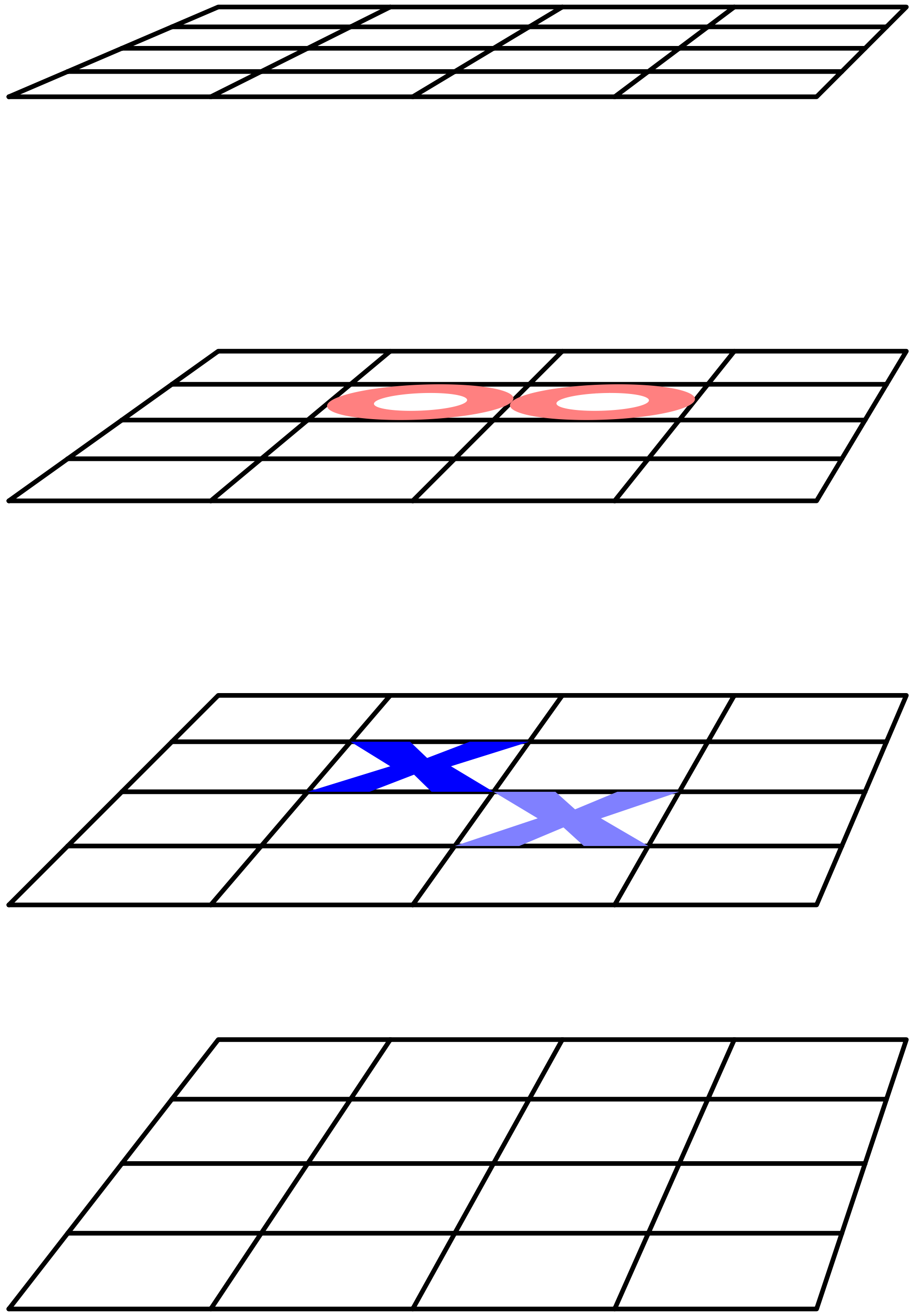
The second player puts the ’X’ in a rich point as a counter strategy.



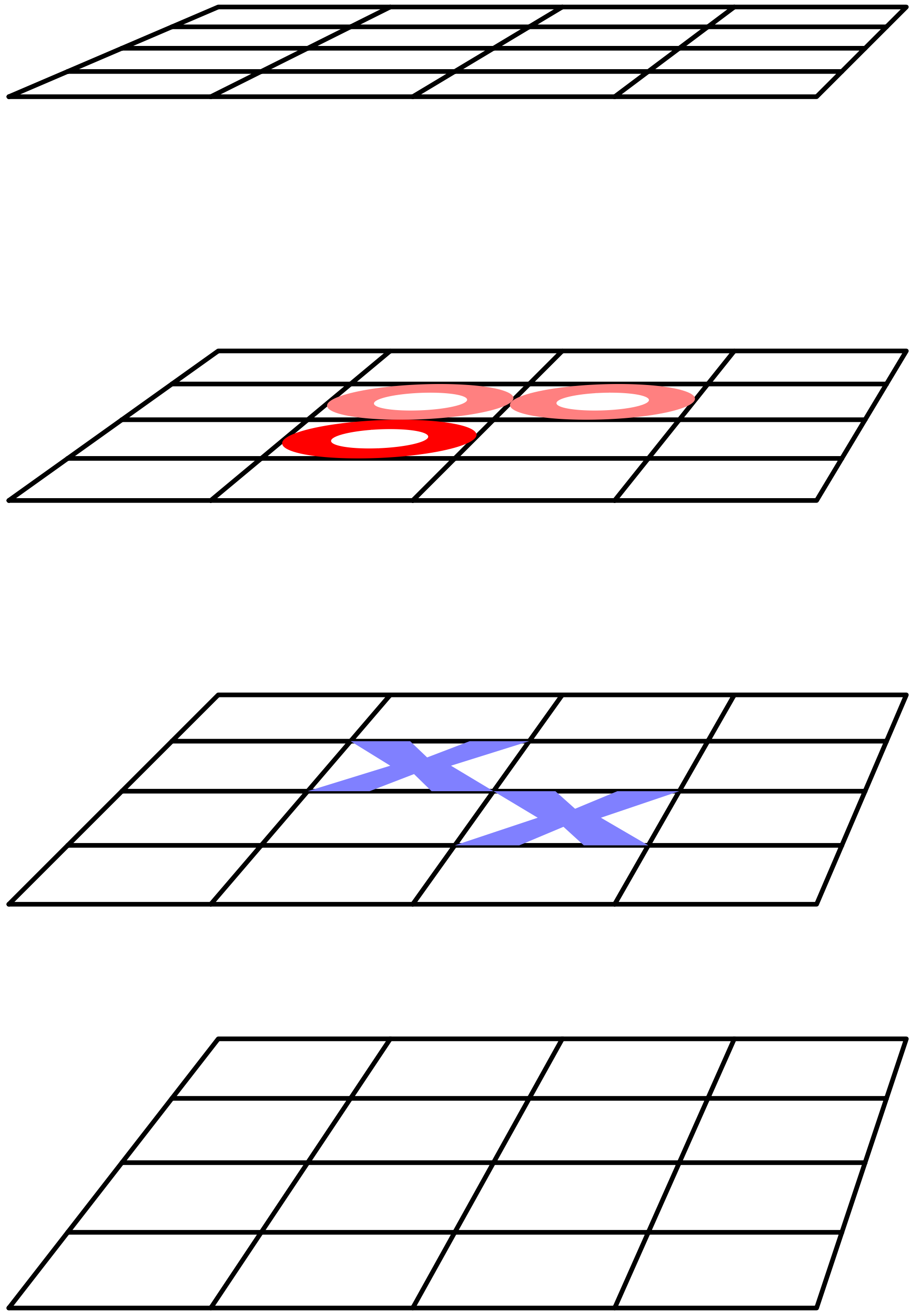
The first player occupies another rich space trying to create a 3-in-a-row threat to the 2nd player.



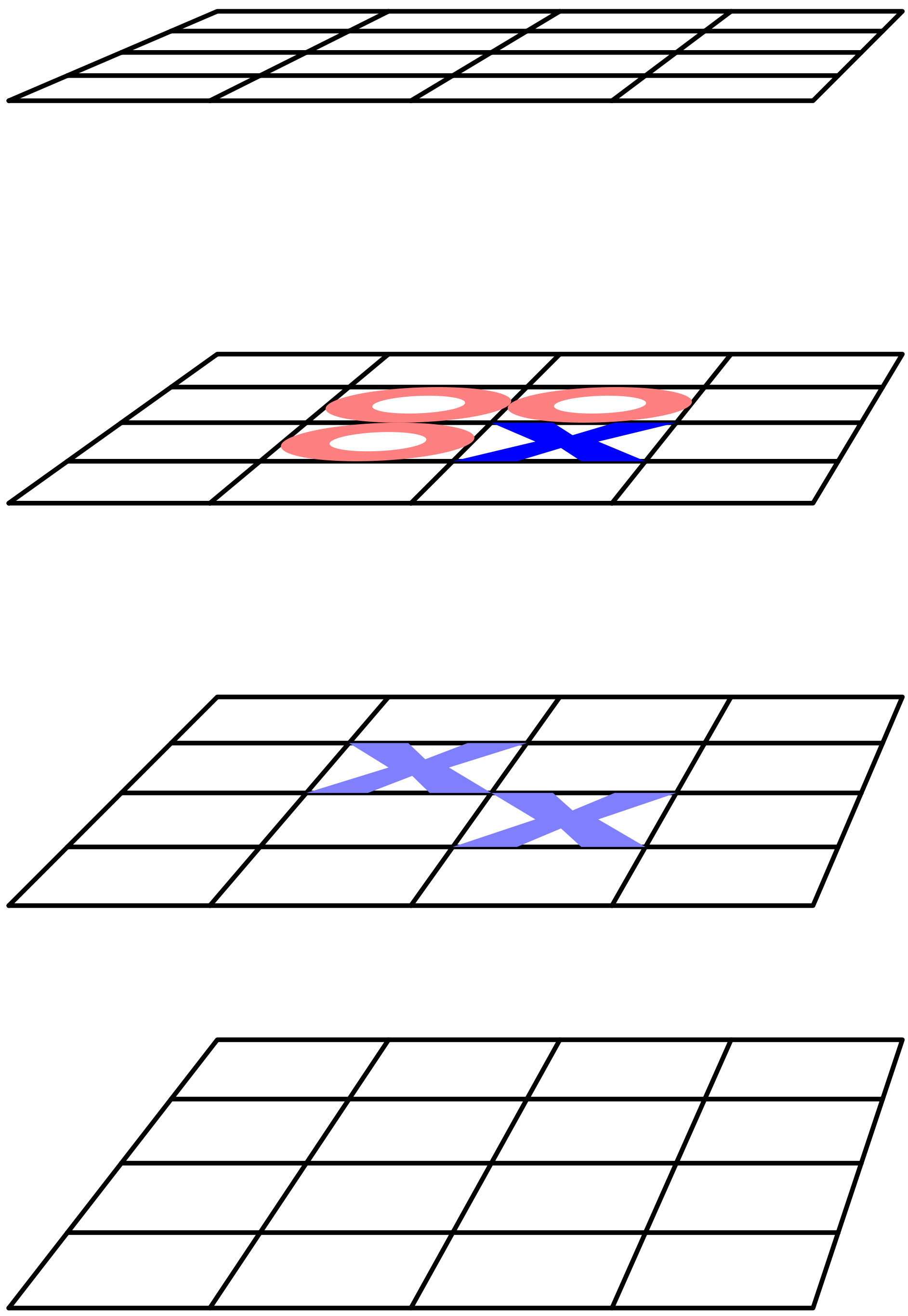
Instead of blocking the first player. 'X' decides to create a 3-in-a-row in order to threaten the first player.



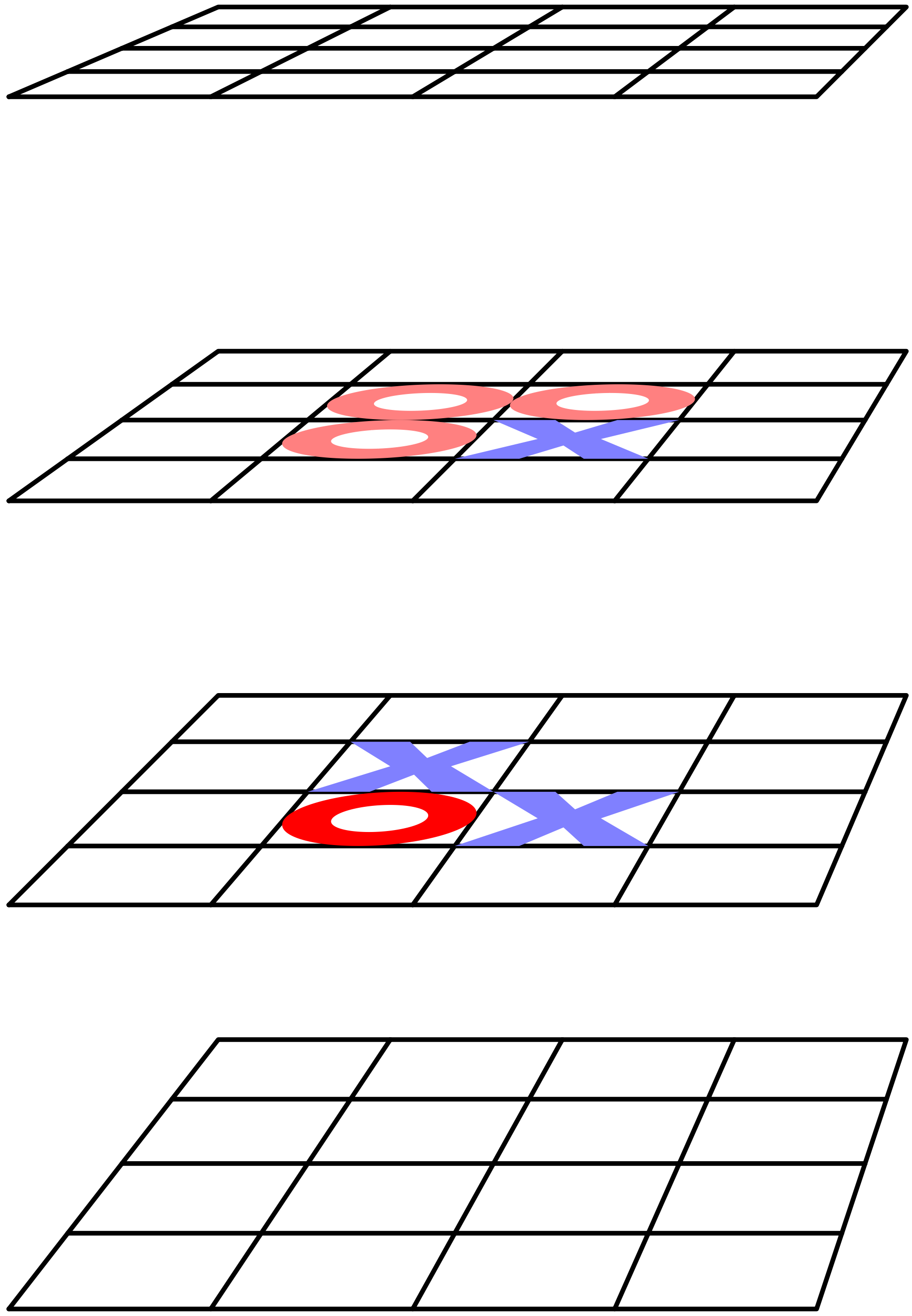
’O’ occupies another rich point and now has three ways to create 3-in-a-row’s.



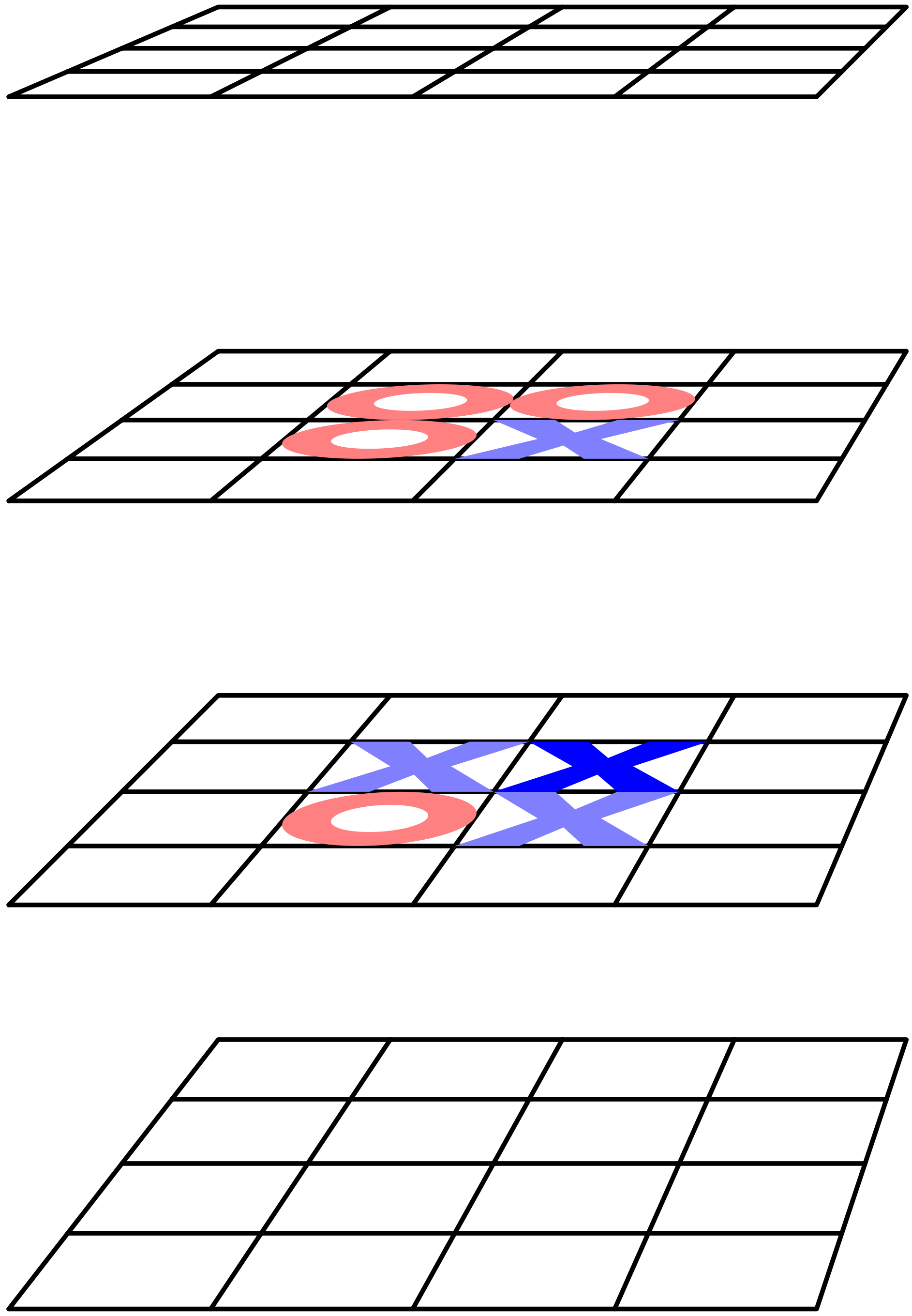
’X’ decides to block off the last rich point in the second plane to prevent ’O’ from being able to force ’X’ off the rich points two rounds from now.



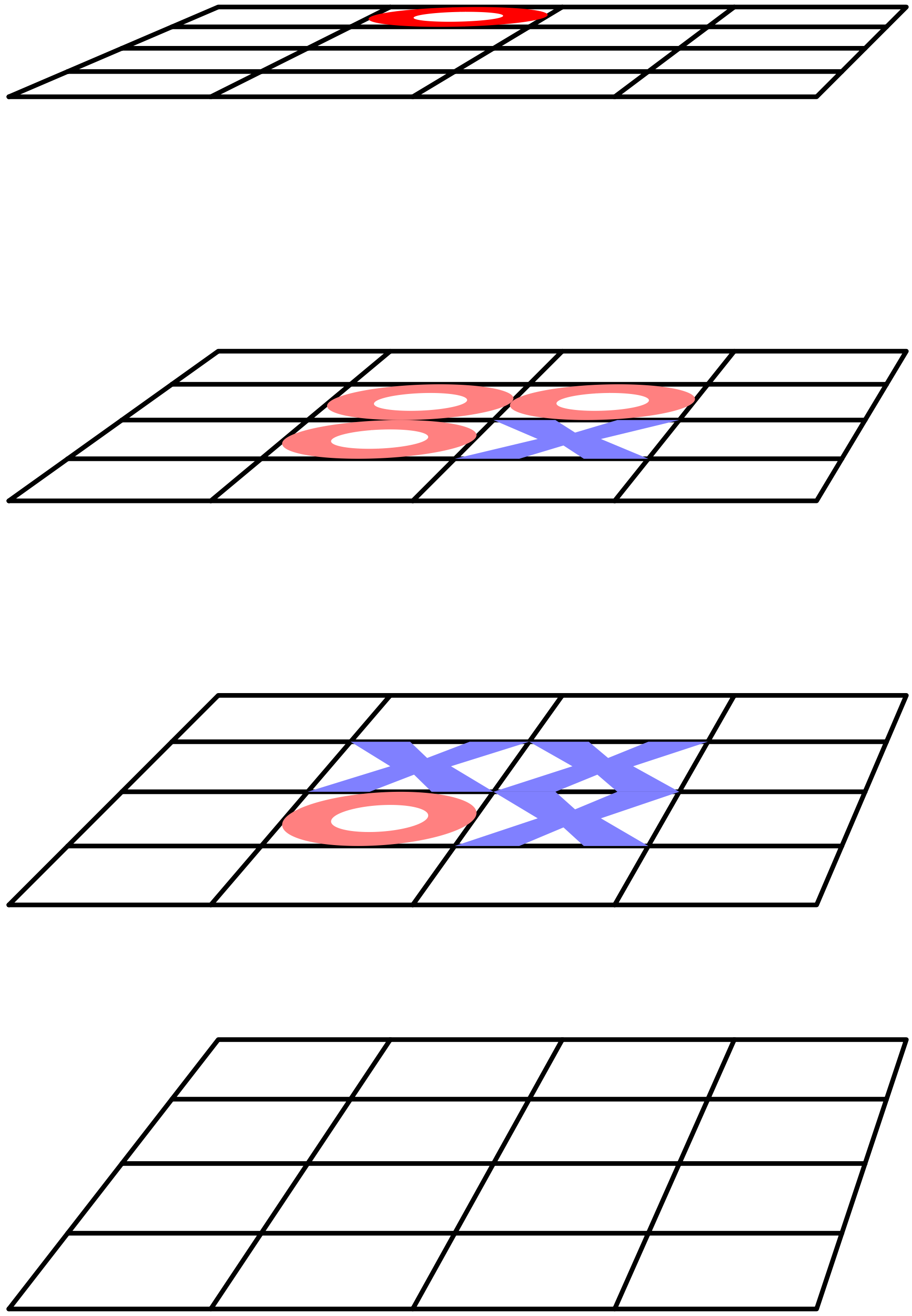
’O’ continues to make itself more opportunities to create 3 in a rows and continues to take up the rich points in order to force ’X’ to take a poor point.



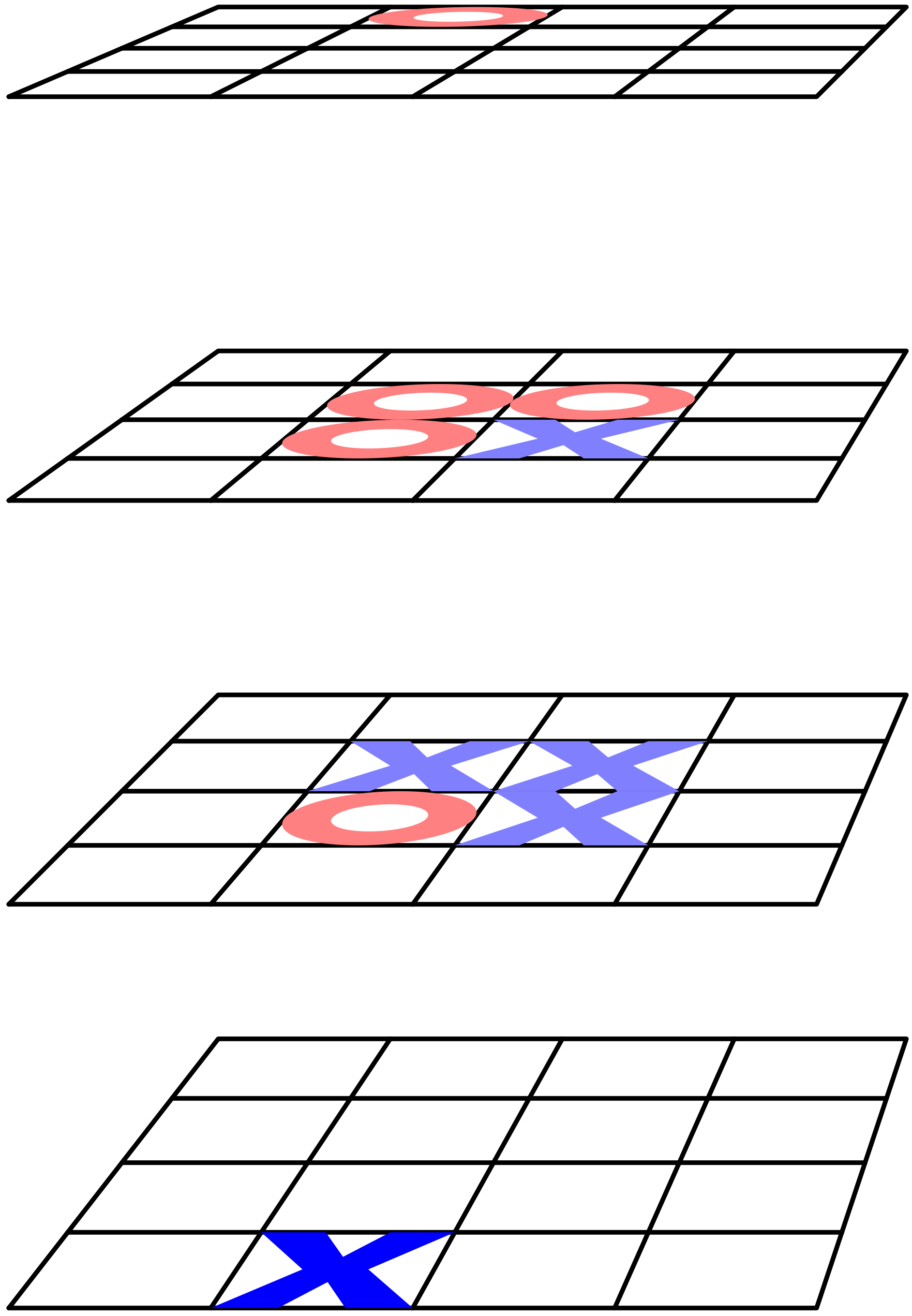
Knowing rich points contains more lines, ’X’ takes hold of the last rich point in the third plane.



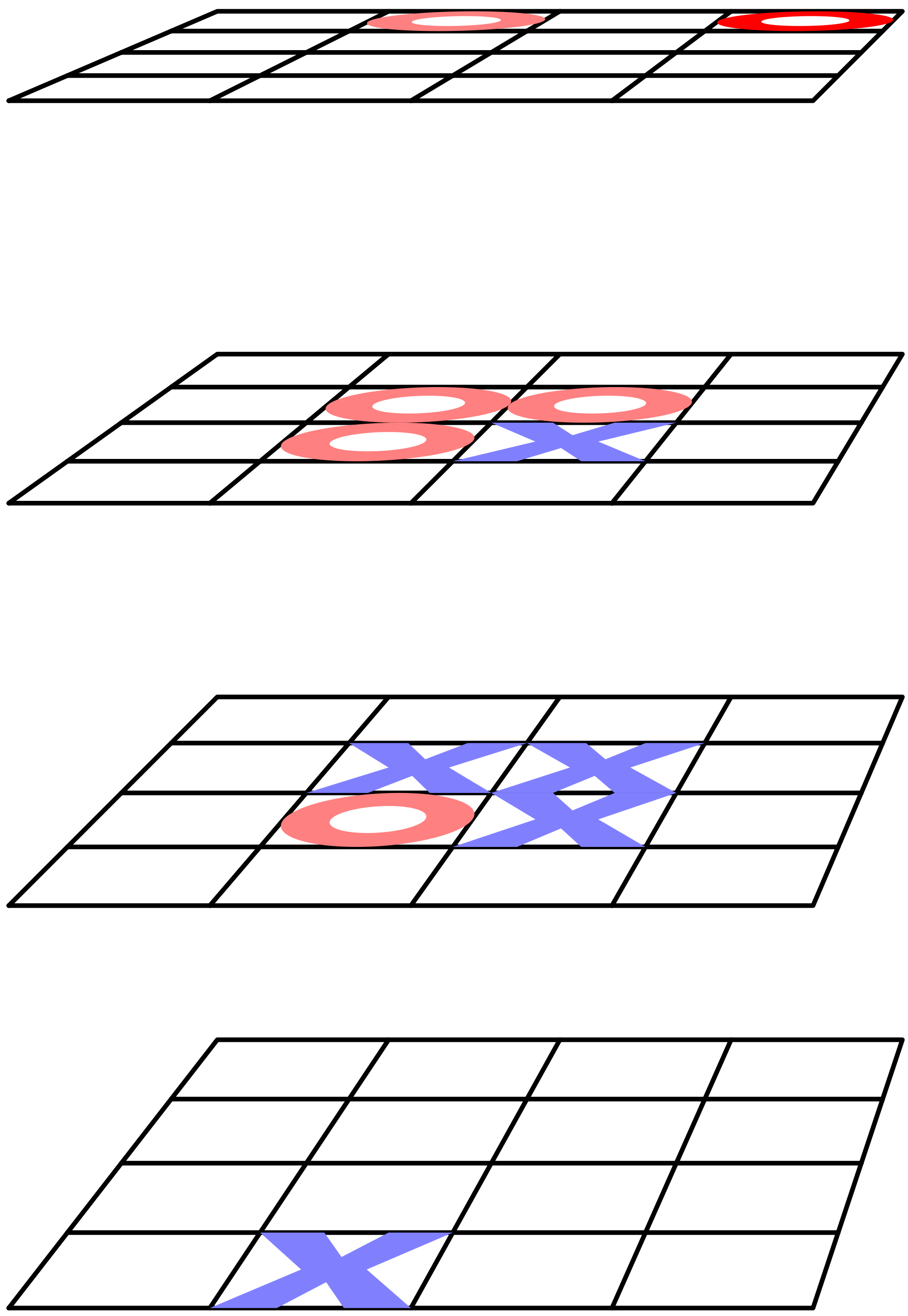
’O’ has now make its way to threaten ’X’ it is now forcing ’X’ to take a poor point to stop ’O’ from connecting four spaces.



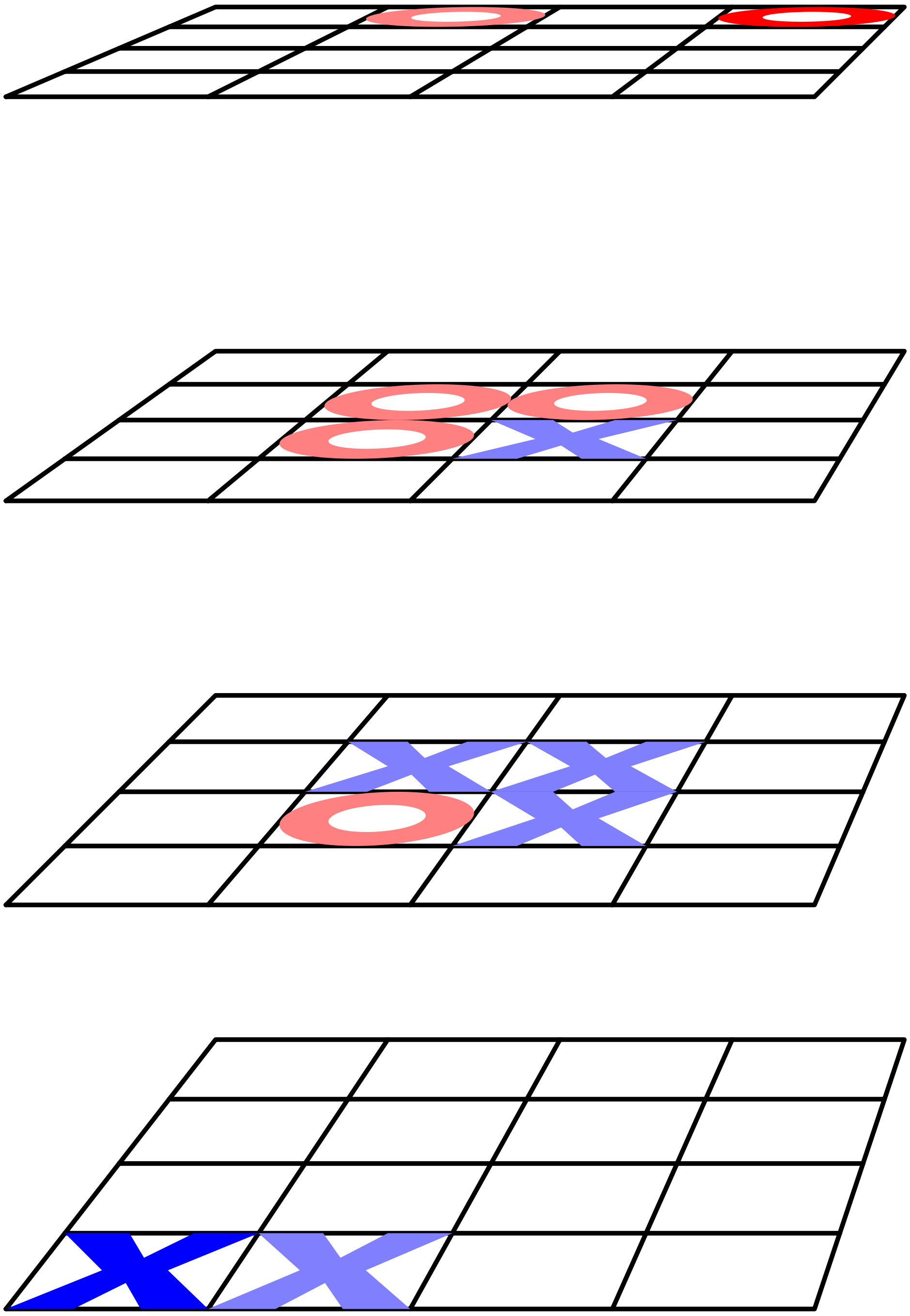
’X’ does what ’O’ had intended.



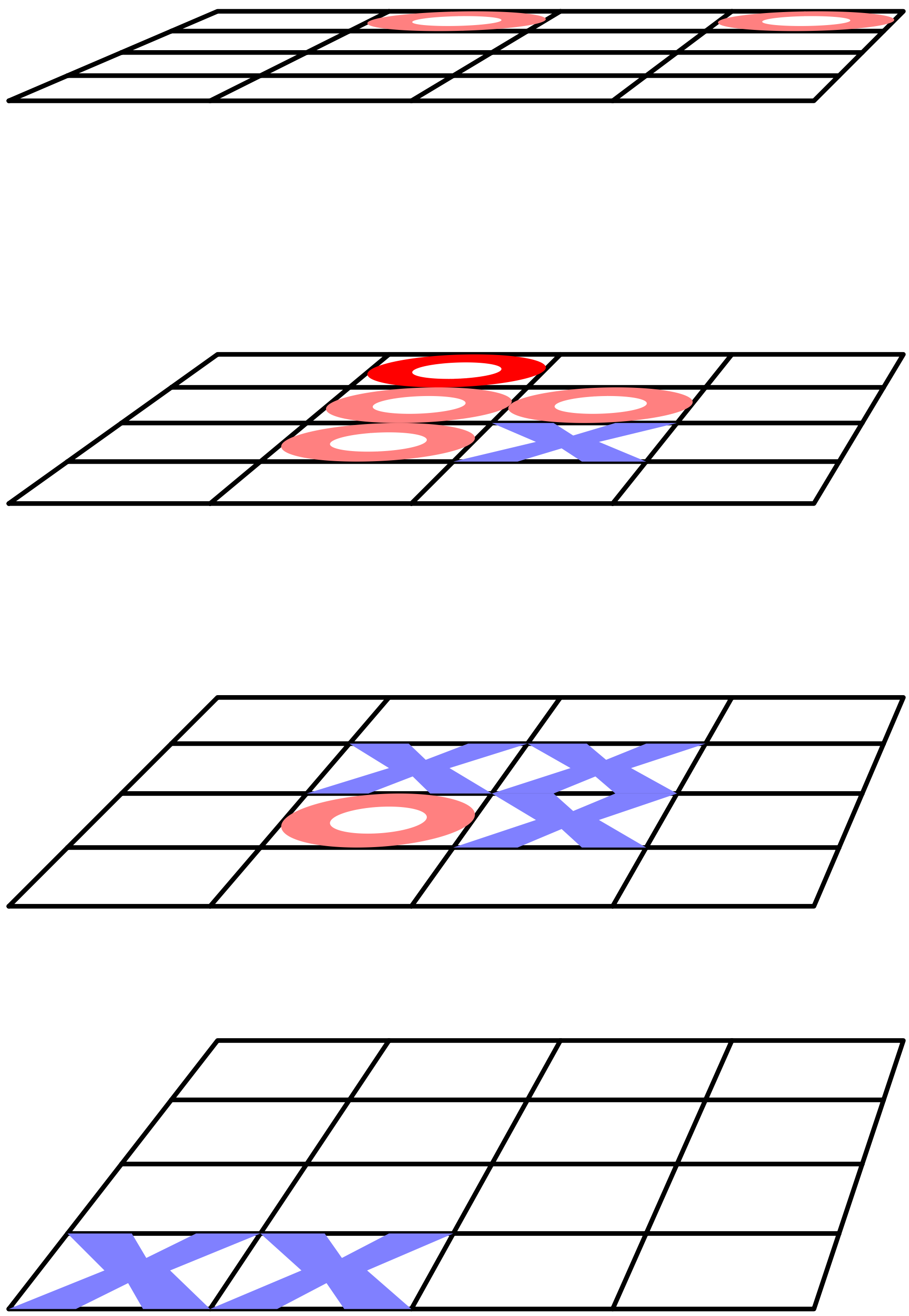
’O’ again threatens ’X’ to block it off.



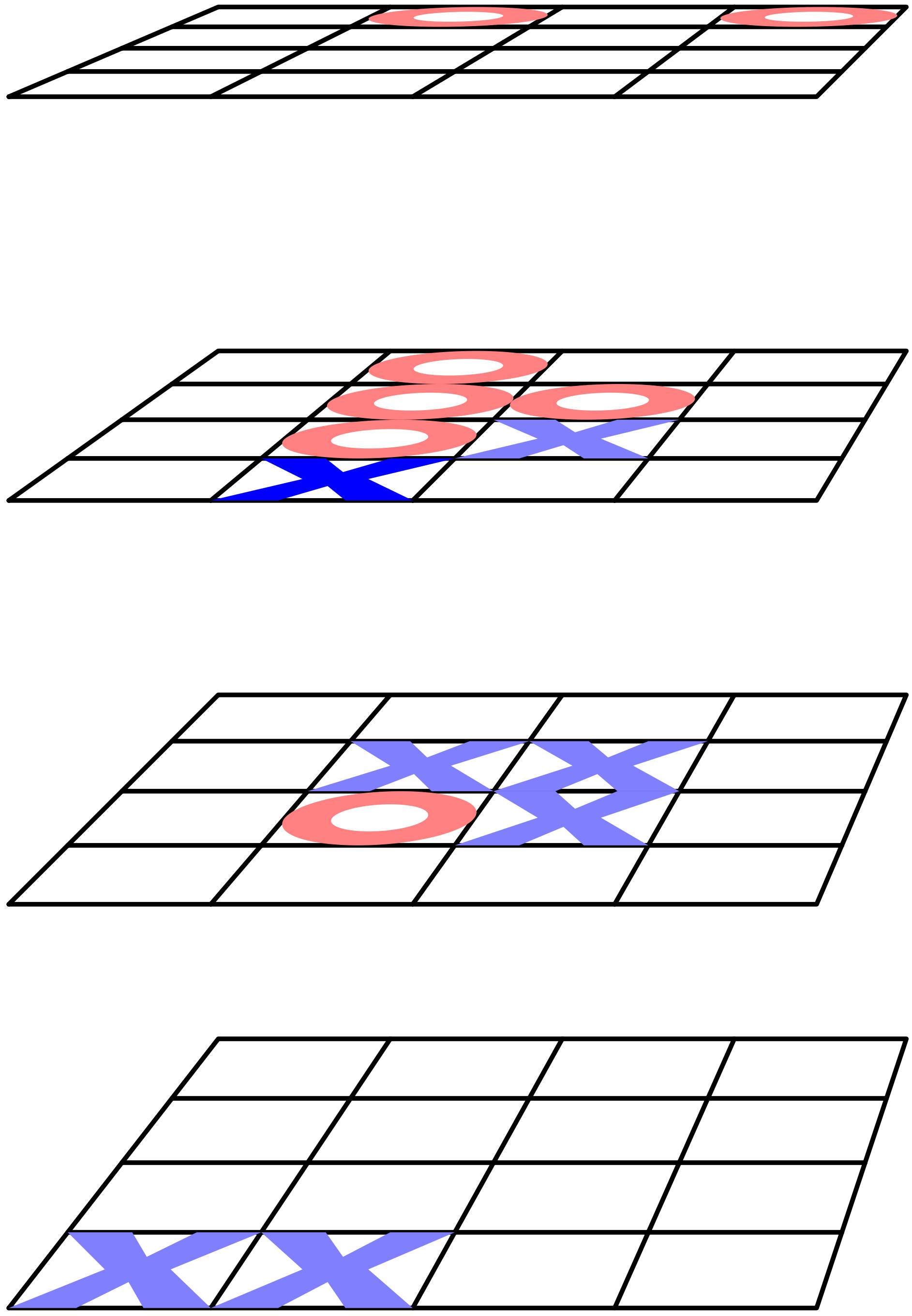
’X’ follows the trap.



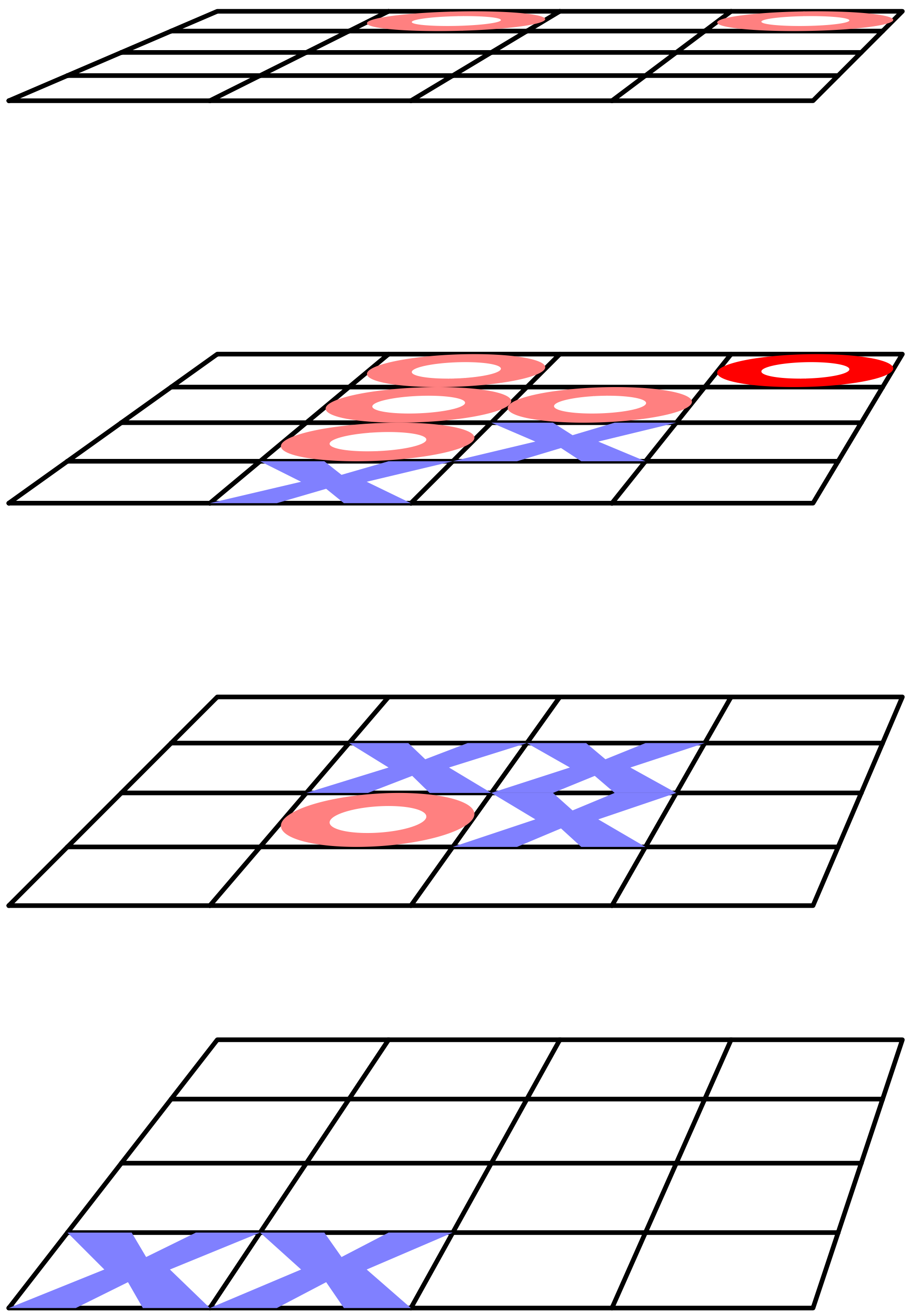
’O’ continues to make its way to creating a fork - a dead end for ’X’



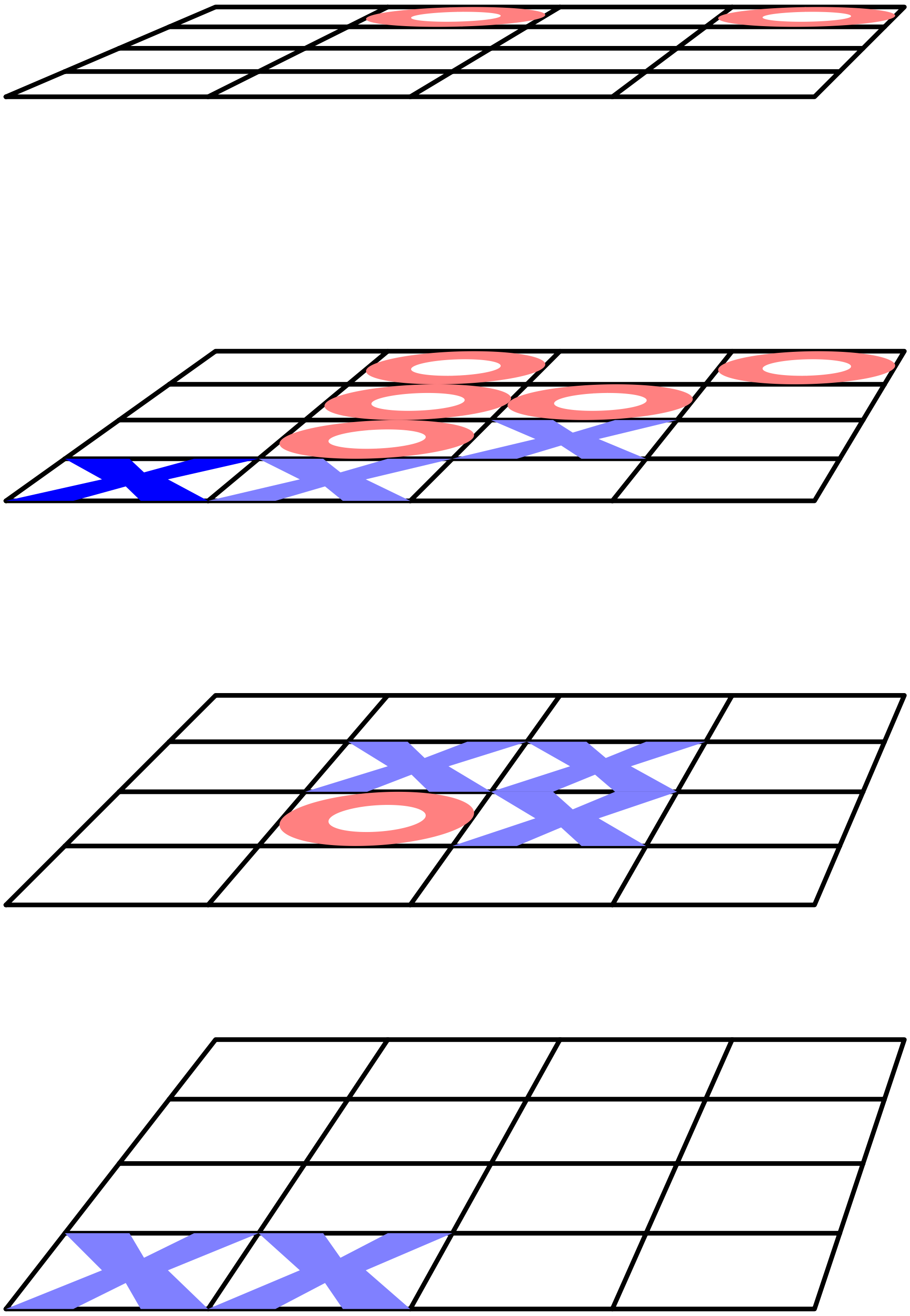
’X’ has no other move than to blocking ’O’s 3-in-a-row.



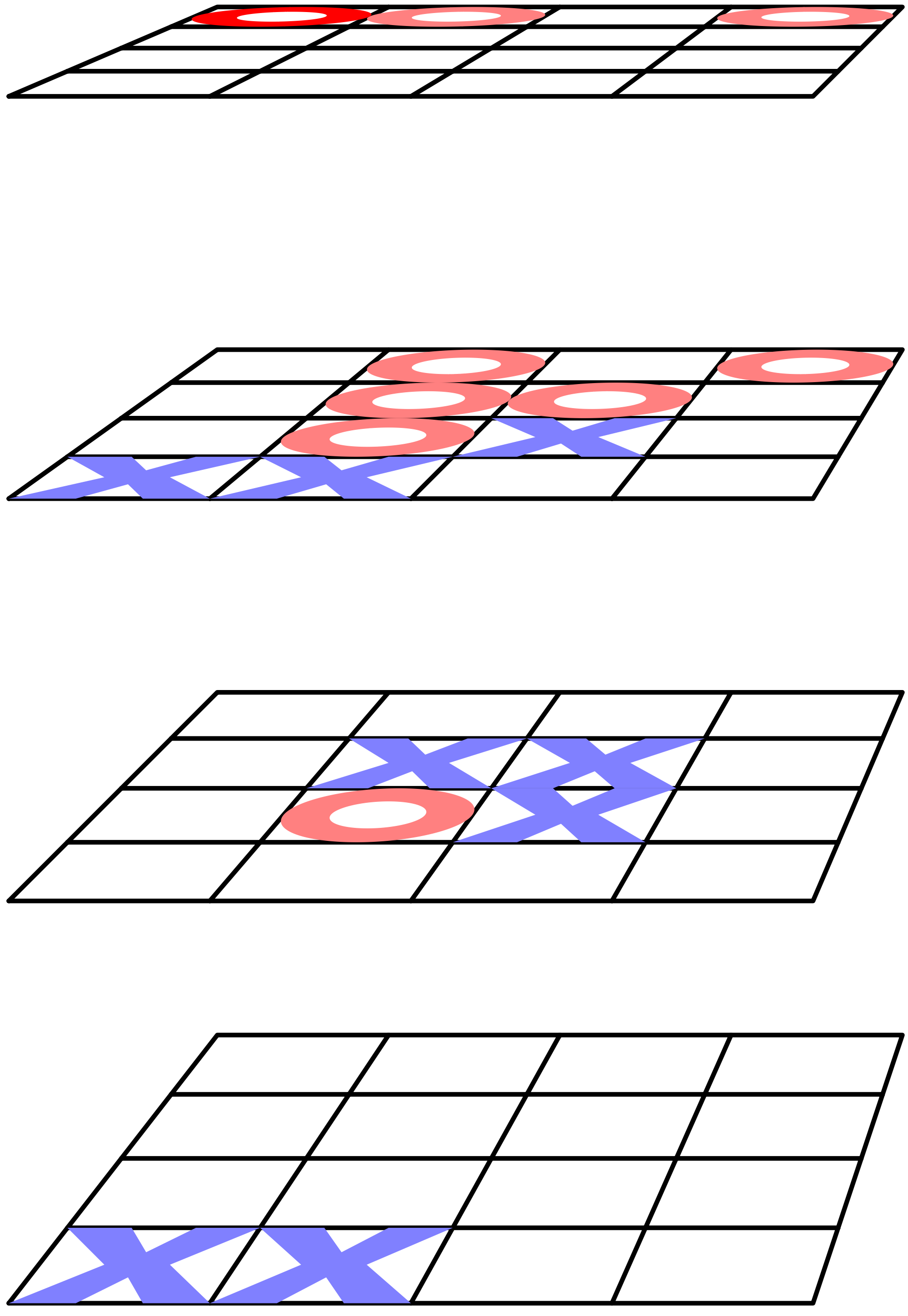
Another threat made by 'O'



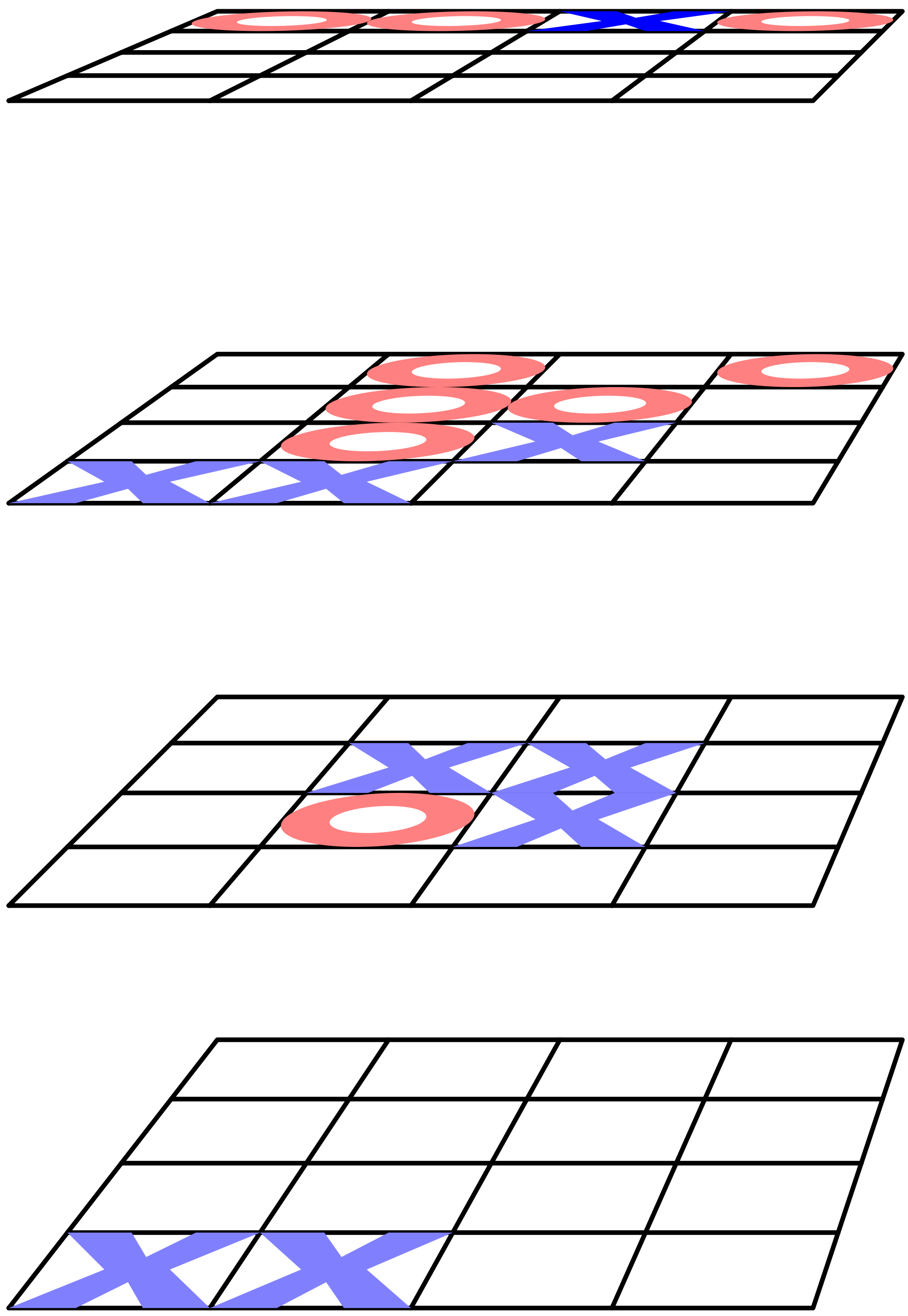
Another defensive play by 'X'



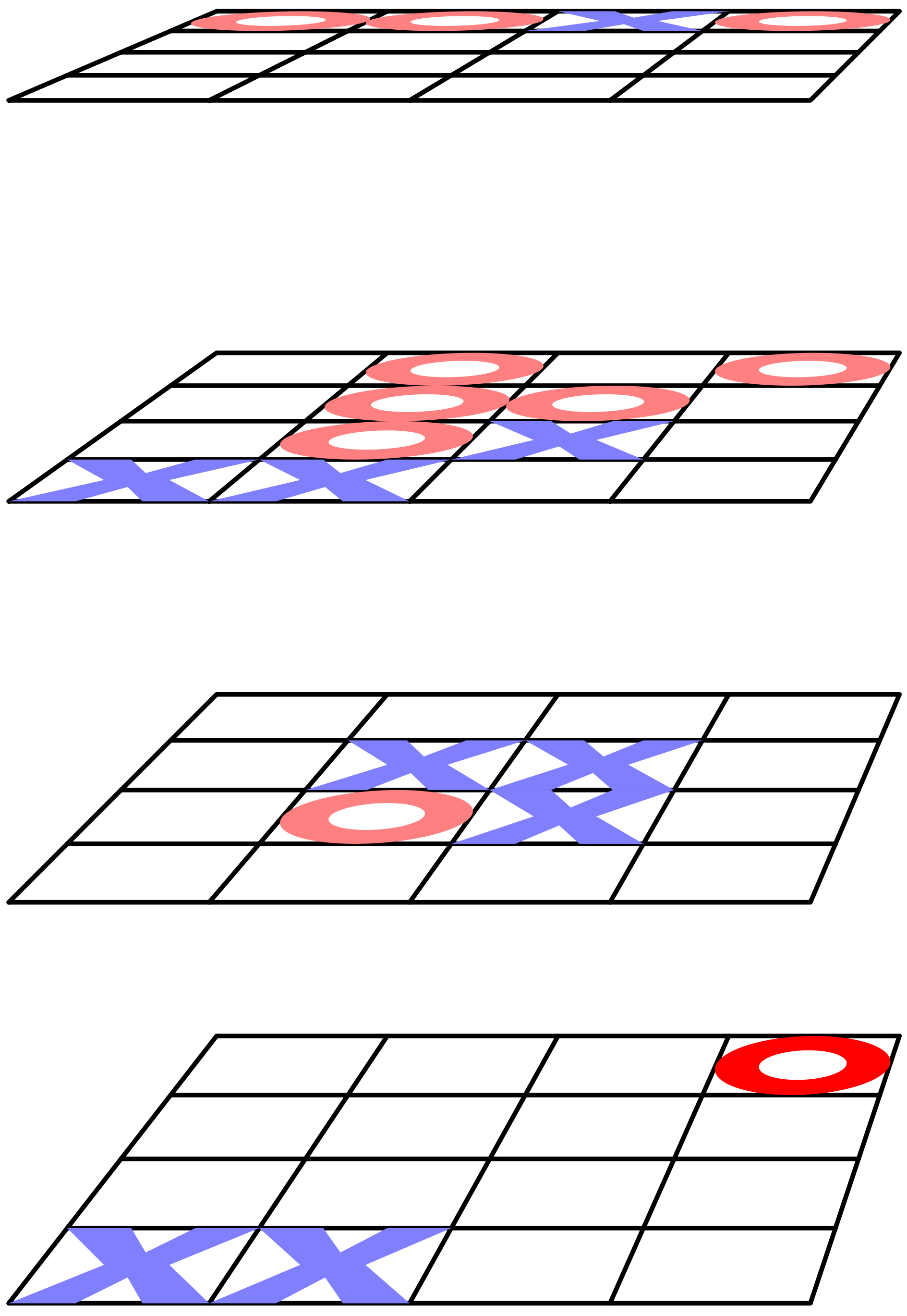
’O’ has begun to put down a double threat finding its way to victory.



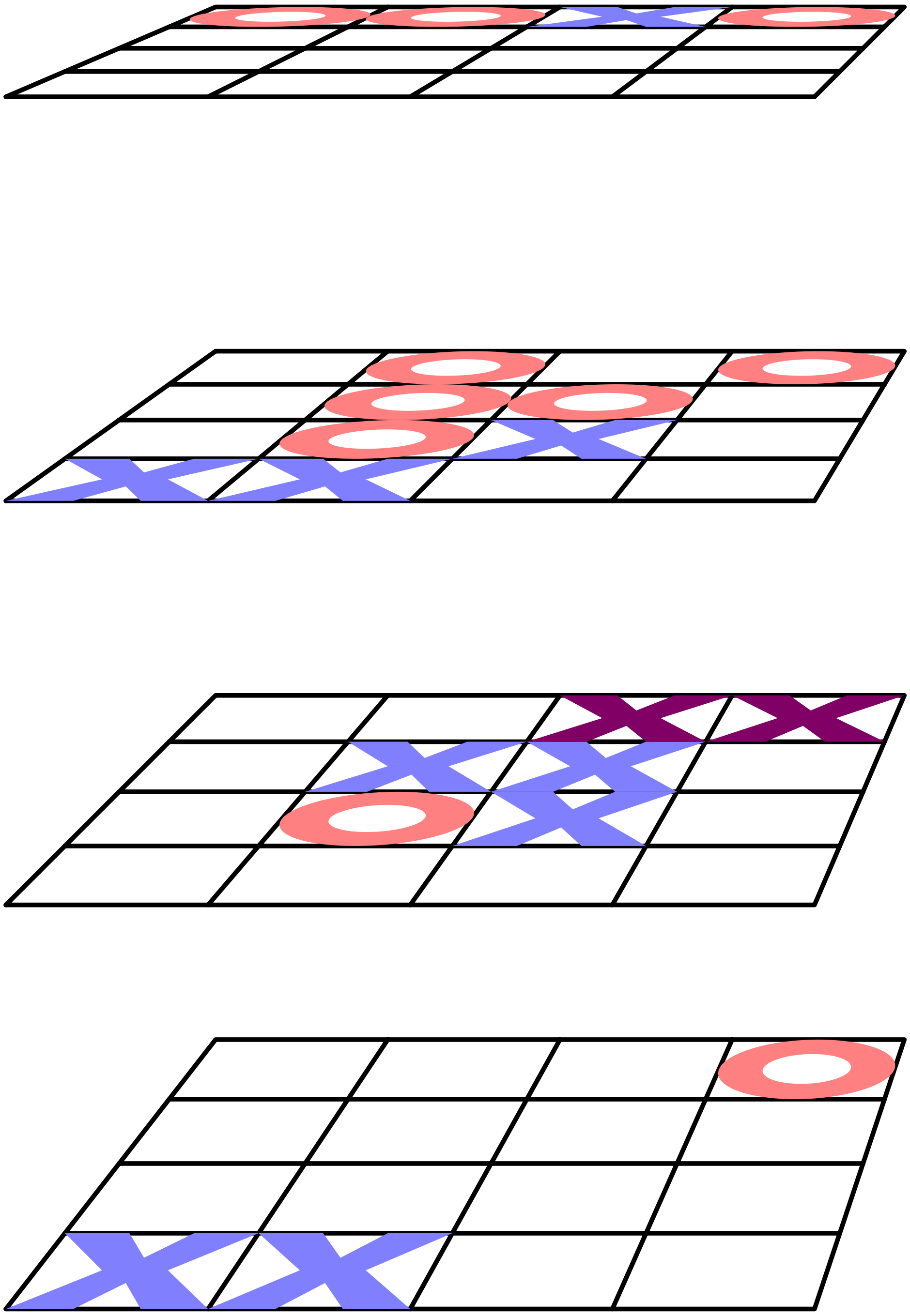
’X’ is now in vulnerable position and its every next move is manipulated by ’O’



This move by ’O’ determins the winner, since now there is no way ’X’ can salvage itself.



To block off 'O', 'X' needs to obtain both of the positions marked in purple simultaneously. Since such move is not allowed in tic tac toes in any dimension, 'O' claims its victory.



The previous play shown is borrowed from Allis and Schoo in "Qubic Solved Again". However, it is possible to play it in a different order. Nonetheless, the first player always has the advantage of forcing the second player into the defensive mode and leading itelf to winning. If this happens, it is called ’a perfect game’ where neither of the player misjudges any move (i.e. no foolish moves).

This topic has much more than I initially realized. There are papers written on it as early as the 1940’s and it is an on-going research topic in recreational mathematics. Researchers are still finding an analytical way to proof the ’first player wins’ notion. The number of different ways of winning has not yet been finalized. There are still a slight discrepency in some of the studies. If I had more time, I would show some of the proofs that have been done. Unforetunately, due to the time limitations, I will draw an end to this project here.

Hope you enjoyed the unfolding of Qubic!

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