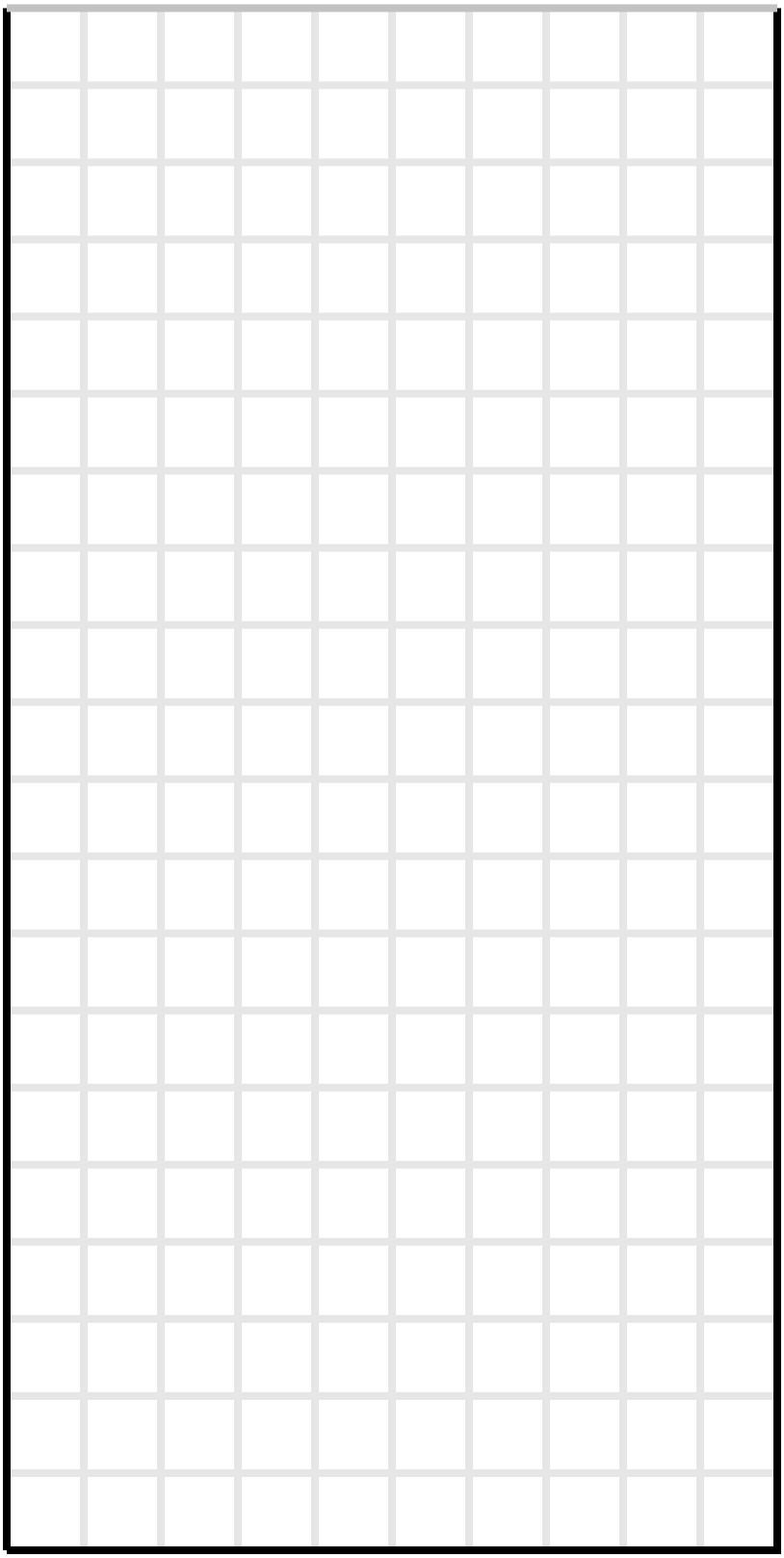
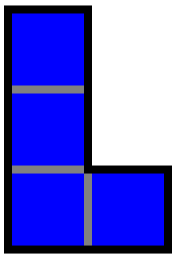
Tetris is a video game that requires the ability to be able to visualize the interaction between different objects and their rotated states. To understand the mathematical mechanisms of Tetris, first we must understand how the game works, and how the pieces can interact with each other. After seeing how the pieces interact we should be able to see that in the end, the game can never continue forever, also known as being NR-Complete. We can then show, mathematically why the game is NR-Complete, and what factors we can change, so the game can continue forever.

The game begins with a 20x10 Grid.

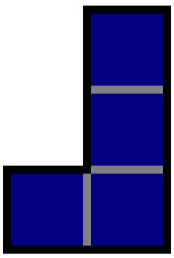


The player is given one of the seven Tetris pieces shown below. Their names are labeled.

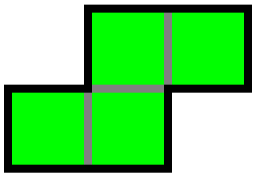
"I"



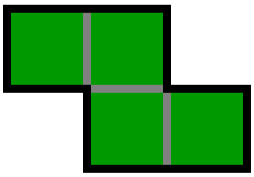
"L"



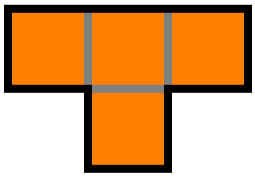
"J"



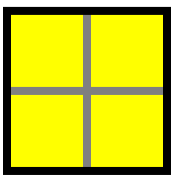
"S"



"Z"

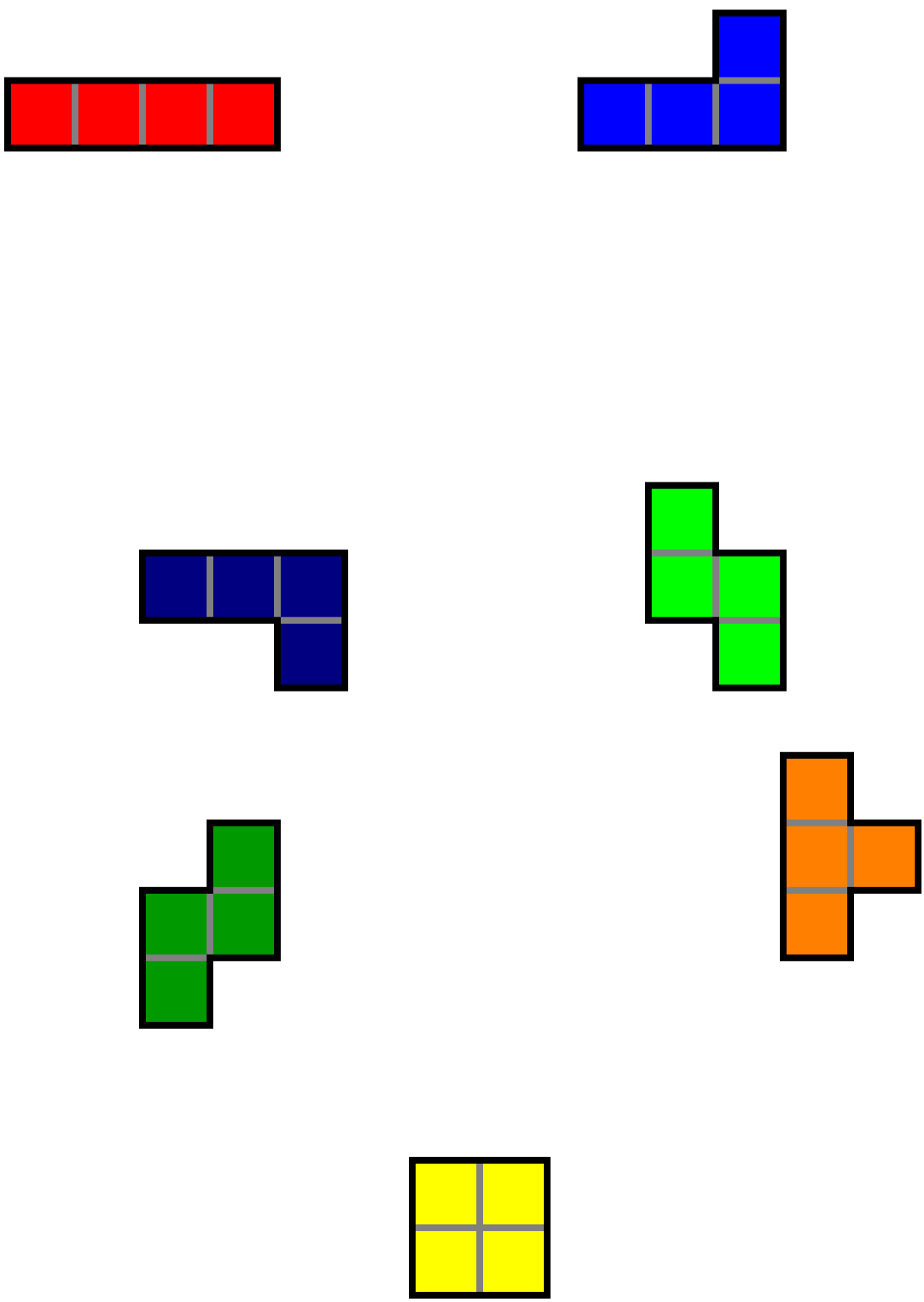


"T"

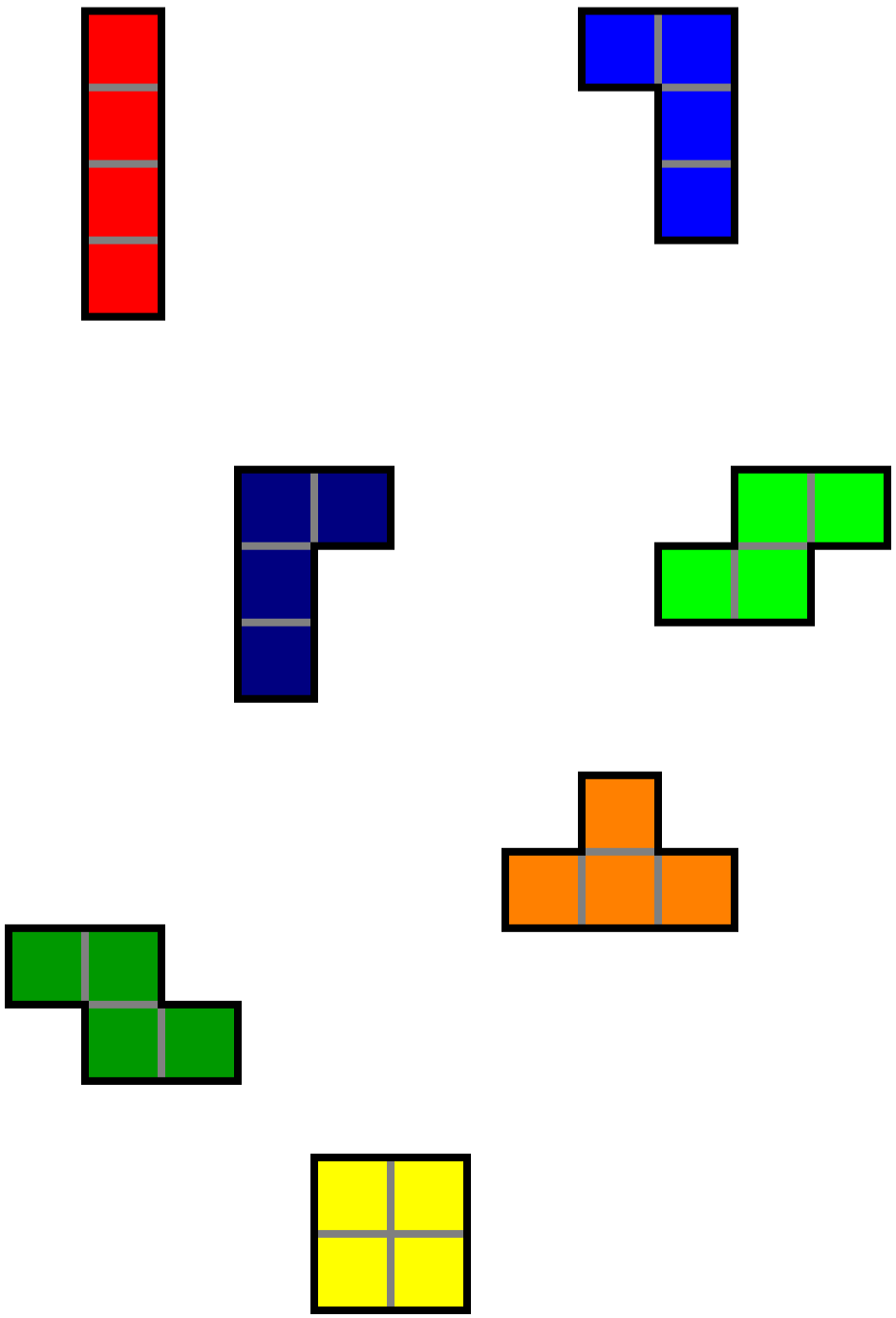


"O"

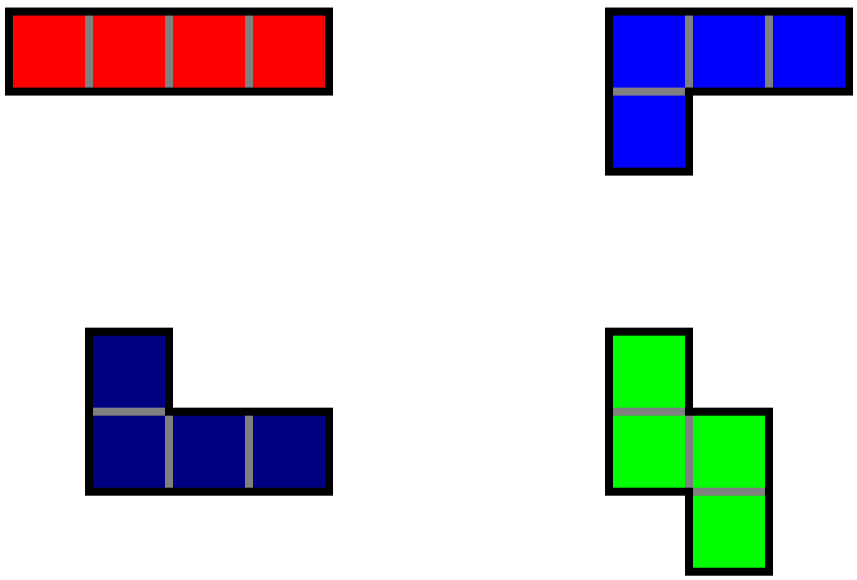
The player can rotate the pieces either counterclockwise...we call these pieces I', L', J', S', Z', T', O' respectively

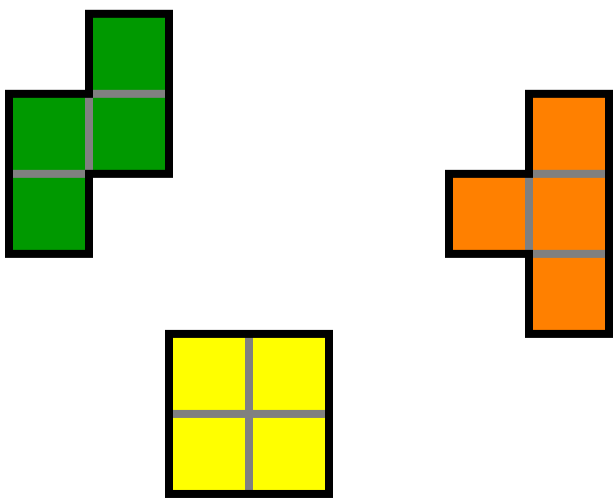


These are I", L", J", S", Z", T", O" respectively



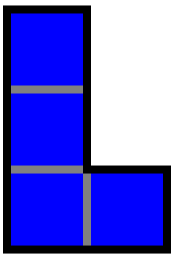
These are I"', L"', J"', S"', Z"', T"', O"' respectively



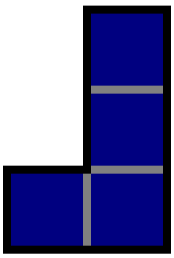


...or clockwise. Note that O"’ = O" = O’ = O.
I" = I, I"’ = I’, S" = S, S"’ = S’ and Z" = Z, Z"’ = Z’.

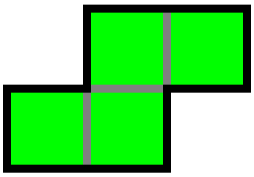
"I"



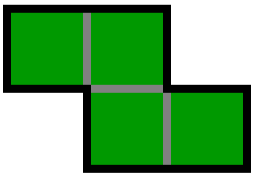
"L"



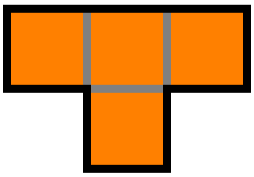
"J"



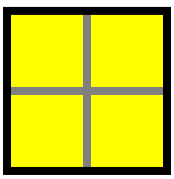
"S"



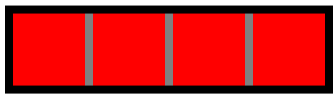
"Z"

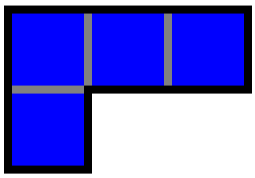


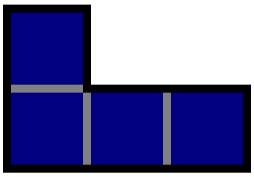
"T"

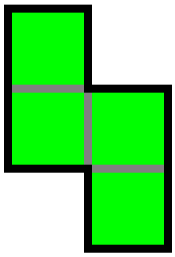


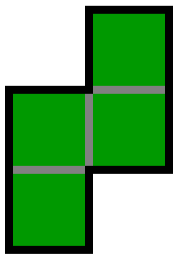
"O"

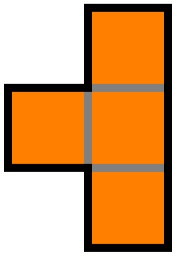


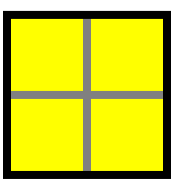


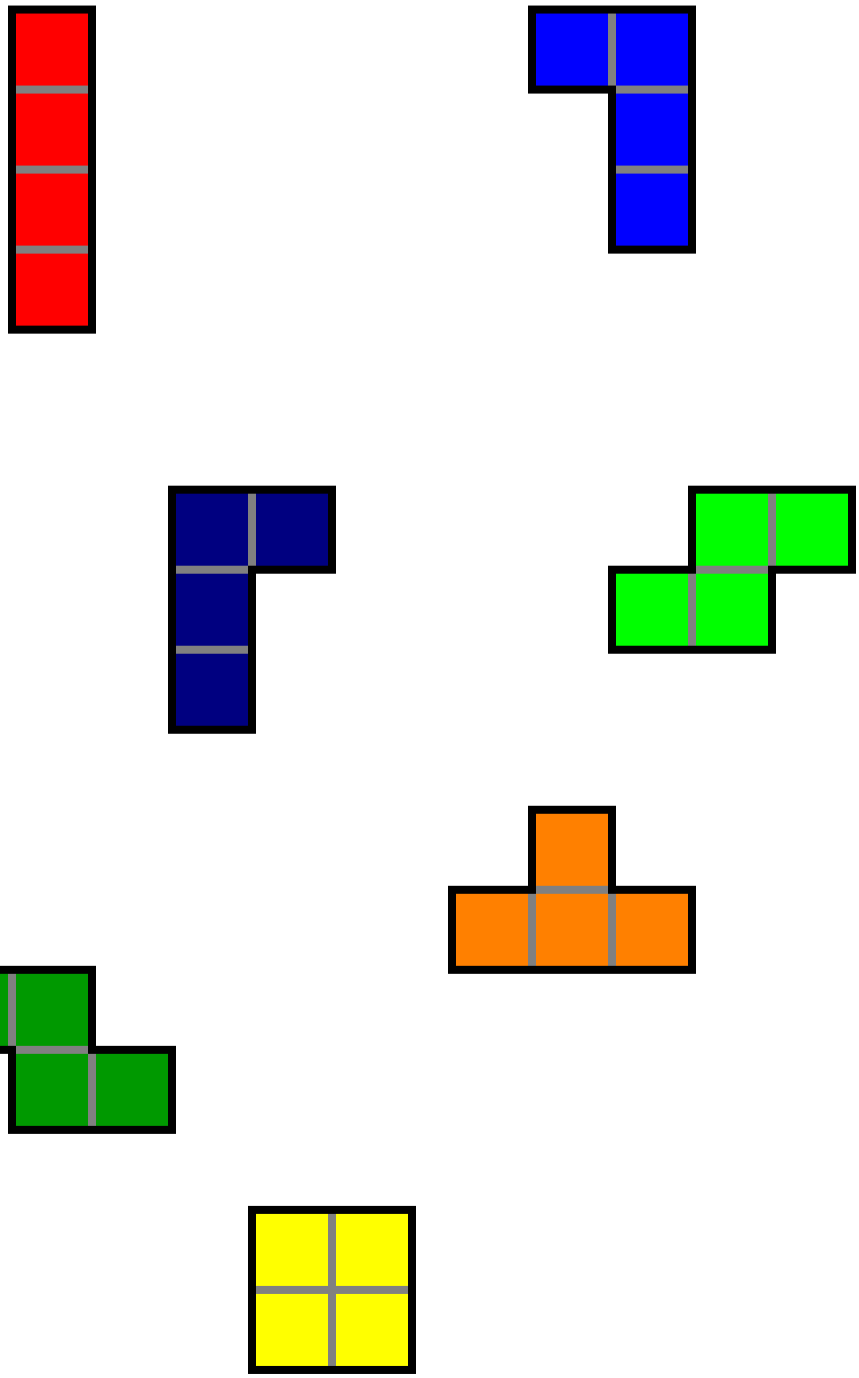


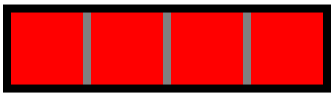


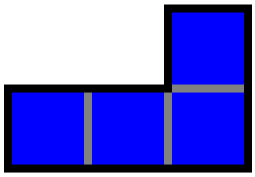


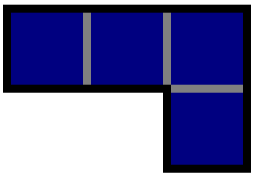


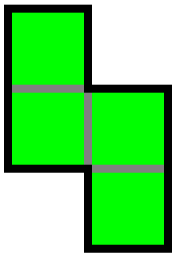


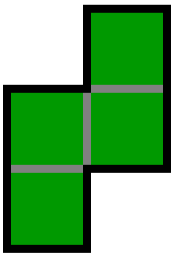


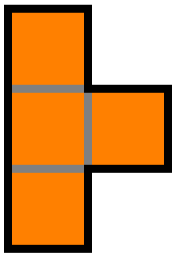


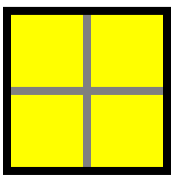






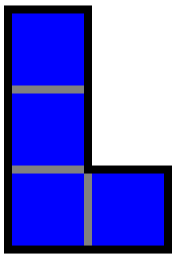




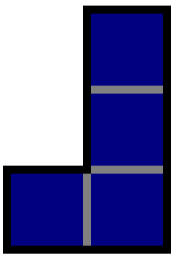


The player can chose to move the pieces left and right as they see fit.

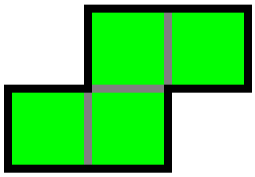
"I"



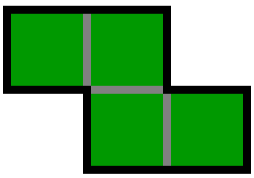
"L"



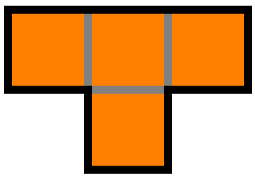
"J"



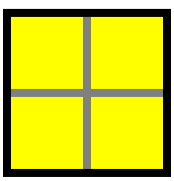
"S"



"Z"

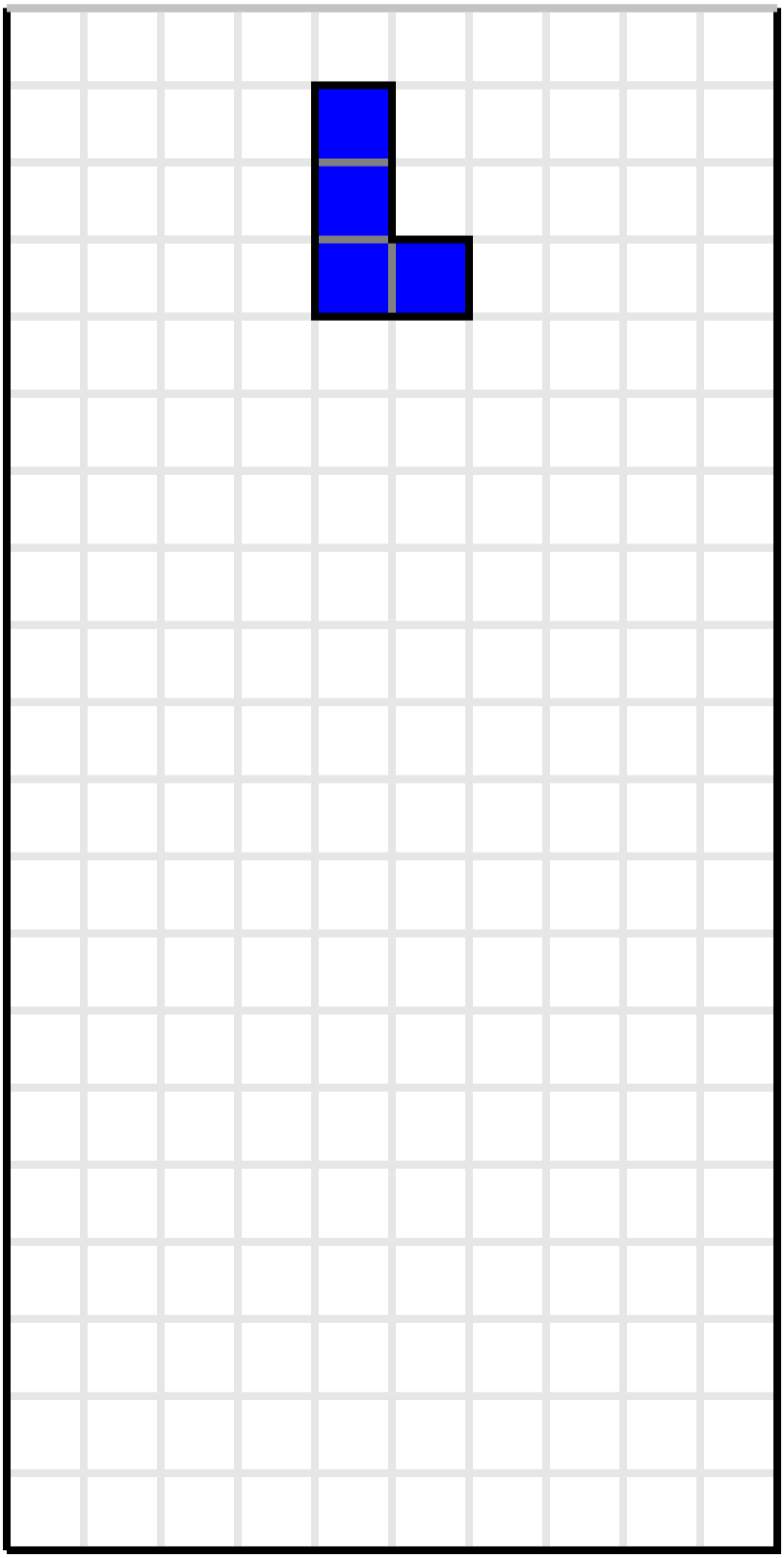


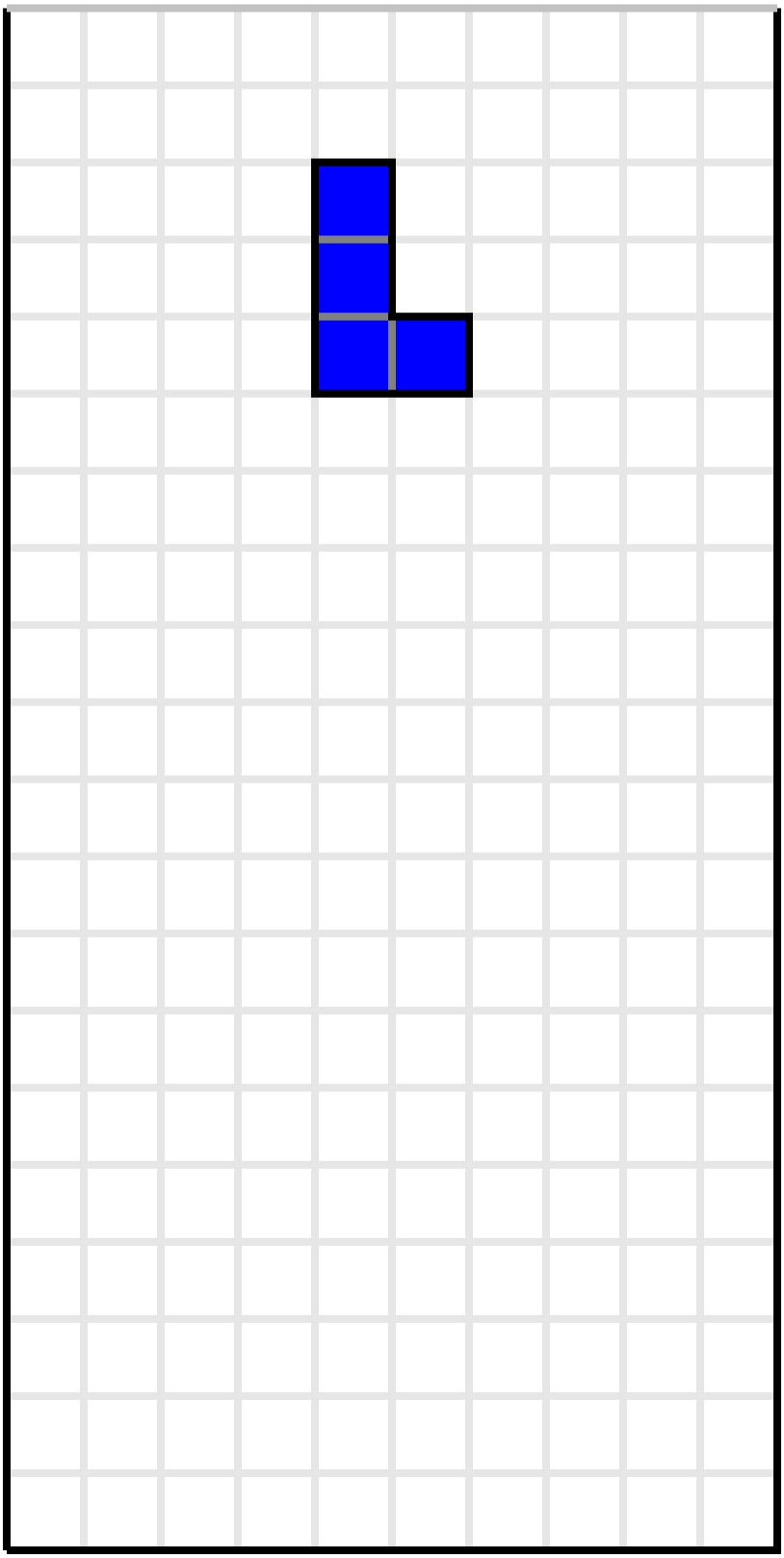
"T"

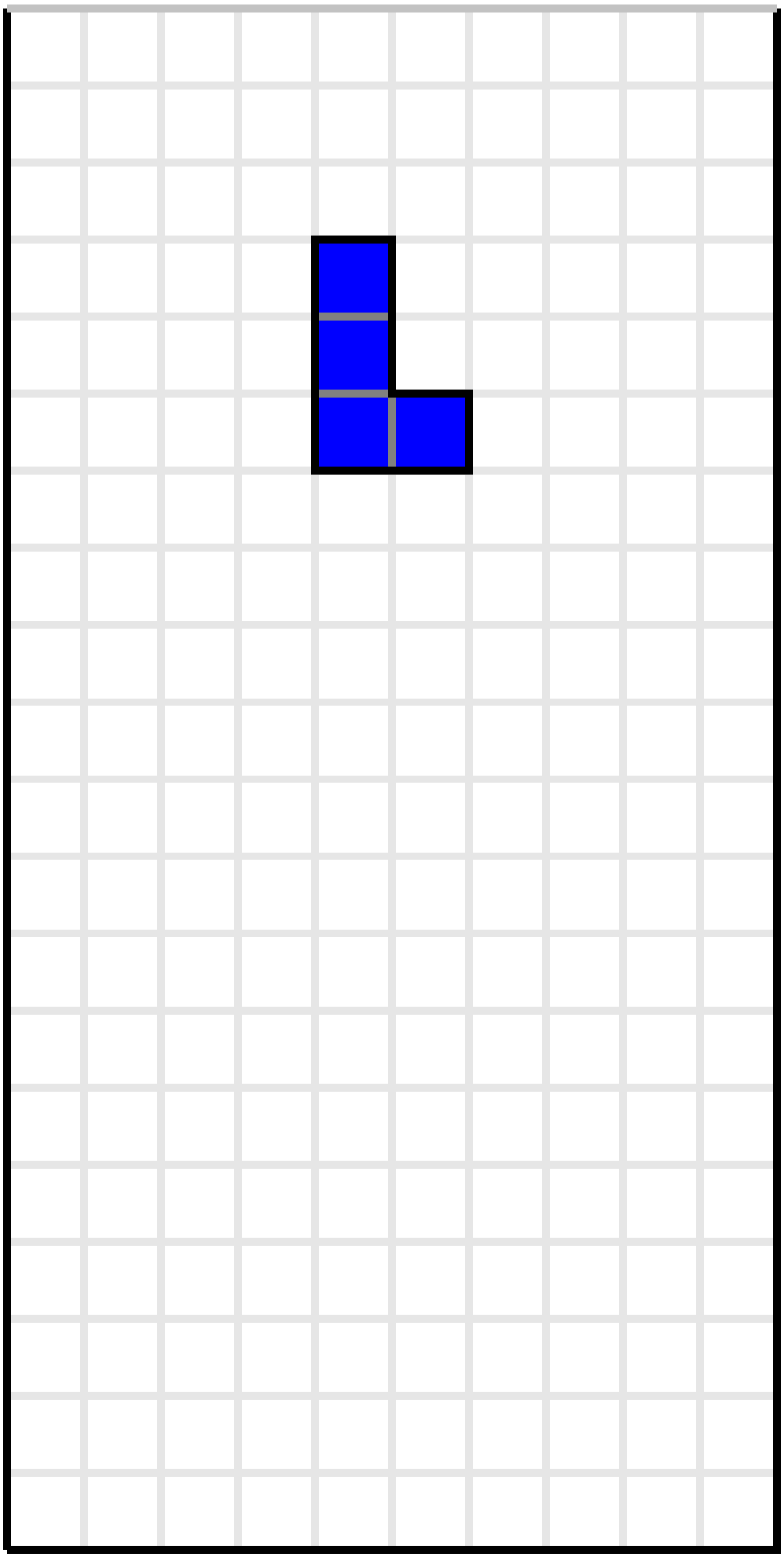


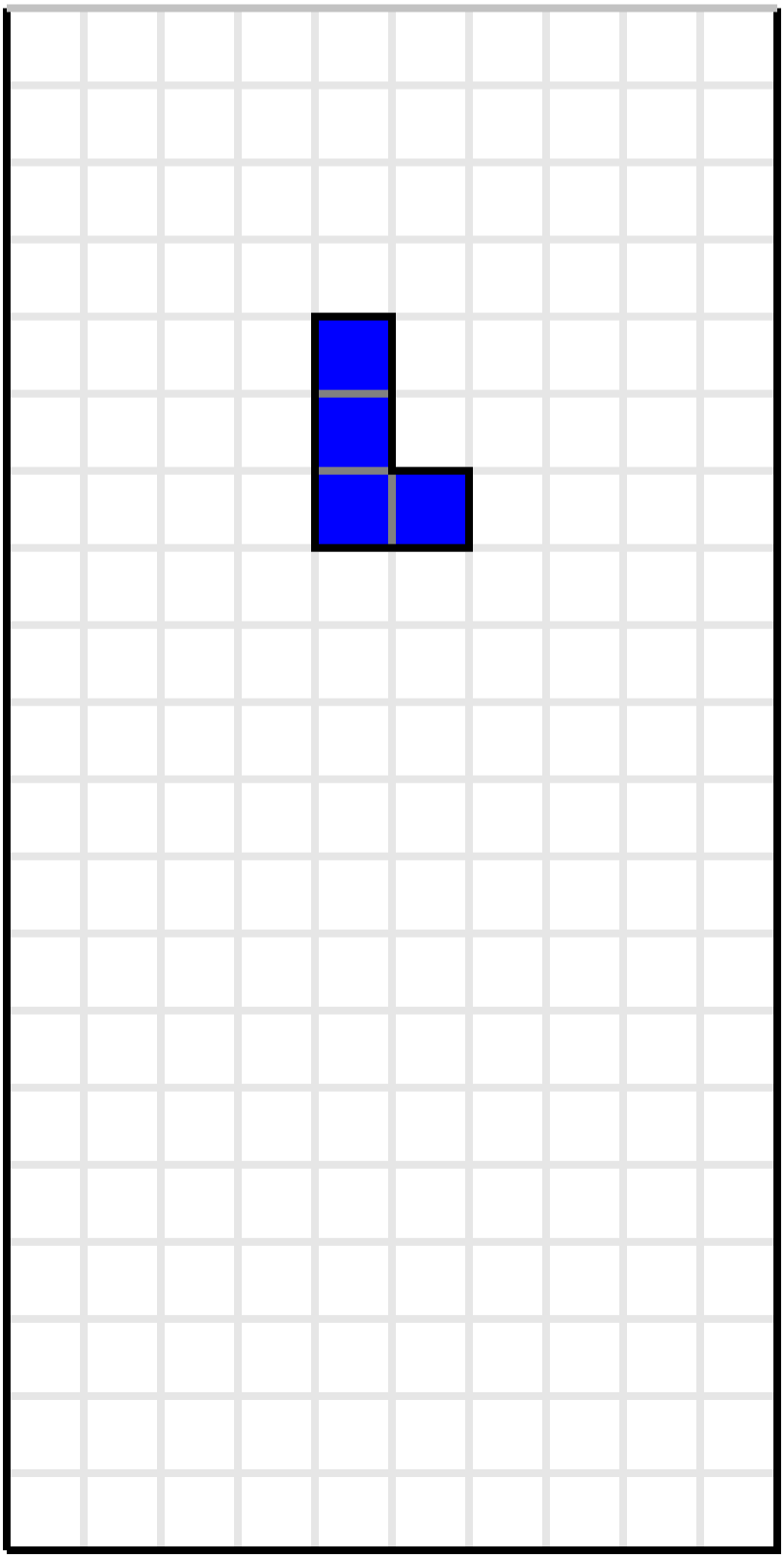
"O"

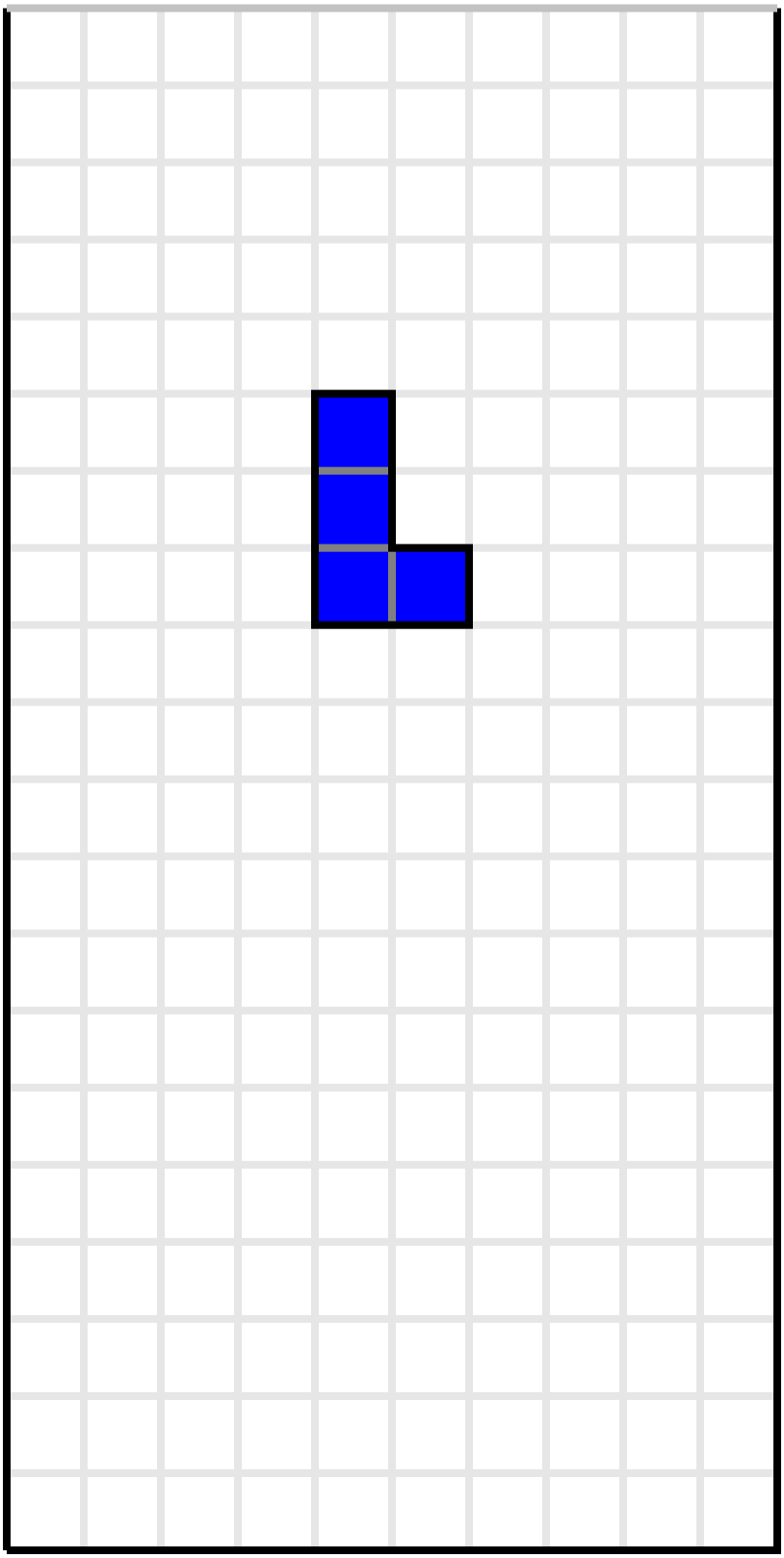
The player is given a piece at the top. The piece can be moved left or right by the player, and will slowly fall..

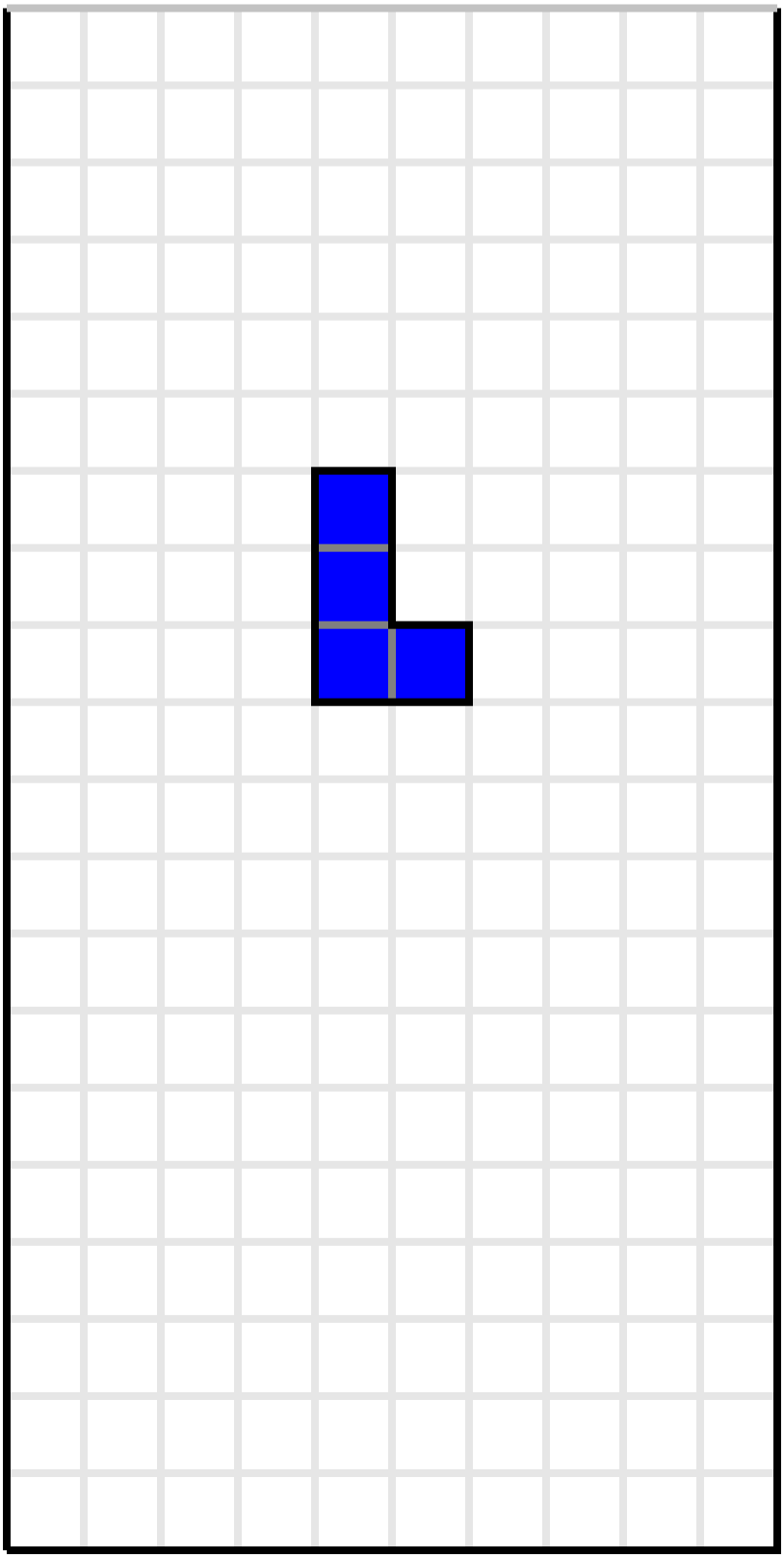


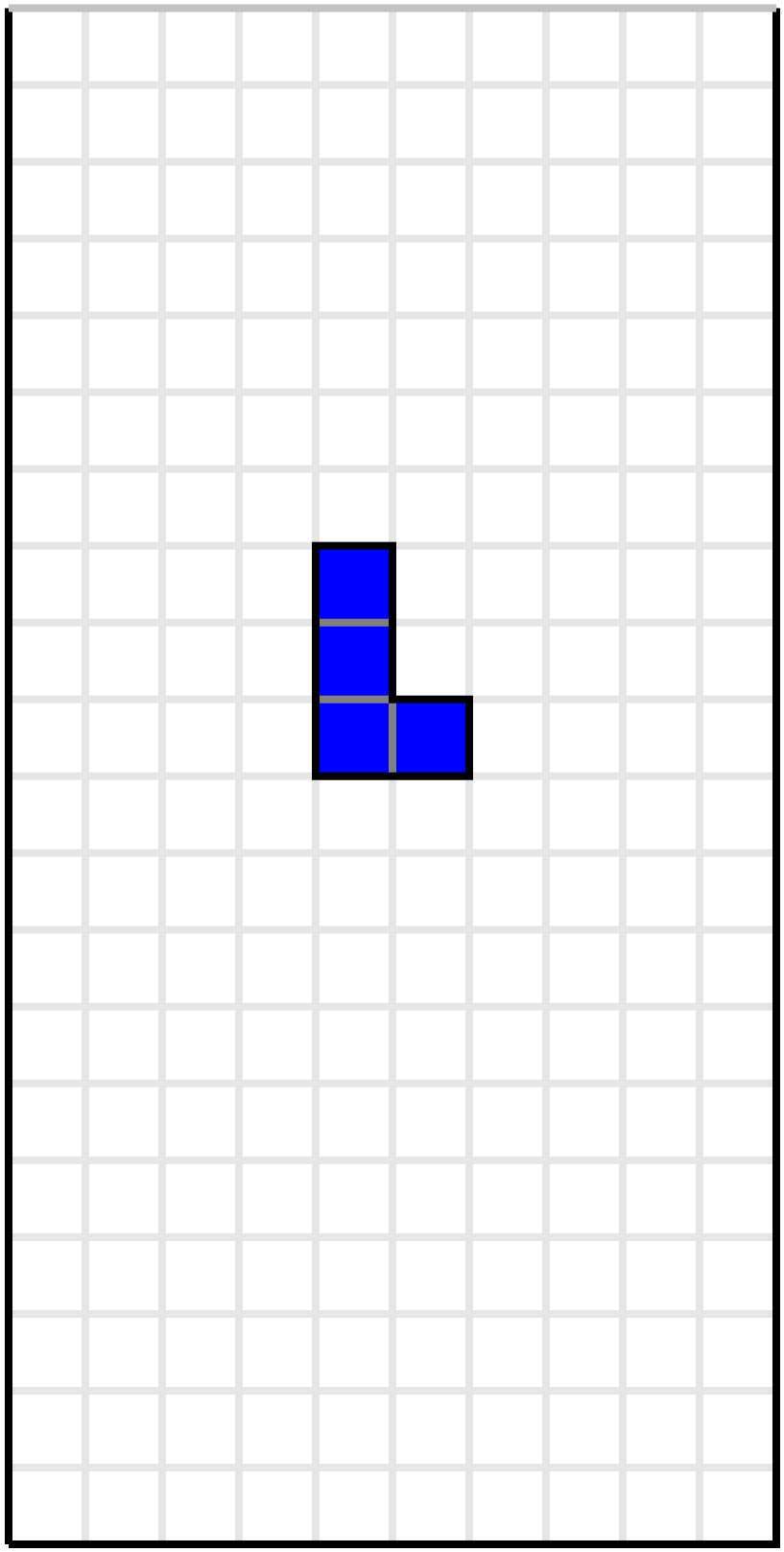


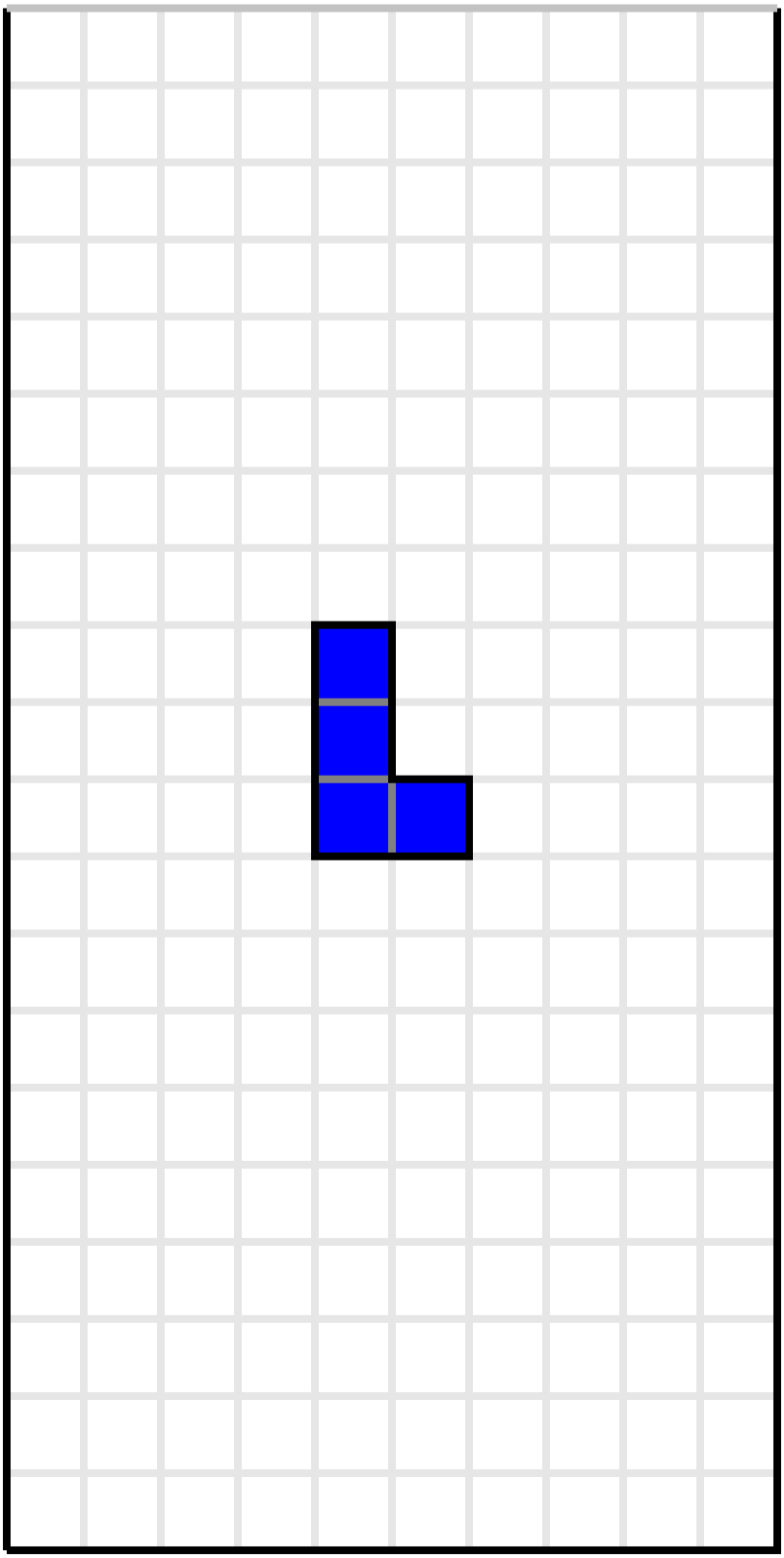


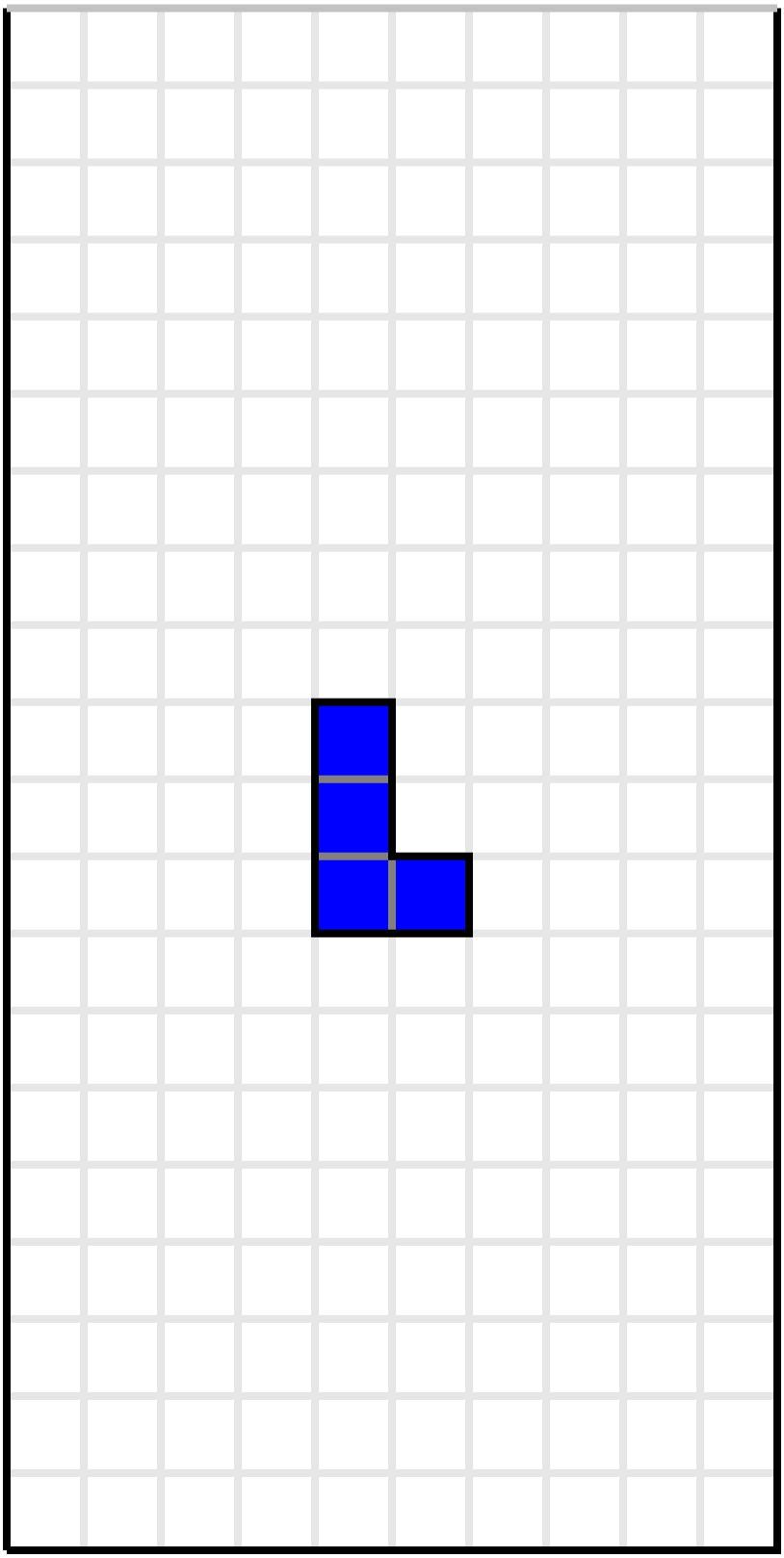


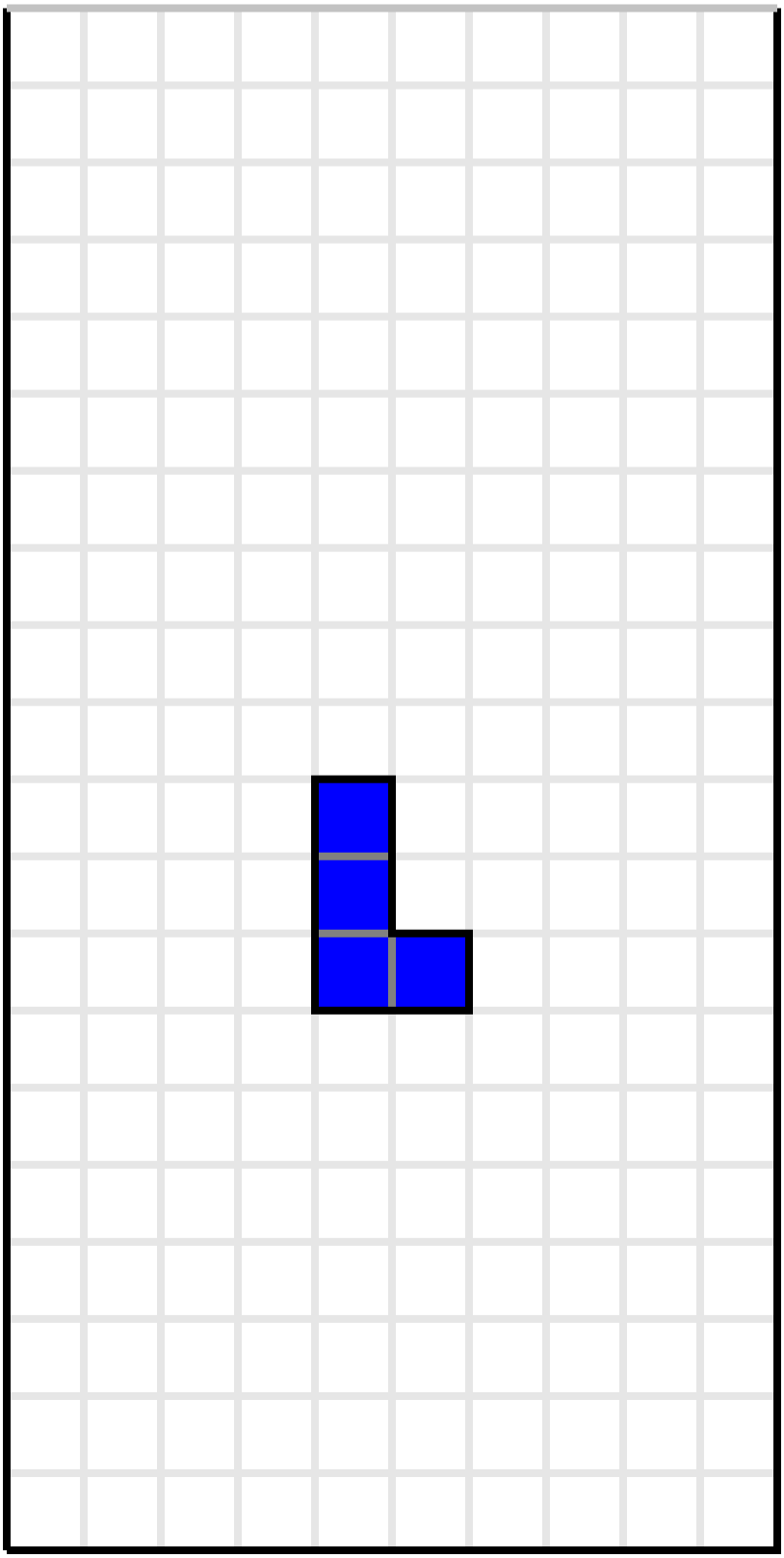


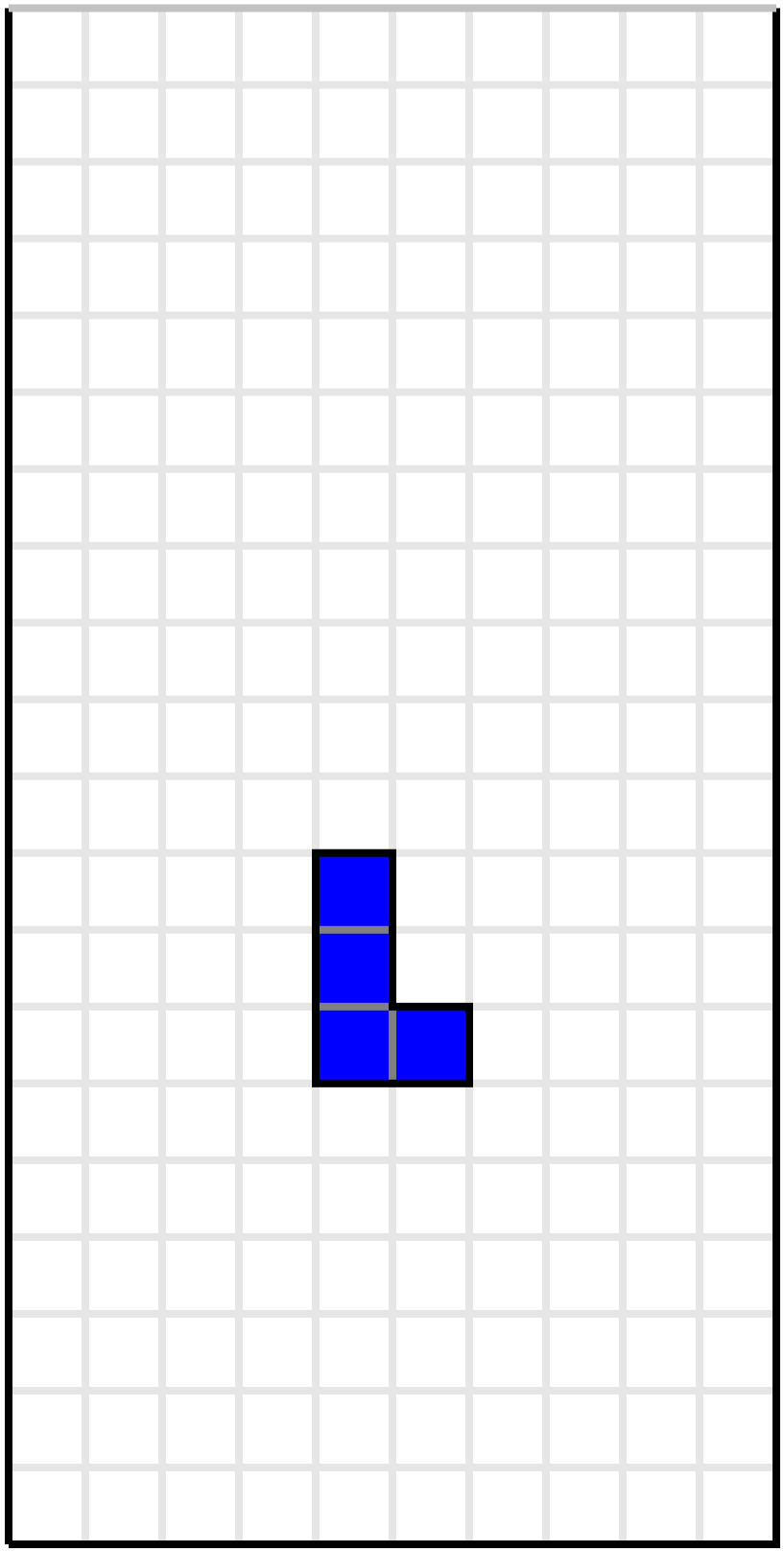


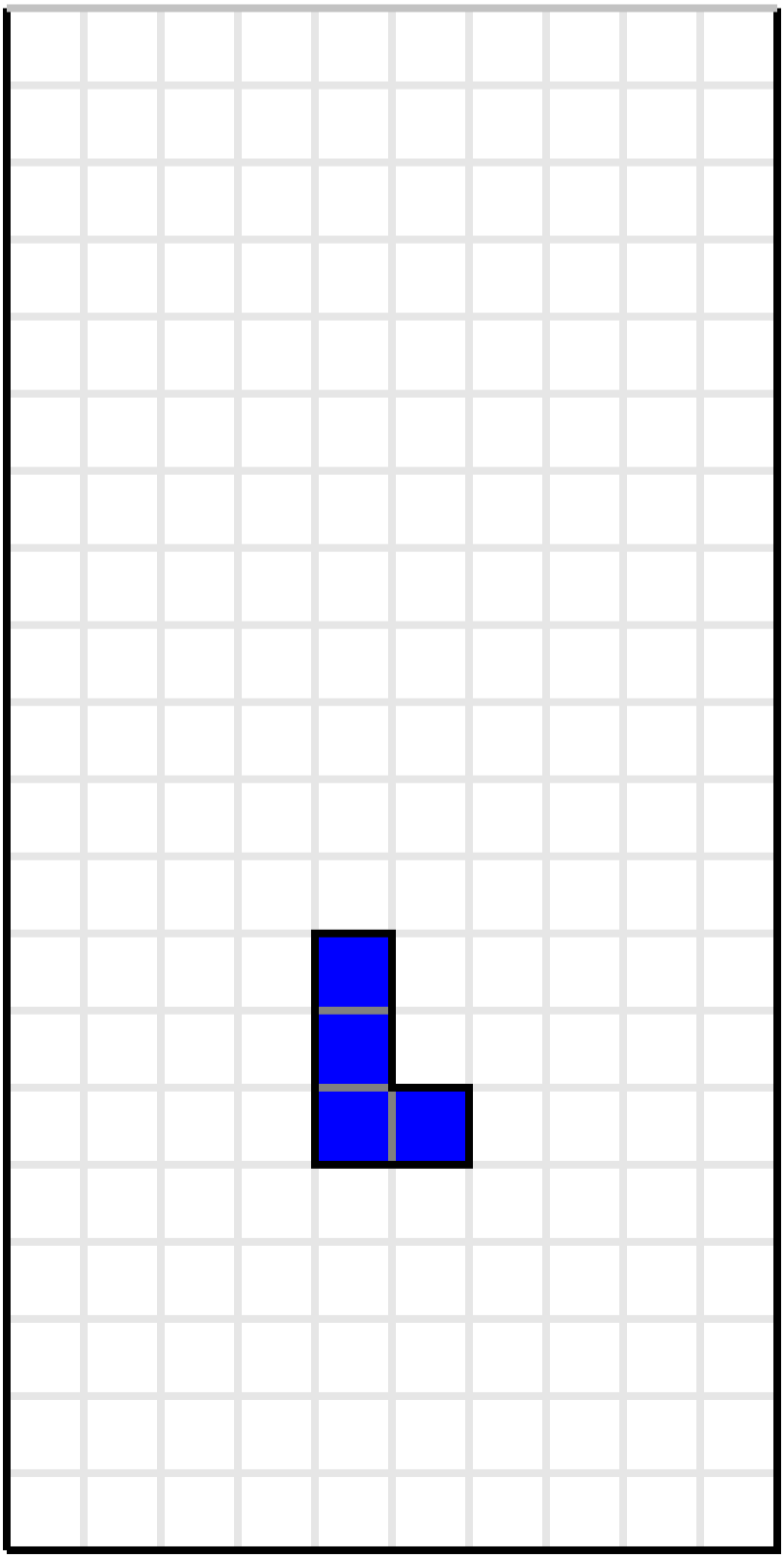


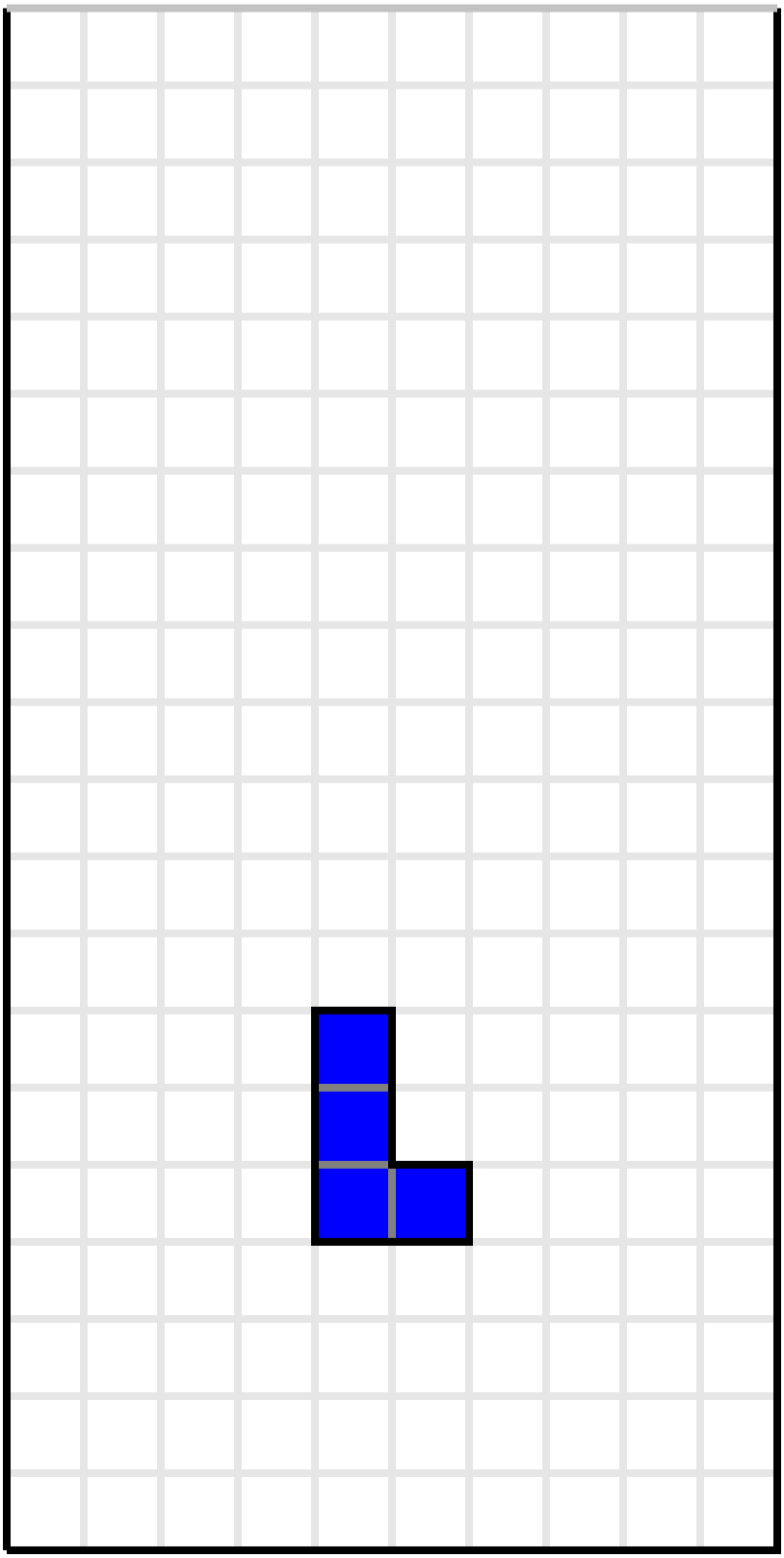


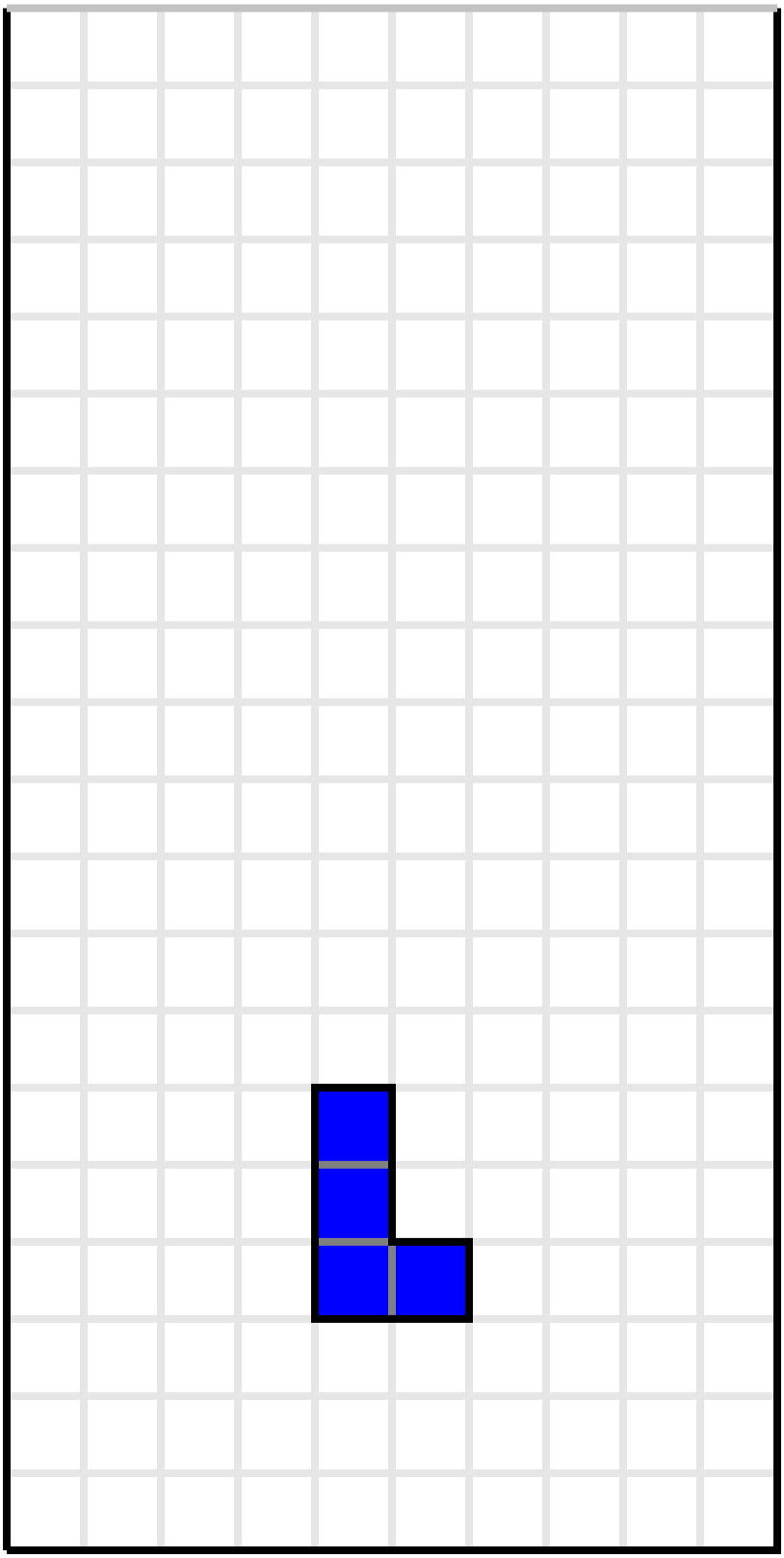


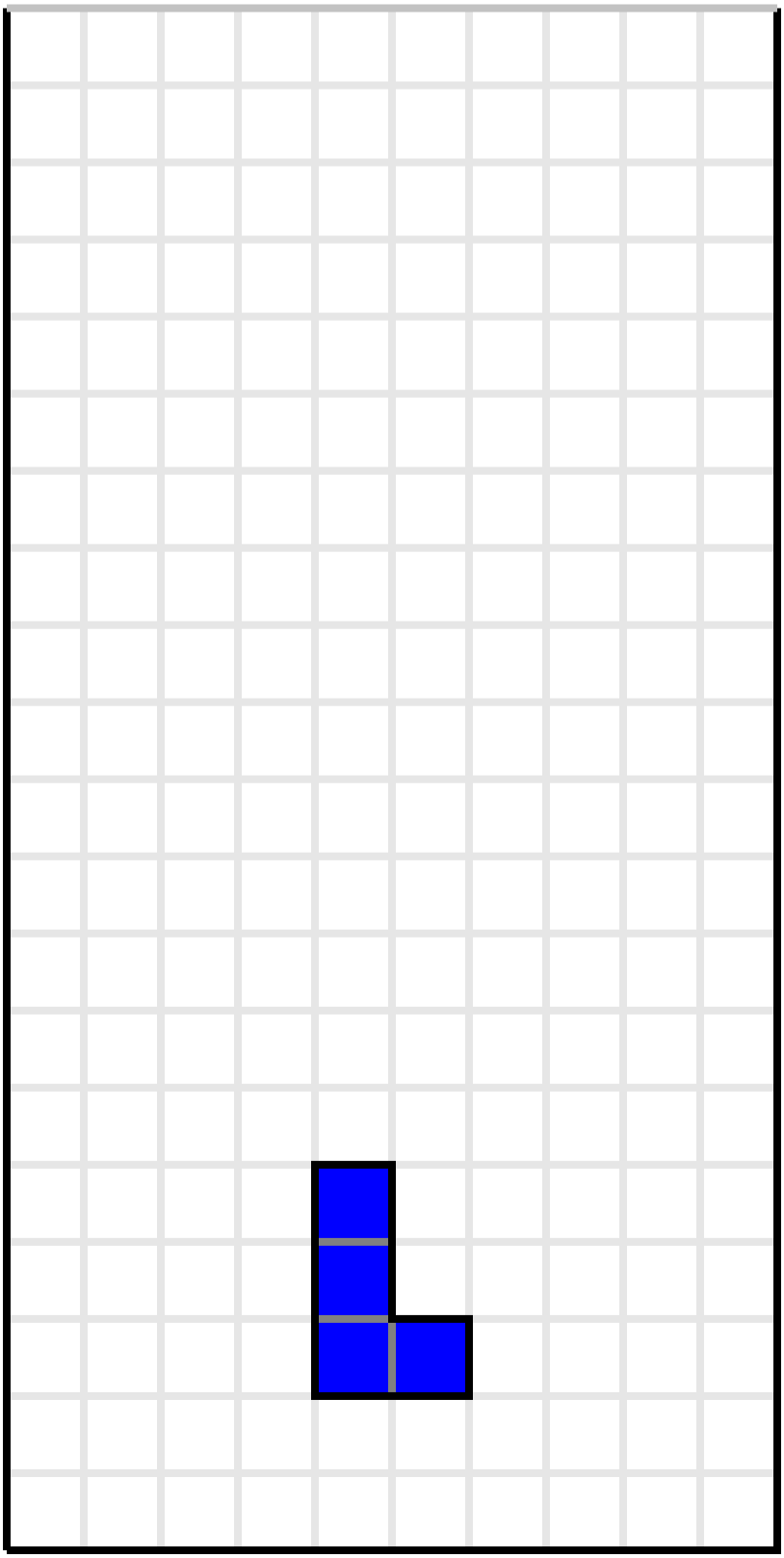


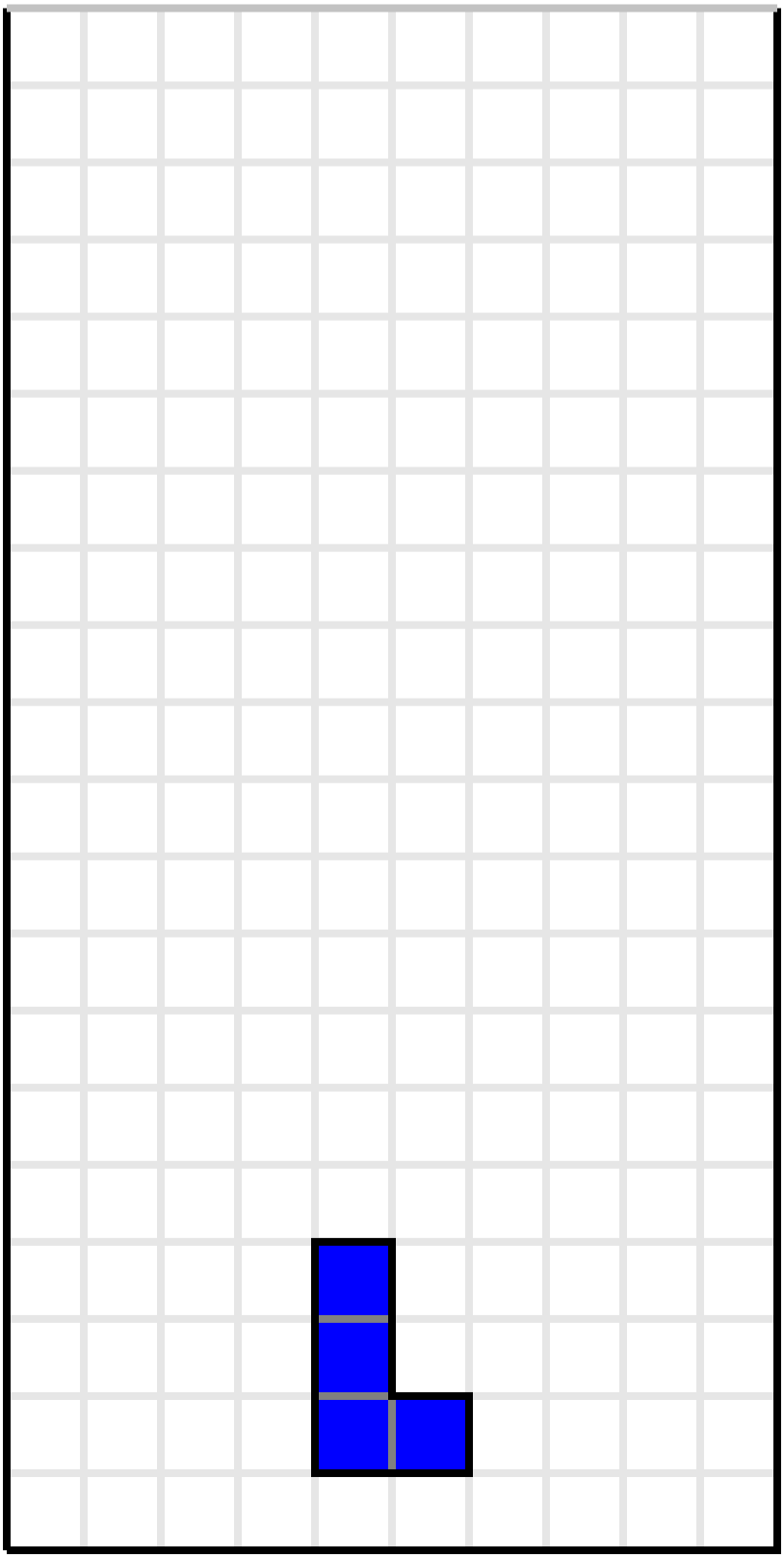






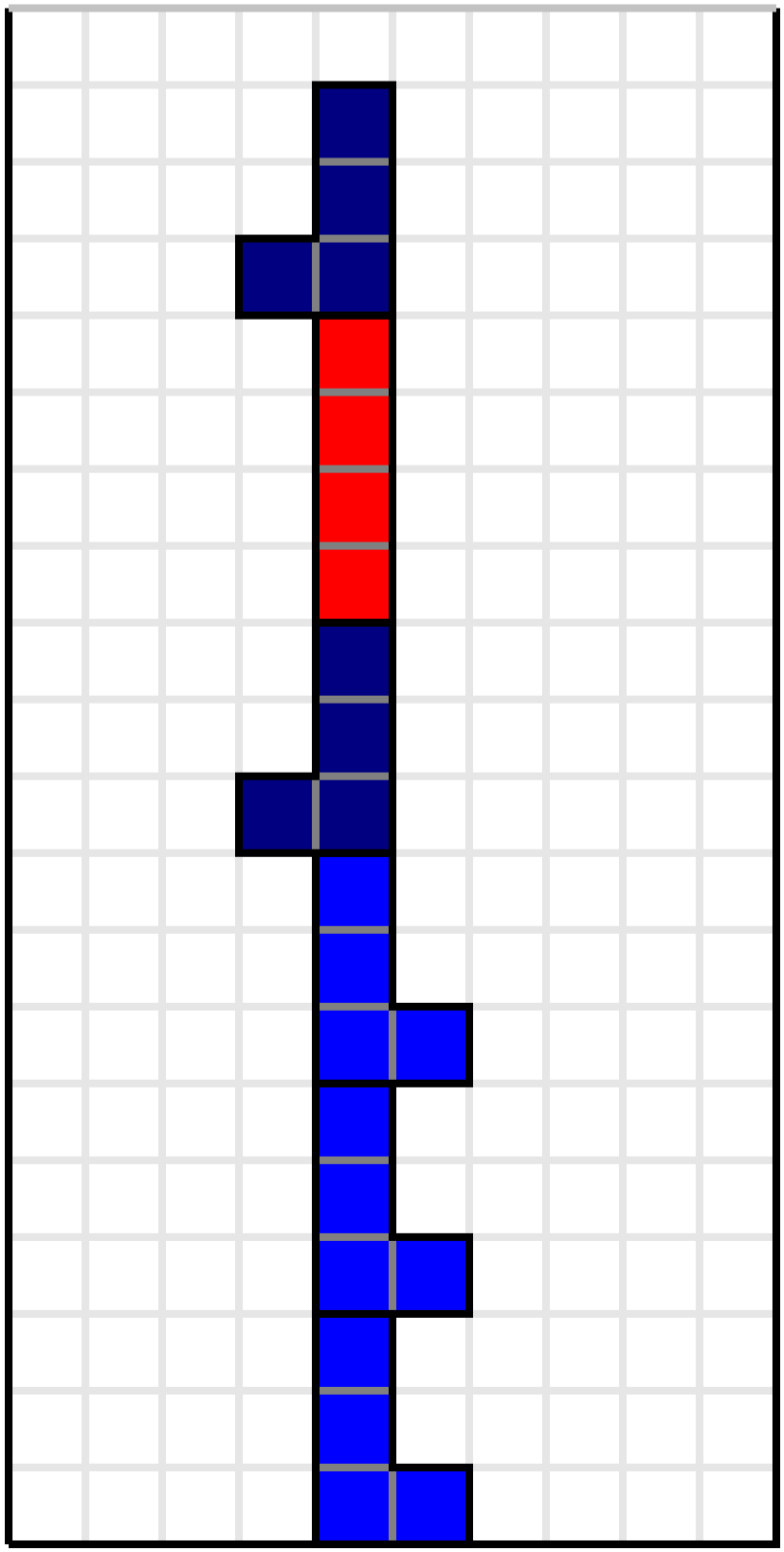




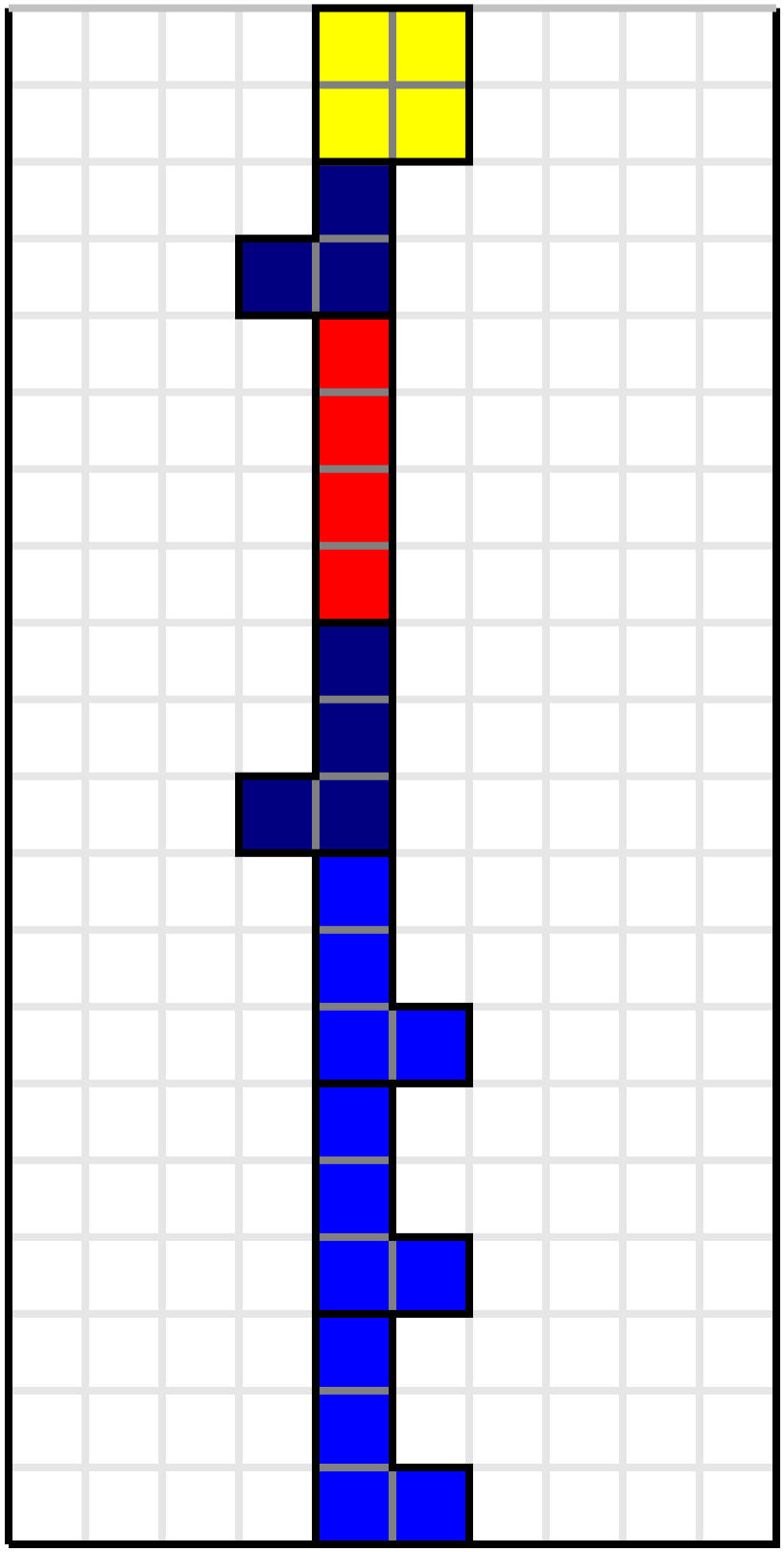


The piece will drop until the lowest edge comes into contact with the top of another piece, or the bottom of the grid.

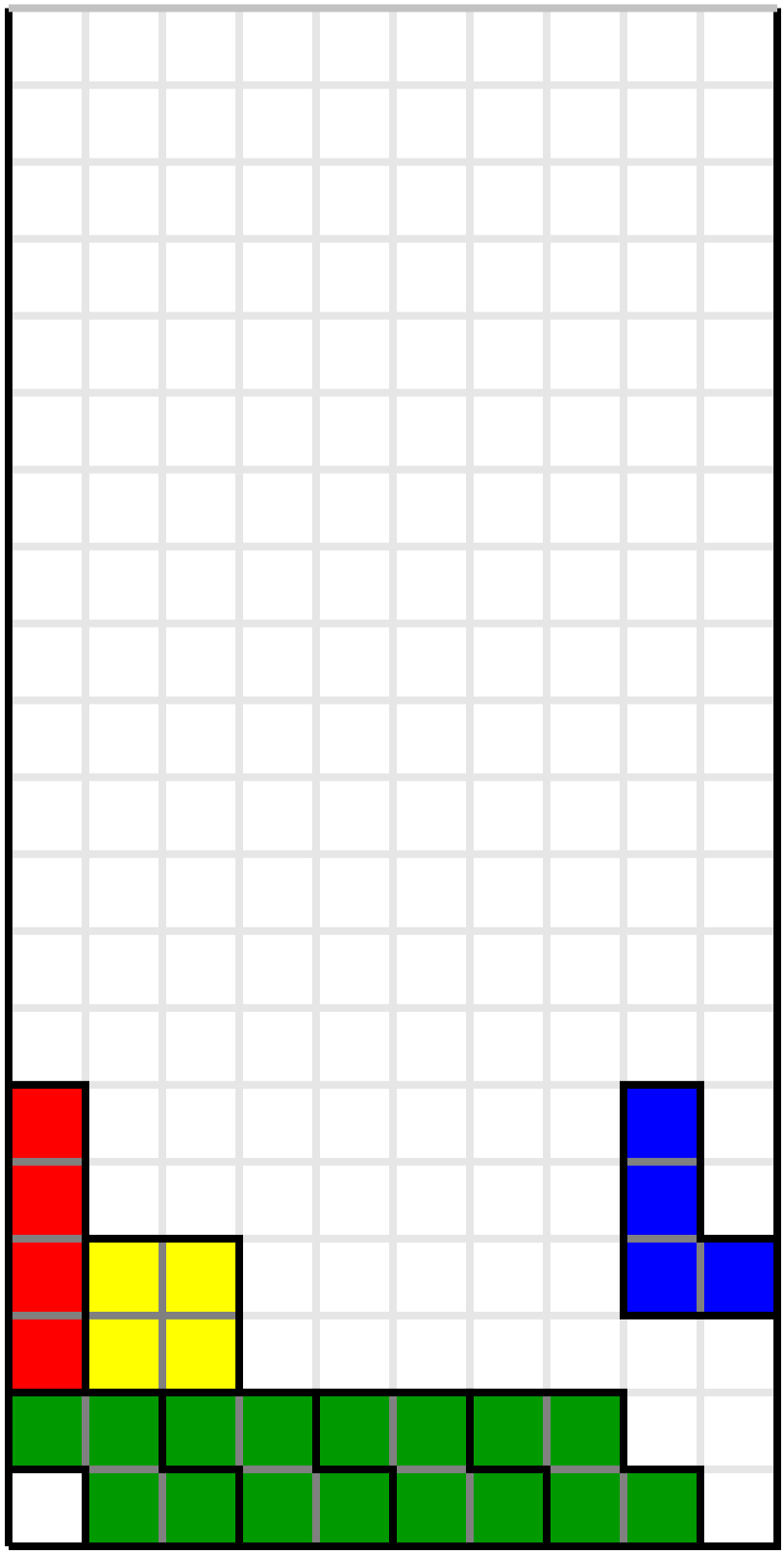
The player loses when the (n + 1) piece is blocked by the nth piece at the top of the grid.



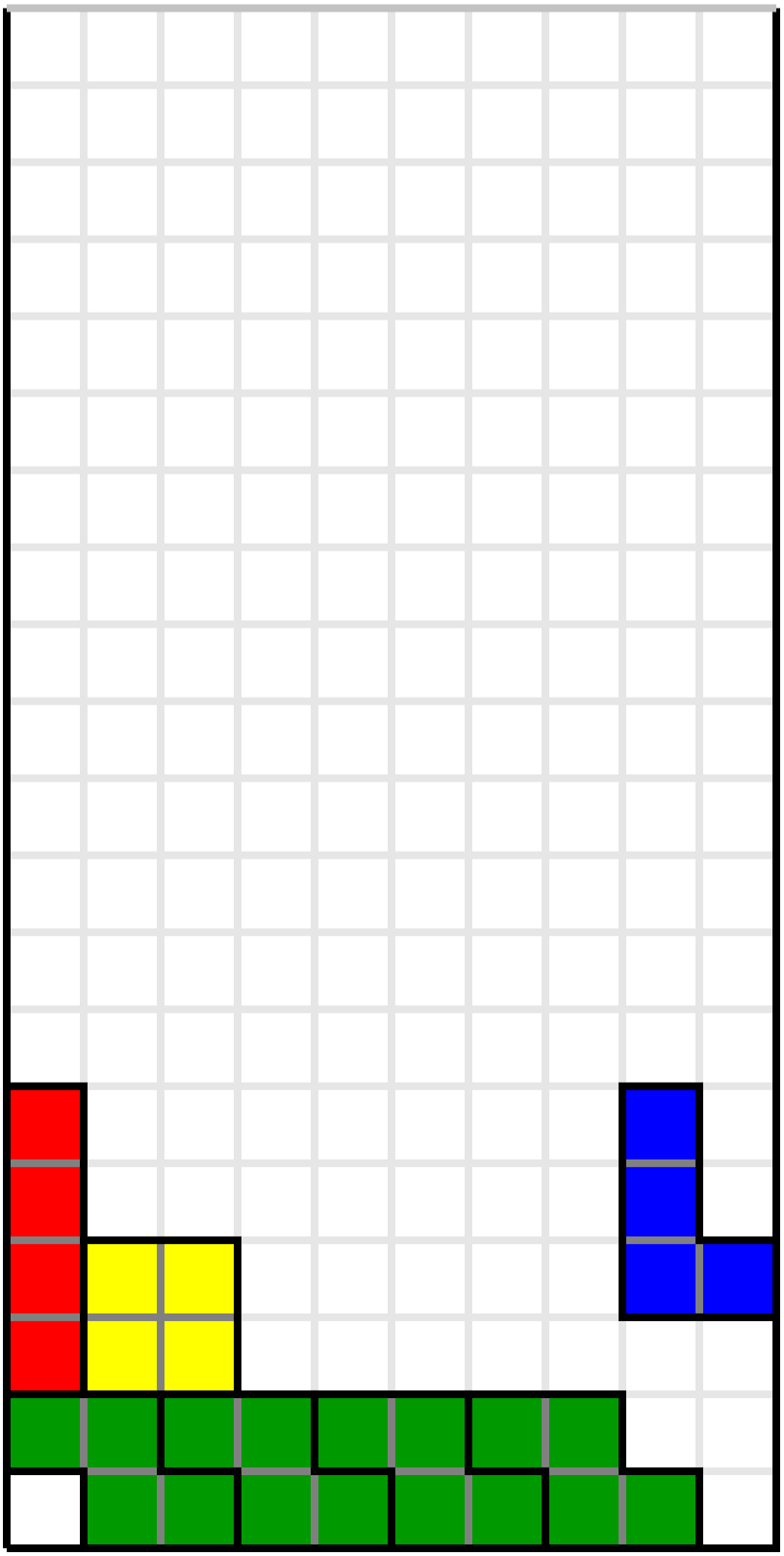
In this case when 7th piece "O" enters, the player cannot move it as its lower edge is already touching the 6th piece "J".



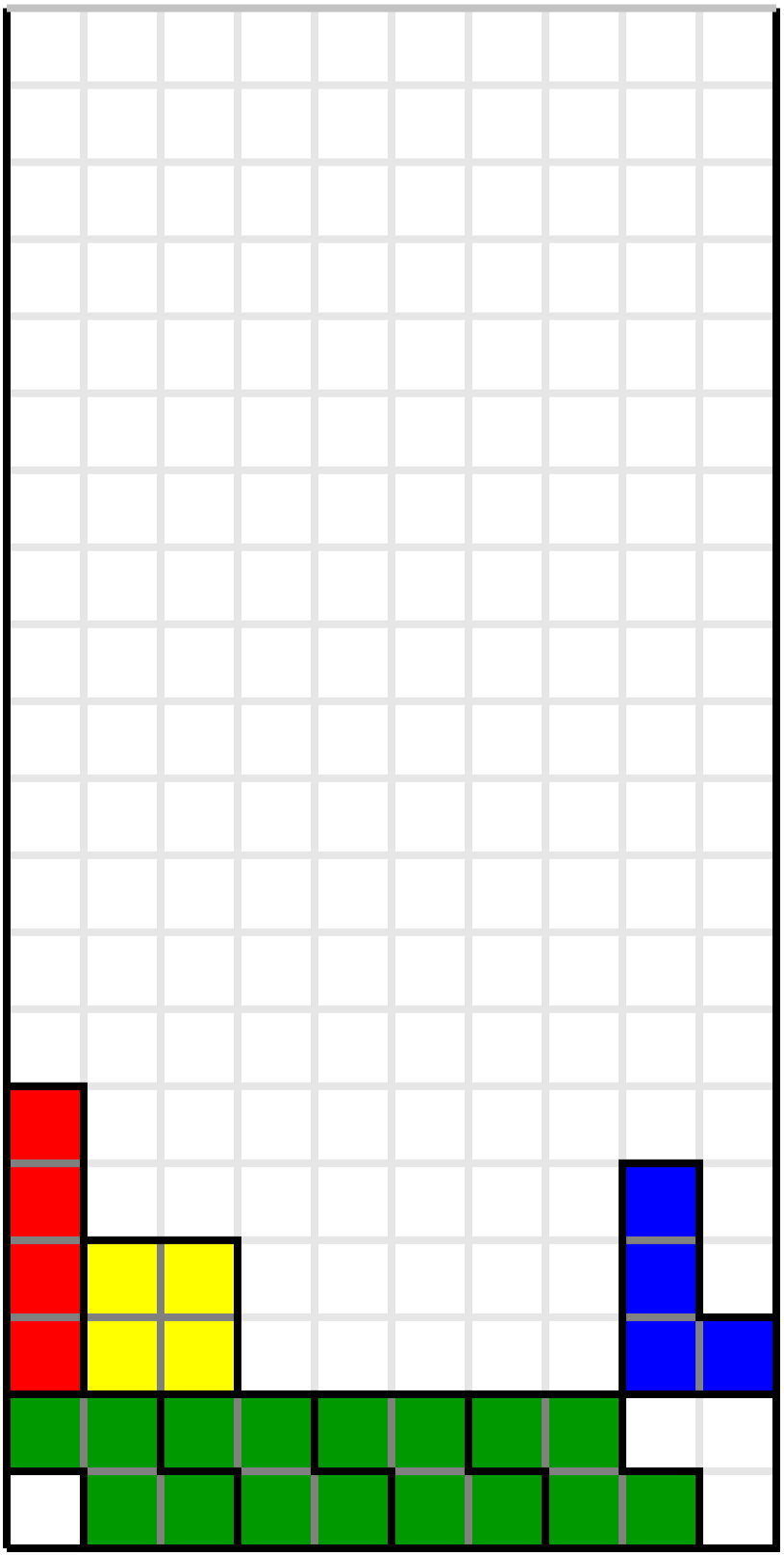
In order for the game to continue longer, the player must create complete rows. If the player completes the ith row, all the rows above the ith row would be lowered, and the ith row removed.



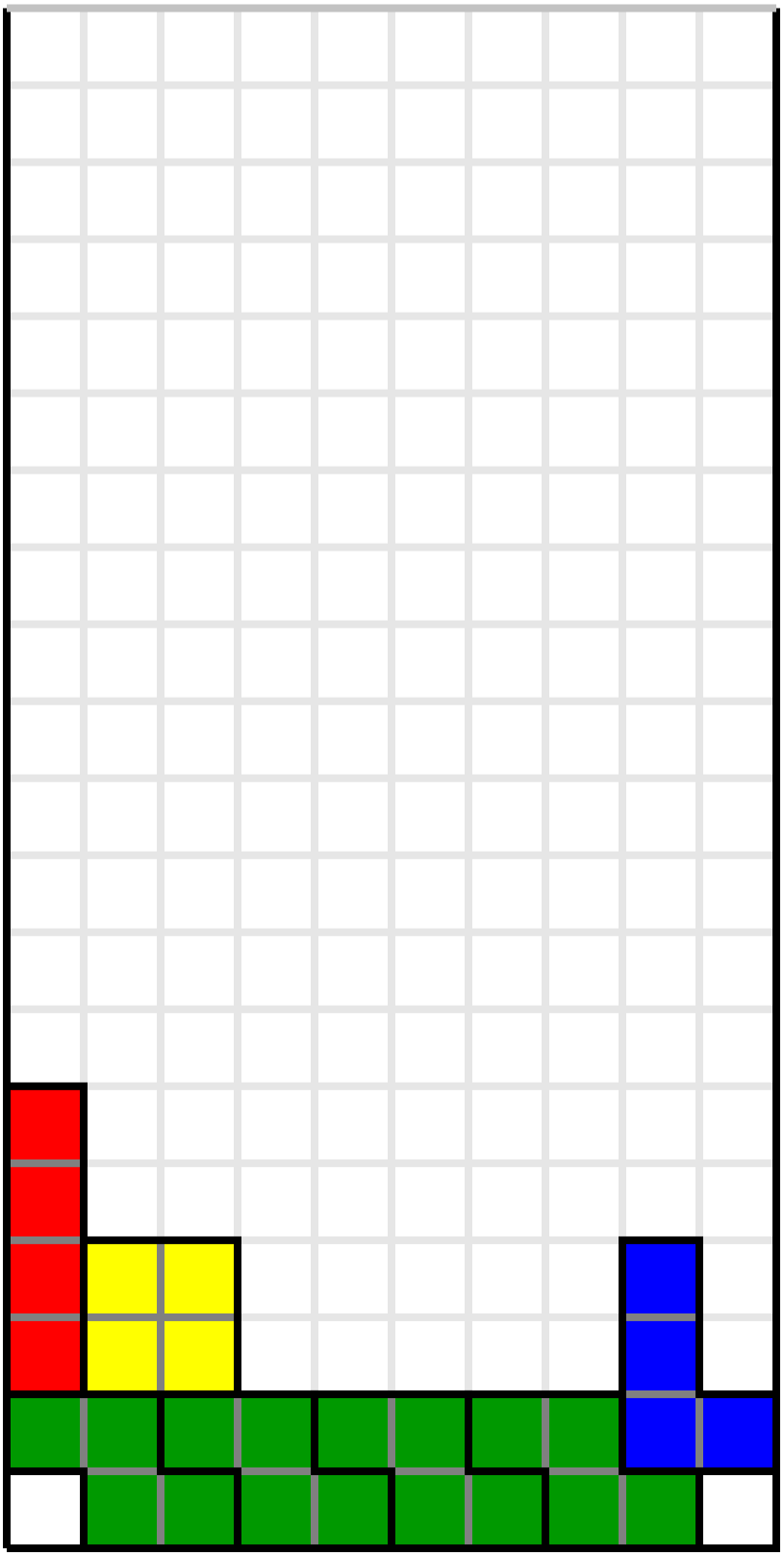
In this case, the 2nd row is removed and all the rows above are lowered by 1.
Note: the "I" does not fall into the space below.



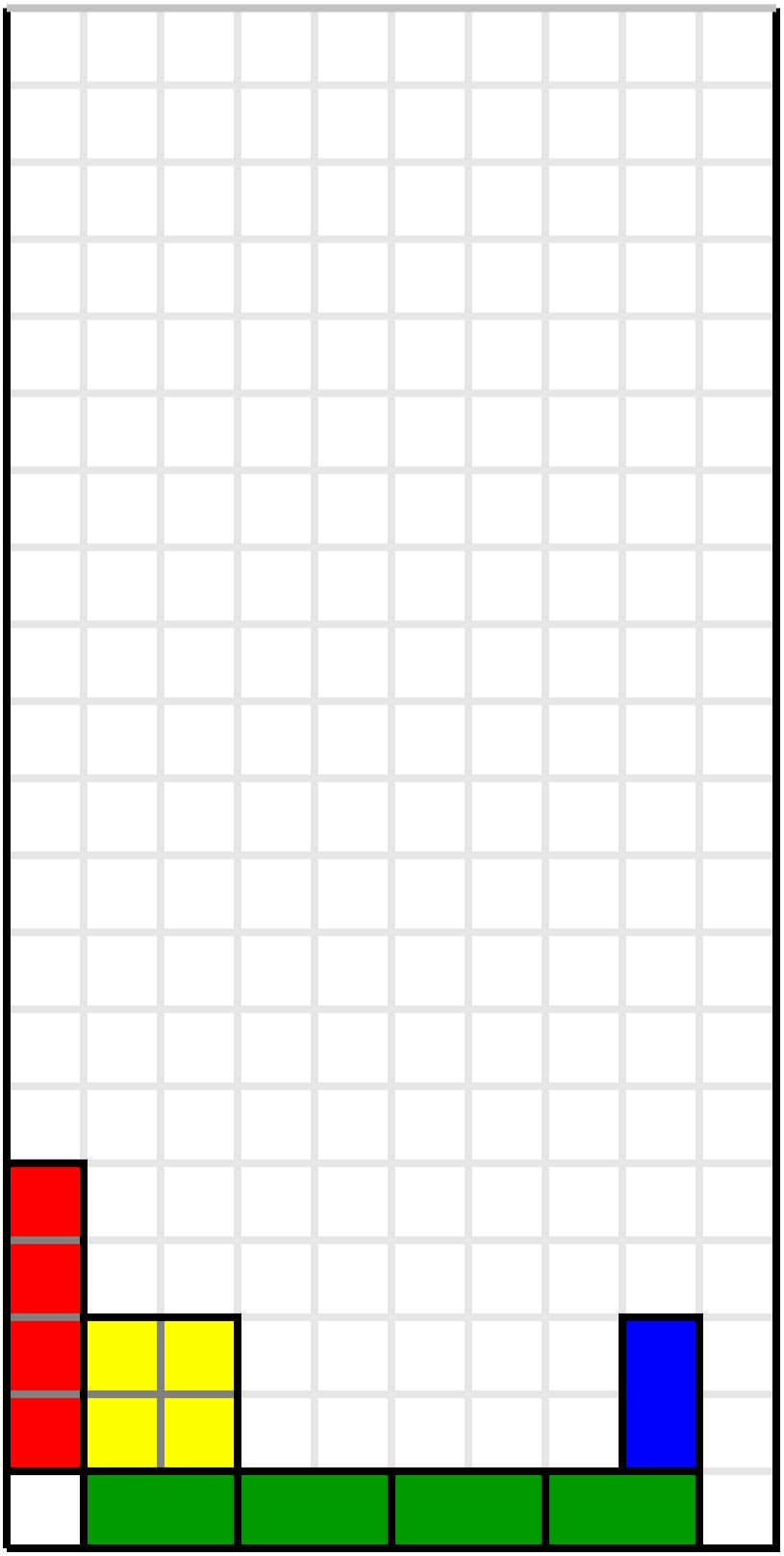
In this case, the 2nd row is removed and all the rows above are lowered by 1.
Note: the "I" does not fall into the space below.



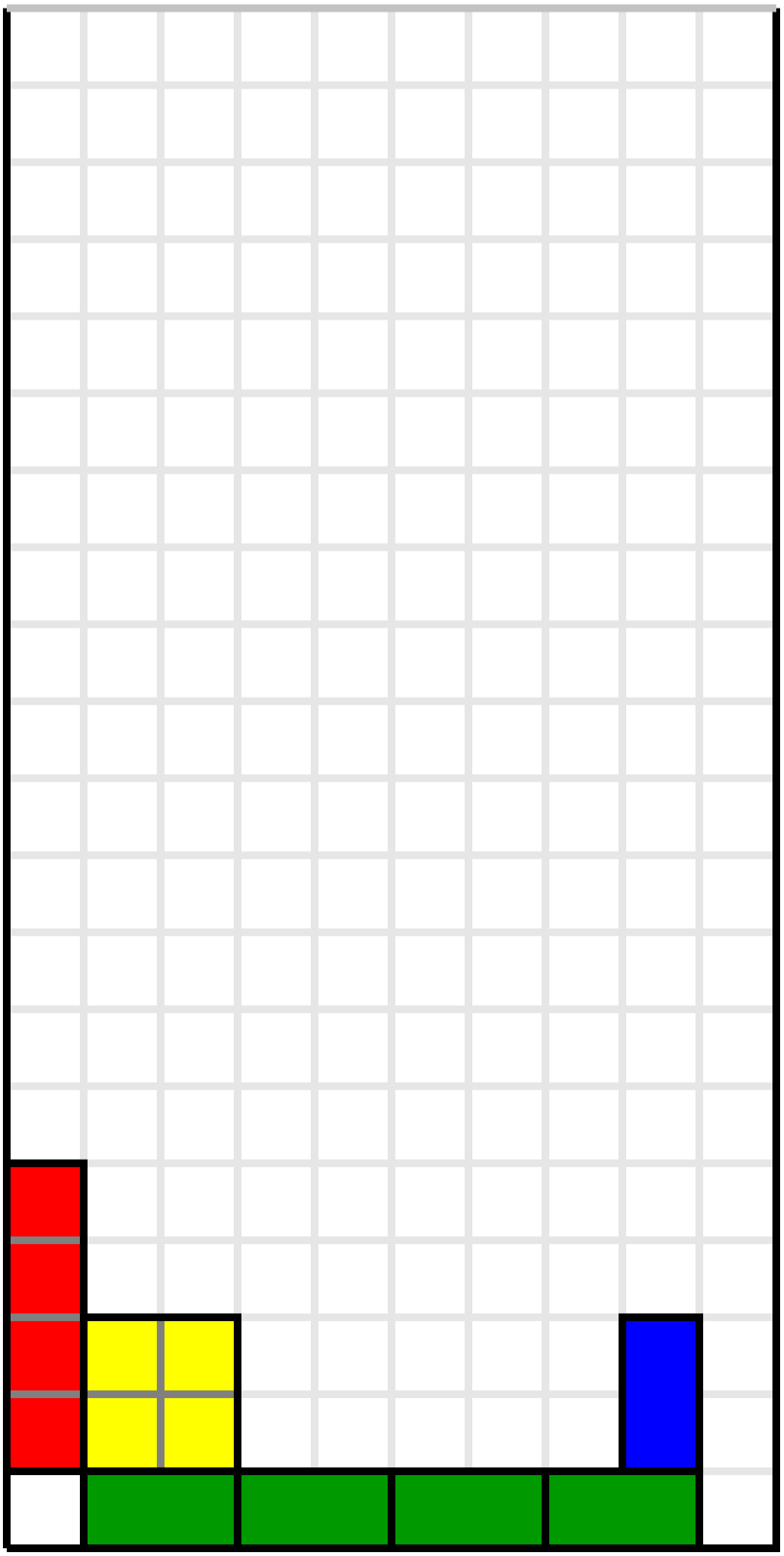
In this case, the 2nd row is removed and all the rows above are lowered by 1.
Note: the "I" does not fall into the space below.



In this case, the 2nd row is removed and all the rows above are lowered by 1.
Note: the "I" does not fall into the space below.



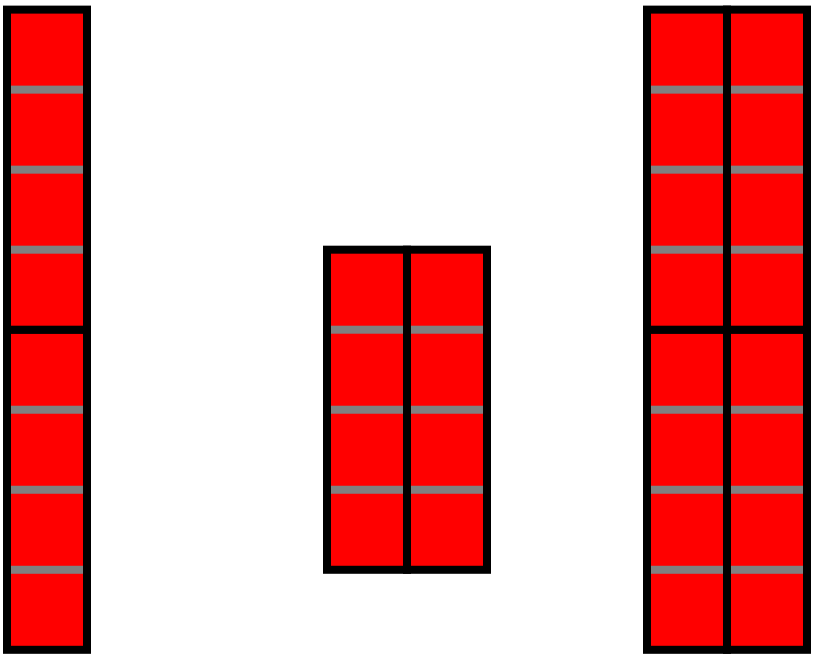
In order to remove the first row, we must fill the hole in the first row, first column. This can only be accomplished by removing the 2nd to 5th rows above, so that a piece can reach the hole.



For the player to maximize the time playing, they limit the holes like the previous page.
To understand how we can do this, we can look at the interaction between pieces.

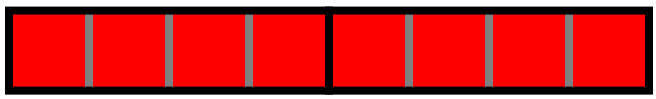
We will start looking at how pieces interact with itself.

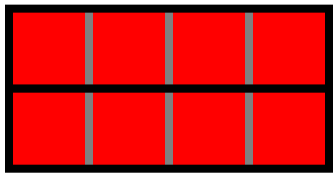
The "I" piece is a very simple piece.
It is easy to see that without any rotation, we can stack them beside each other, or on top of each other or both.

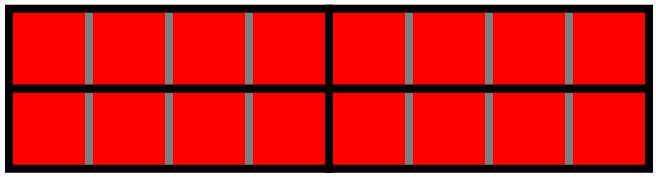


We can see that if the pieces are used in this way, there would be no holes left over.

We can also rotate the "I" piece and see that there will also be no holes left over.
Note that rotating again would give the same results as not rotating.

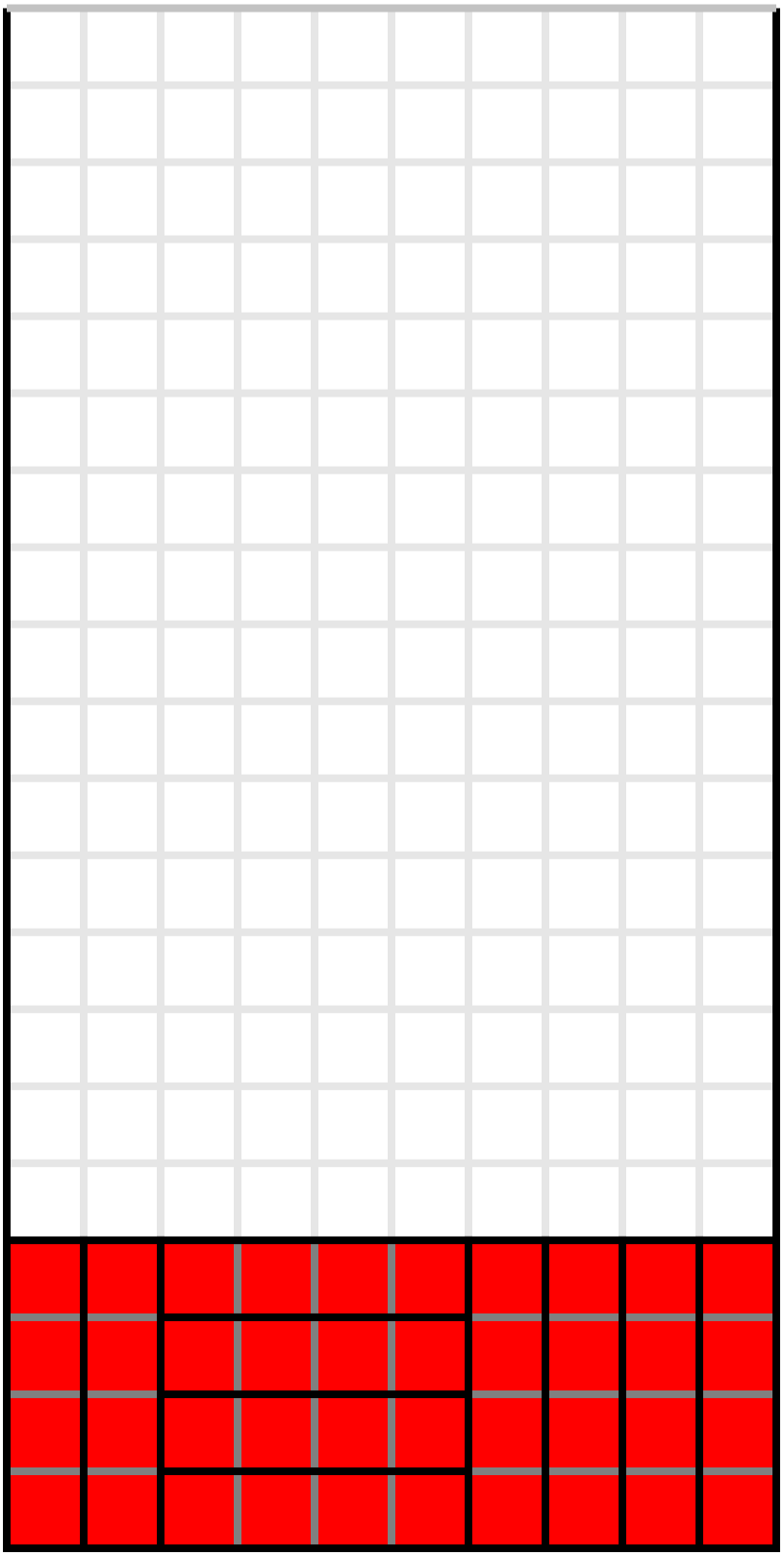






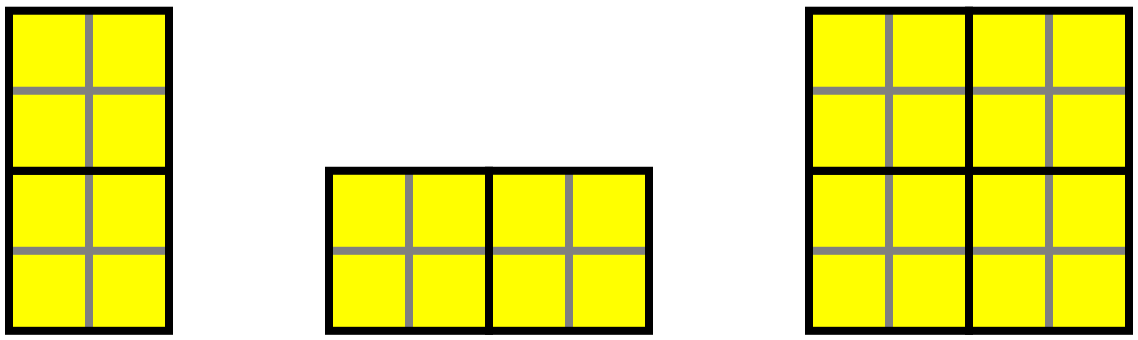
However...

Because the base of the grid is not evenly divisible by 4 (10/4 = 2.5), we must use a combination of rotated and non-rotated "I"'s to evenly fill up a row (or 4 rows in this case).



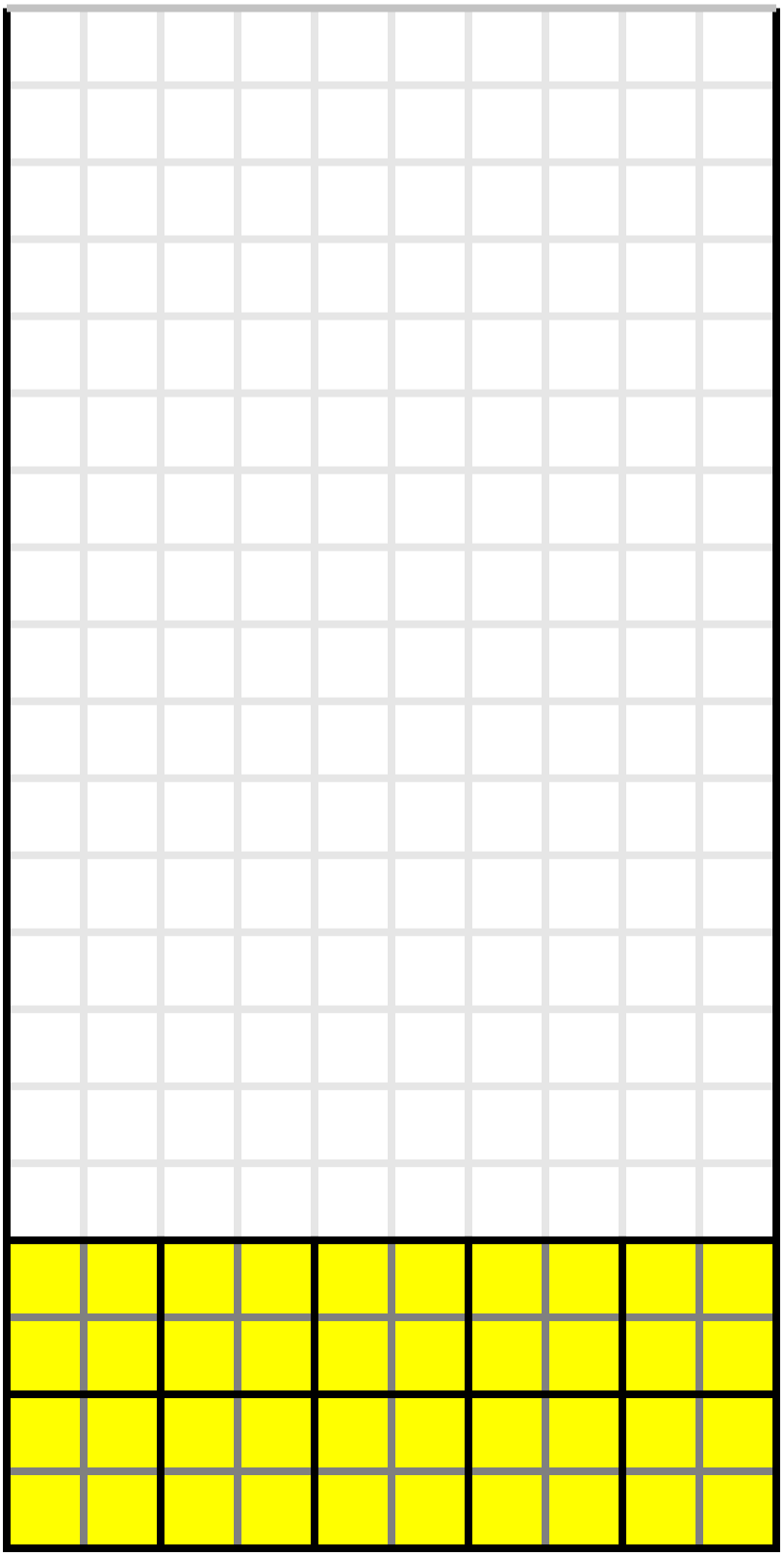
Since 4 rotated "I"'s stacked on each other is the same as 4 "I"'s beside each other, we can use 10 non-rotated "I"'s instead to fill up 4 rows in the grid.

The "O" piece is also very simple.
Because even when the "O" piece is rotated it does not change shape.

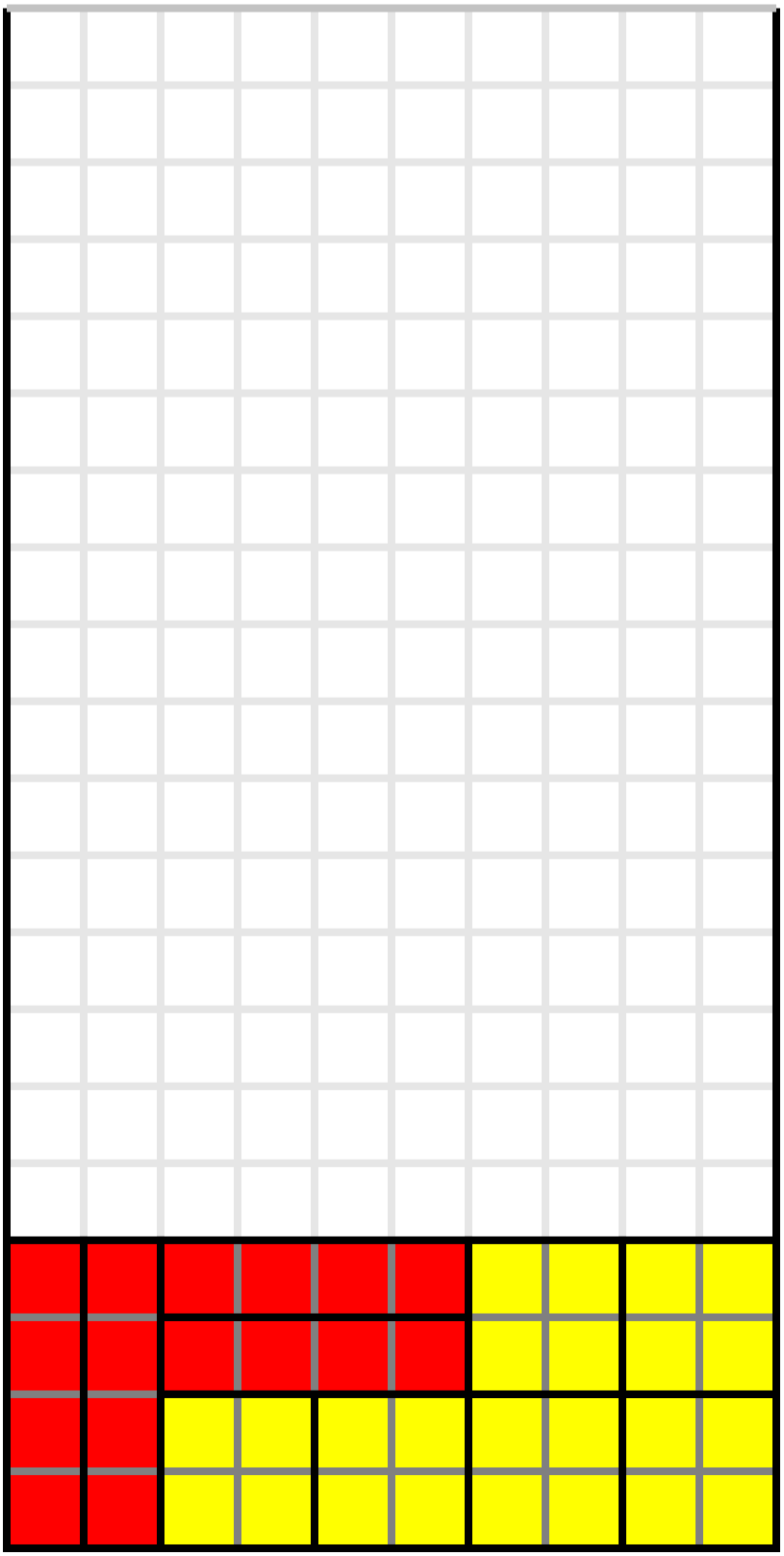


We can see that if the pieces are used in this way, there would be no holes left over.

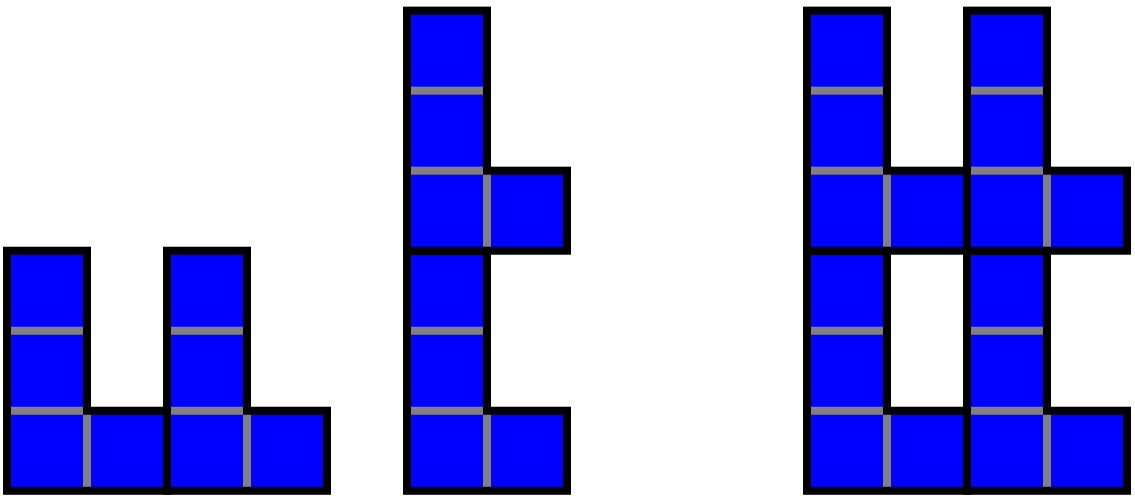
Since 2 evenly divides 10 (10/2 = 5) we can see that 10 "O"'s can be placed so that 4 rows are evenly filled, with no holes.



We can also see that 2 "O"s give us a 2x4 set same as two "I"'s. Thus, the 2 "O"'s and 2 "I"'s can be used interchangeably.

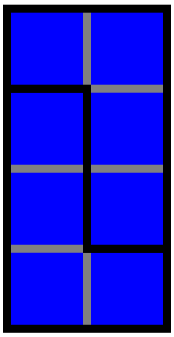


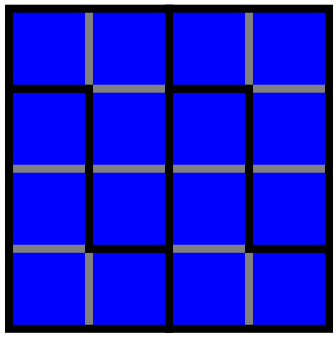
The "L" piece does not work as well as the "O" or "I". Because of its shape, placing it on top of itself or beside will lead to holes being formed.



We must find a different way to limit the holes as these pieces interact...

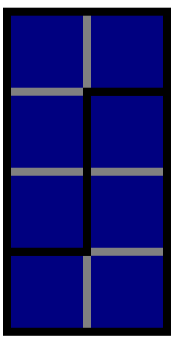
If we rotate the "L" twice, we can see that it can fit on top of the un-rotated "L", forming a 2x4 set, similar to 2 "O"'s or 2 "I"'s.

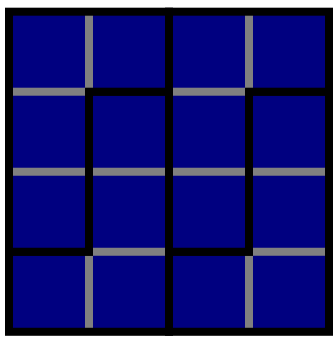




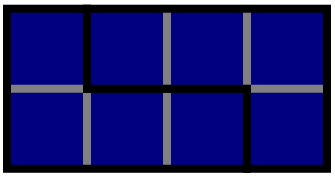
We can use these pieces to put into our grid. But first, we will look at the "J" piece.

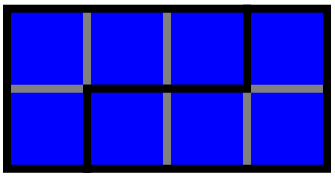
Similar to the "L", the "J" needs to be rotated twice to prevent holes. This will give us the same 2x4 set, that we try to target.



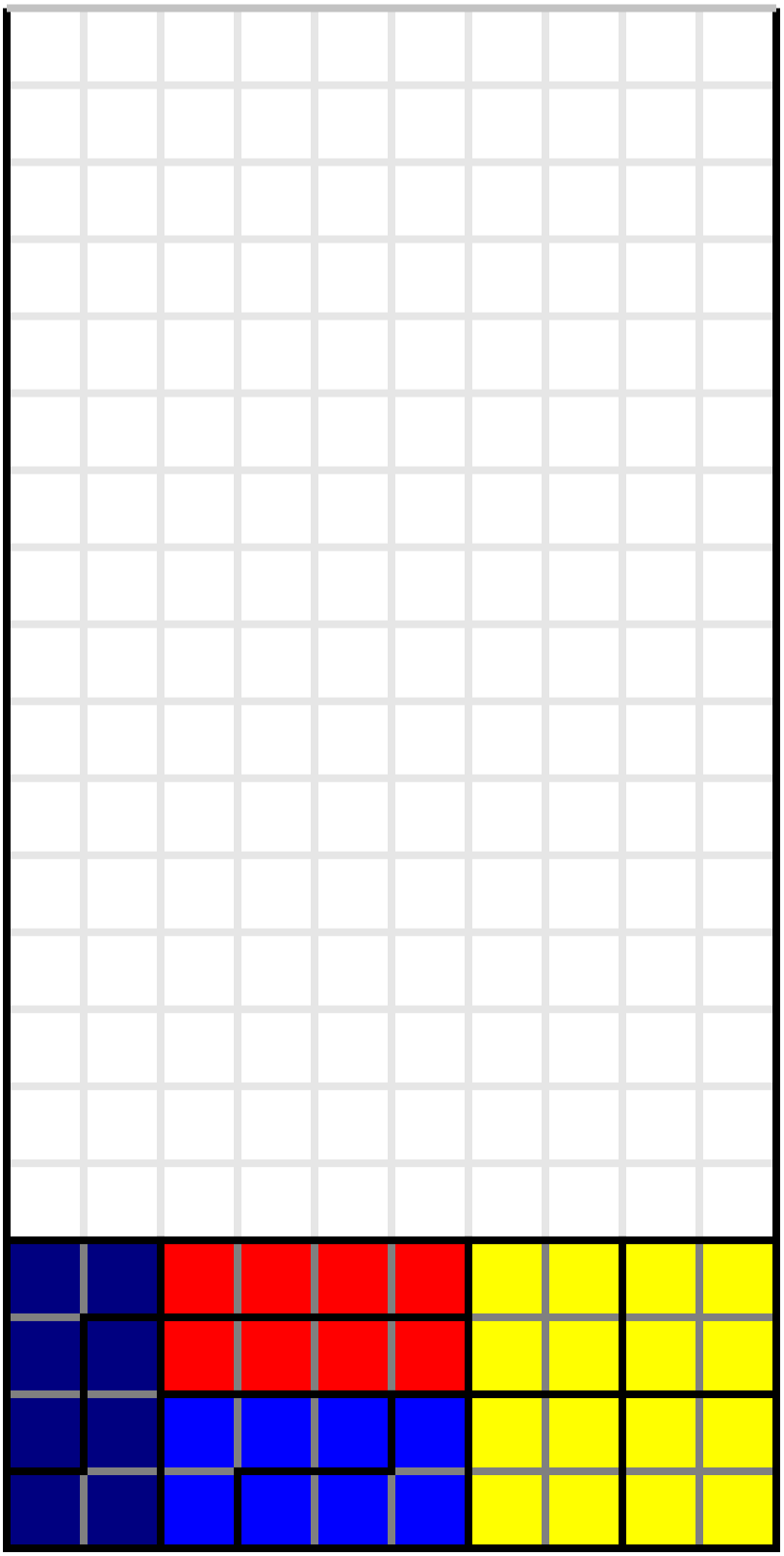


We can also rotate these pieces (as a pair) but because the base is 4 in width, we cannot fill the grid with rotated pieces.

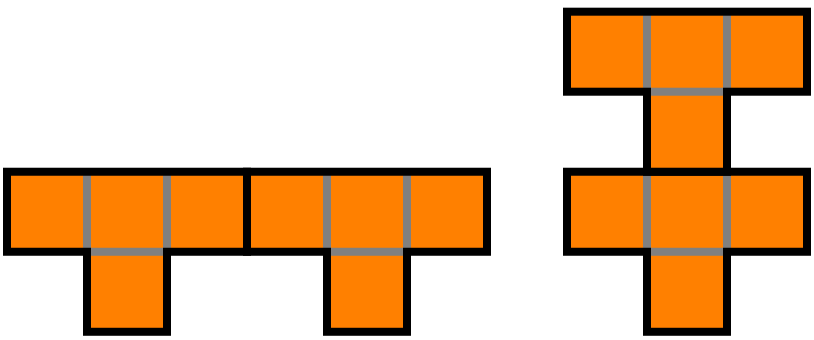




The "J" and the "L" pieces are slightly more complicated than the "O" and the "I" but only due to the need to have them rotated.

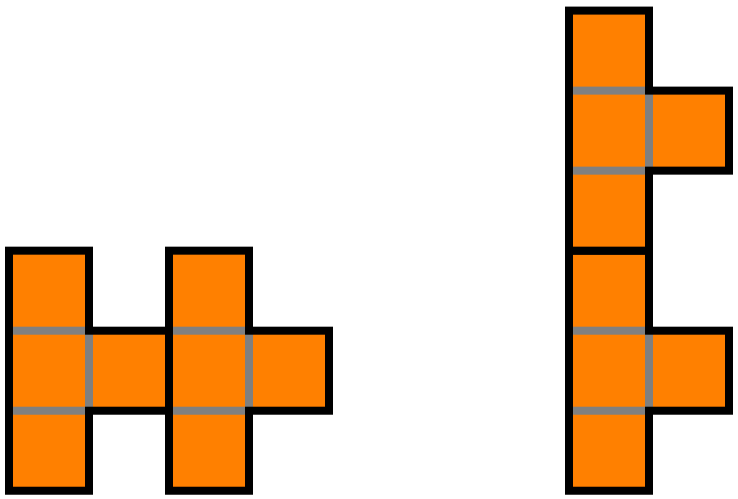


The final three pieces, "T", "S" and "Z" are a more difficult, as they tend to provide problems when only interacting with themselves. Looking at the "T".



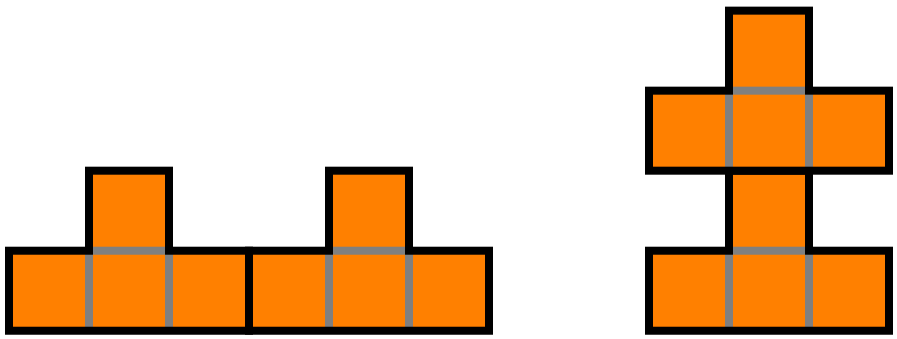
As we can see, these provide holes. We can try and do a rotate or two.

By rotating it once counterclockwise, we are left with the same predicament as we can see holes will form.



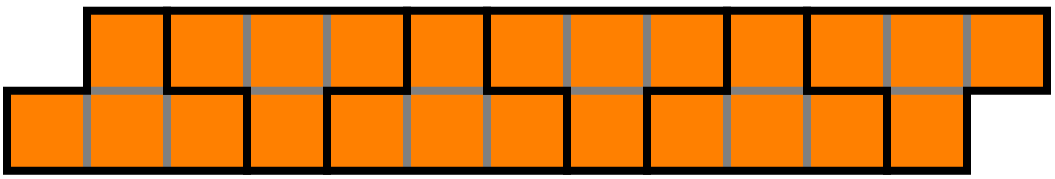
We will try to rotate again.

We can see that placing the upsidedown T's beside each other form no holes, but if we continue and place them on top of each other holes will form.



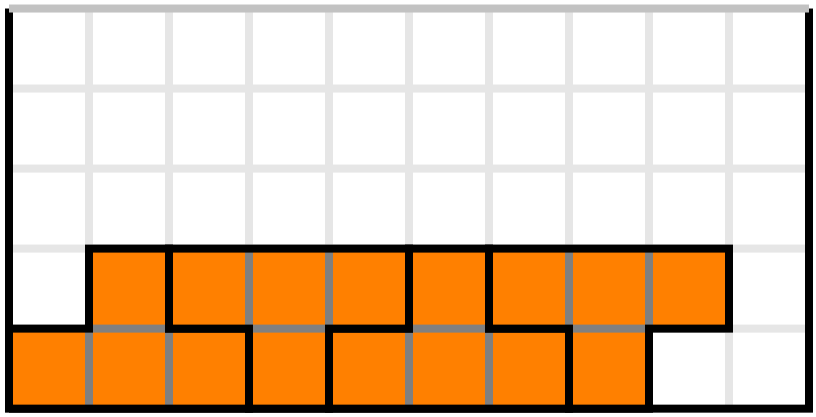
Finally, we will try to use a combination of various rotated and non-rotated T's.

We will try using two times rotated T's with non-rotated T's in an alternating pattern continue and place them beside each other.

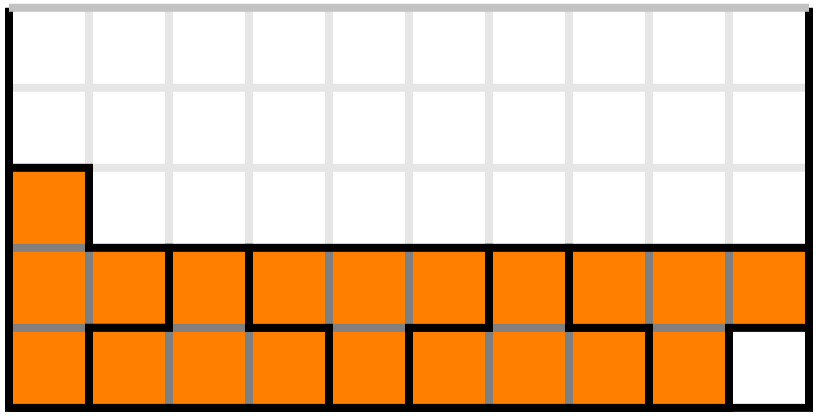


We can see that only using these pieces, holes will still exists. We can try again on a Tetris grid.

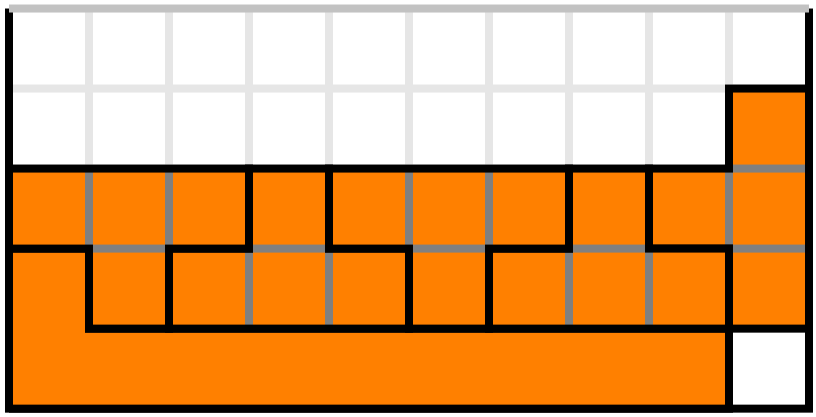
We can see that there will be holes with this setup. We can try another sequence.



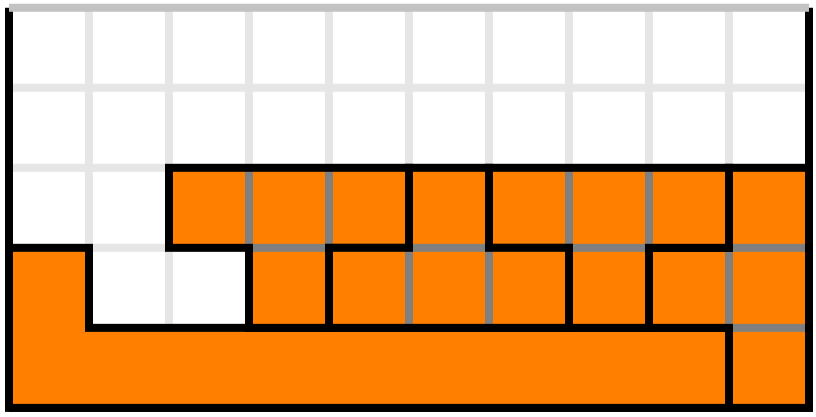
We can see that this seems to work, so we will continue this pattern. Note, the 2nd line is removed.



We can see that there will be holes with this setup. We will try another sequence.



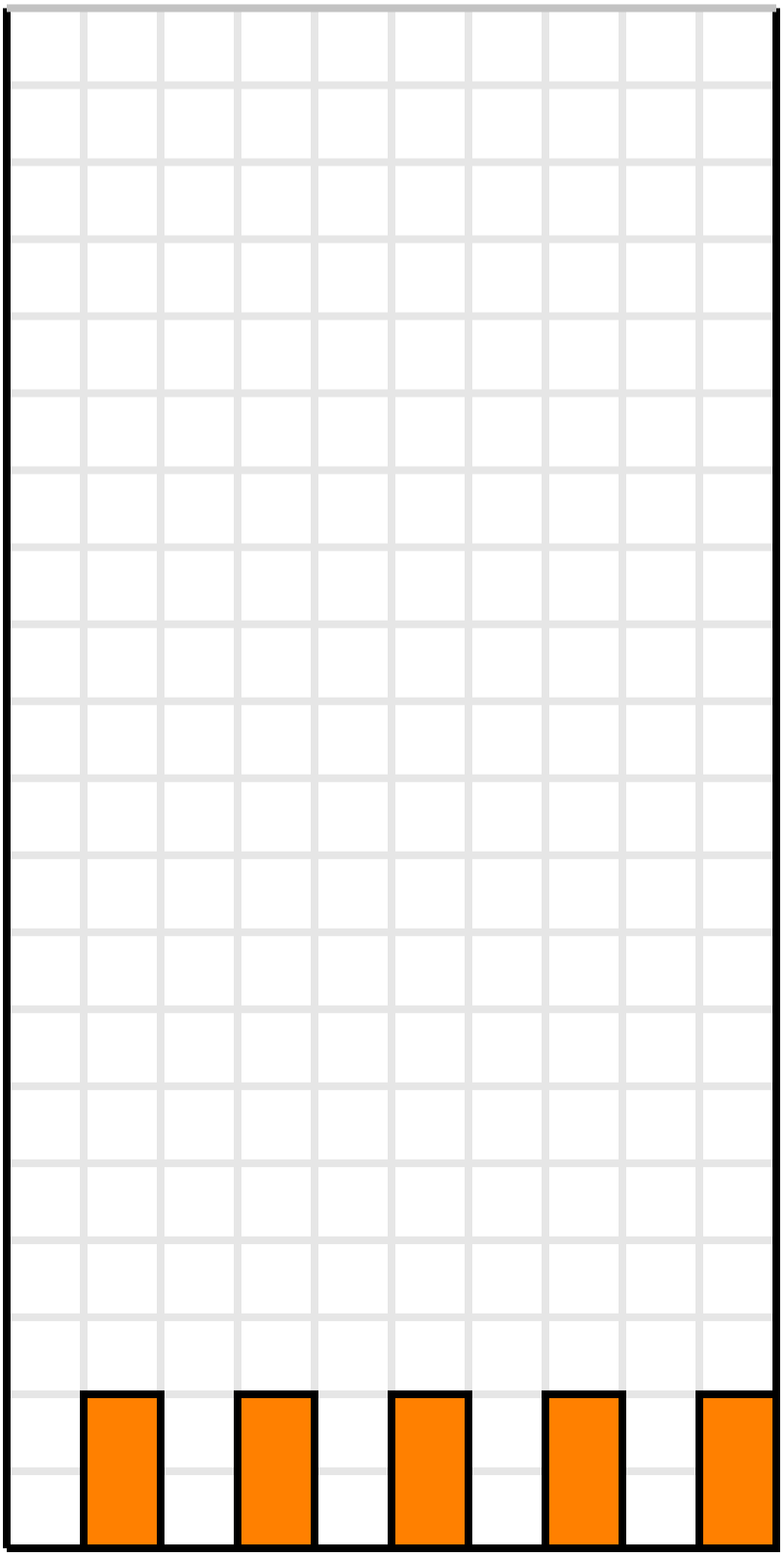
Continuing this pattern will just create holes in different spots.



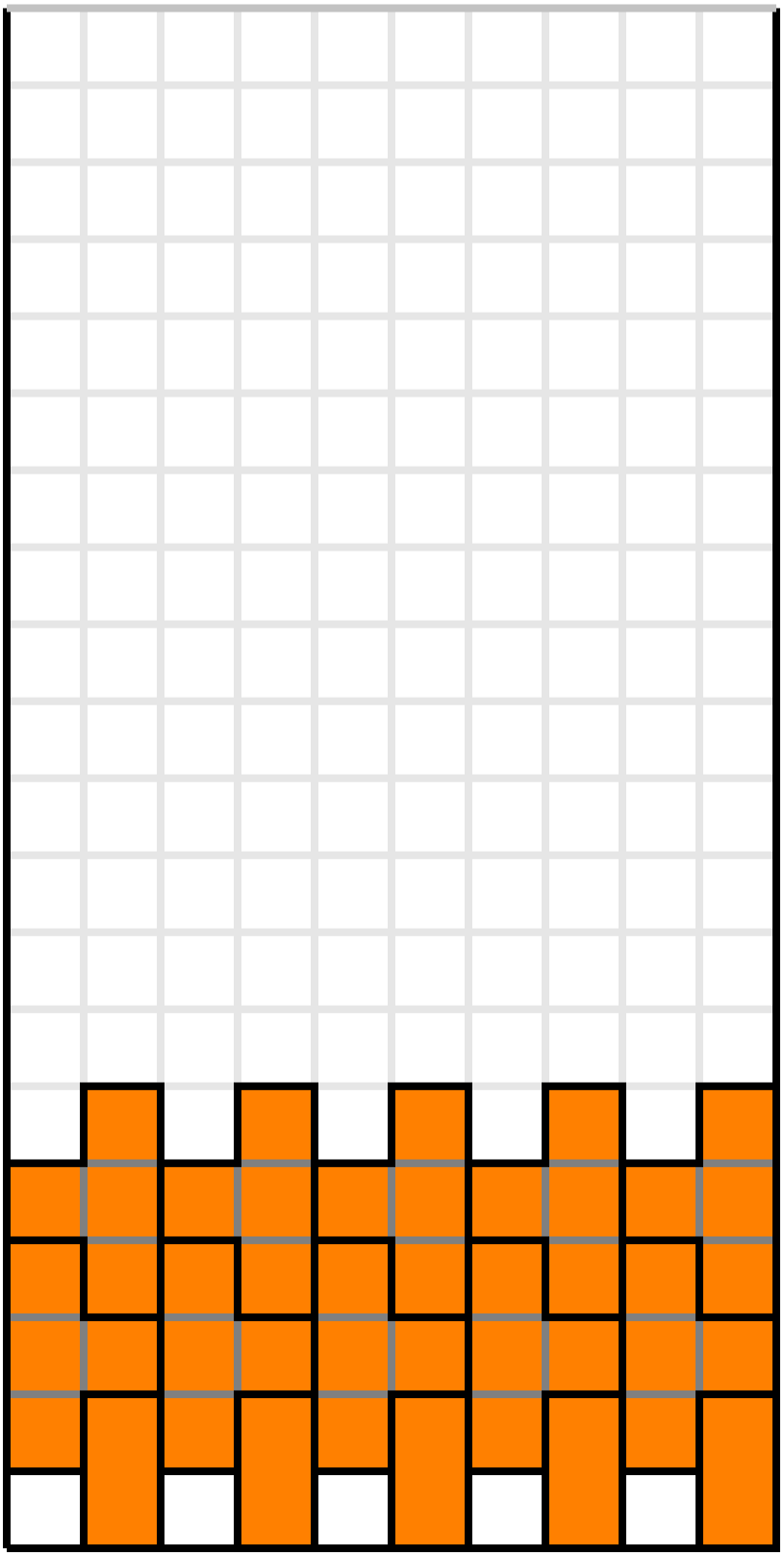
We can see that there is no way to prevent holes. However, there is a way to include holes but the pattern will continue forever.

We will use rotated "T"’s in the following manner. This will remove the 2nd row.

This leaves with the following pattern. We will call this pattern T*.

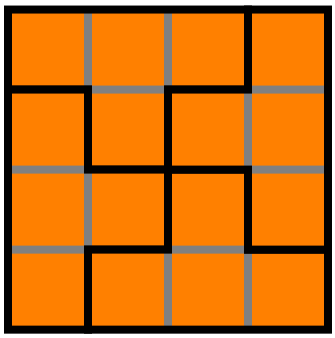


With the remaining pieces, we will use more "T"s rotated in the opposite direction and on top continue with more "T"’s as shown below.



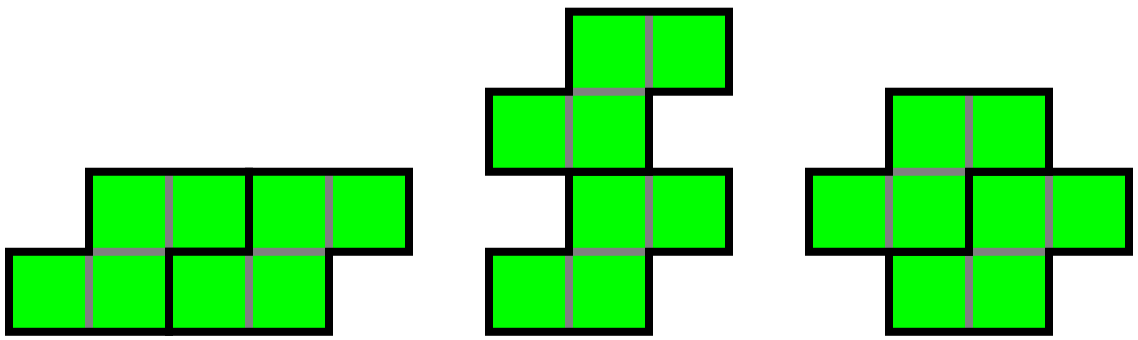
By removing the 4 rows from before we are left with T*. We can continue this pattern forever, although holes will always exists.

We can look at another pattern: one that features a "T" with each rotation (0, 90, 180, 270 degrees). and their interactions.



These give us 4x4 pieces. However, our grid has a base 10 and so we cannot use this pattern as 10/4 = 2.5 and we cannot have half of this 4x4 piece.
But we can use this to replace any 4x4 sets from a combination of "I"'s, "O"'s, "L"'s or "J"'s.

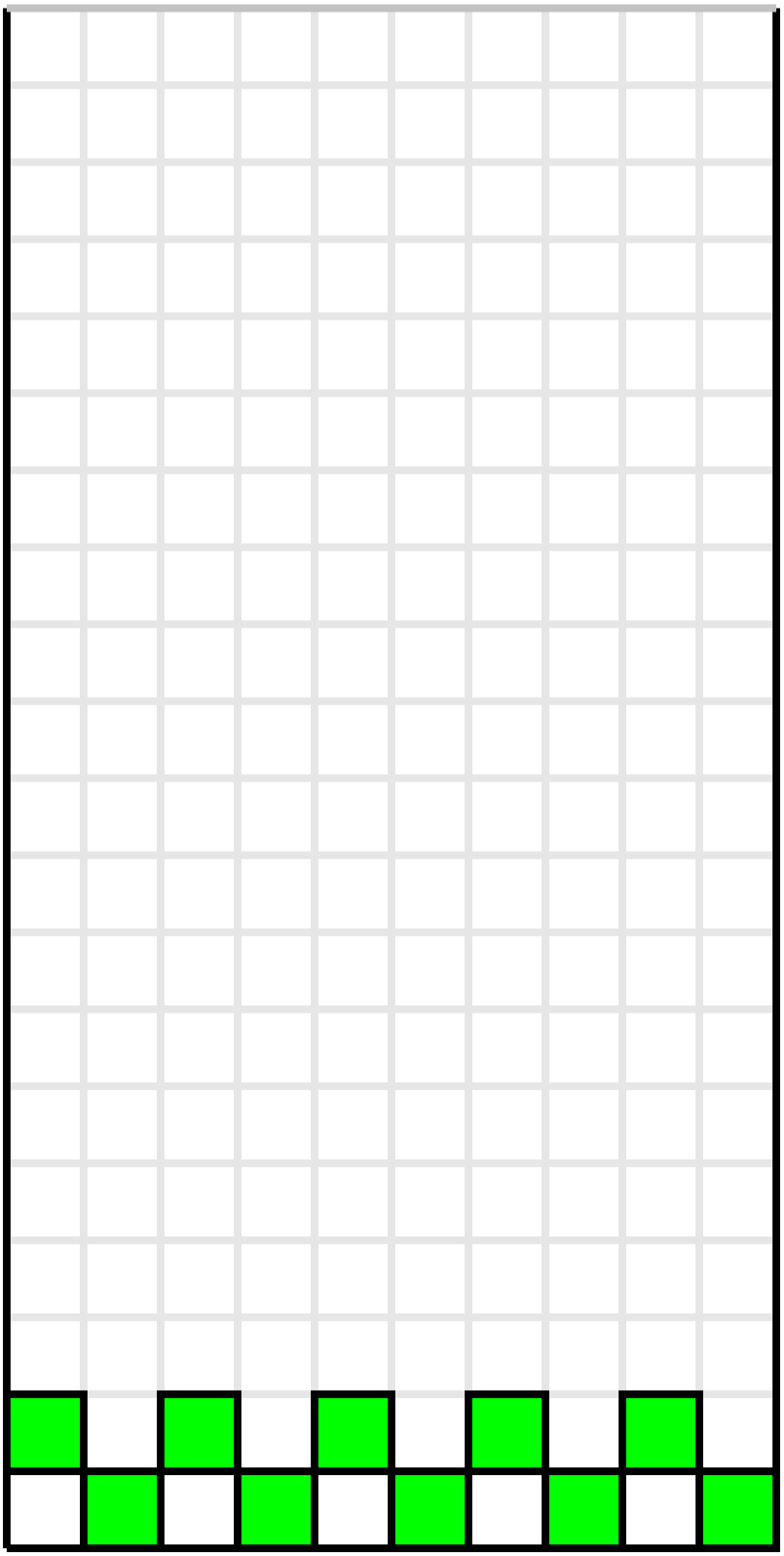
The "S" and "Z" are similar to the "T".
We can quickly see that whatever we do with the "S" we can simply mirror with the "Z".
We will focus on the "S".



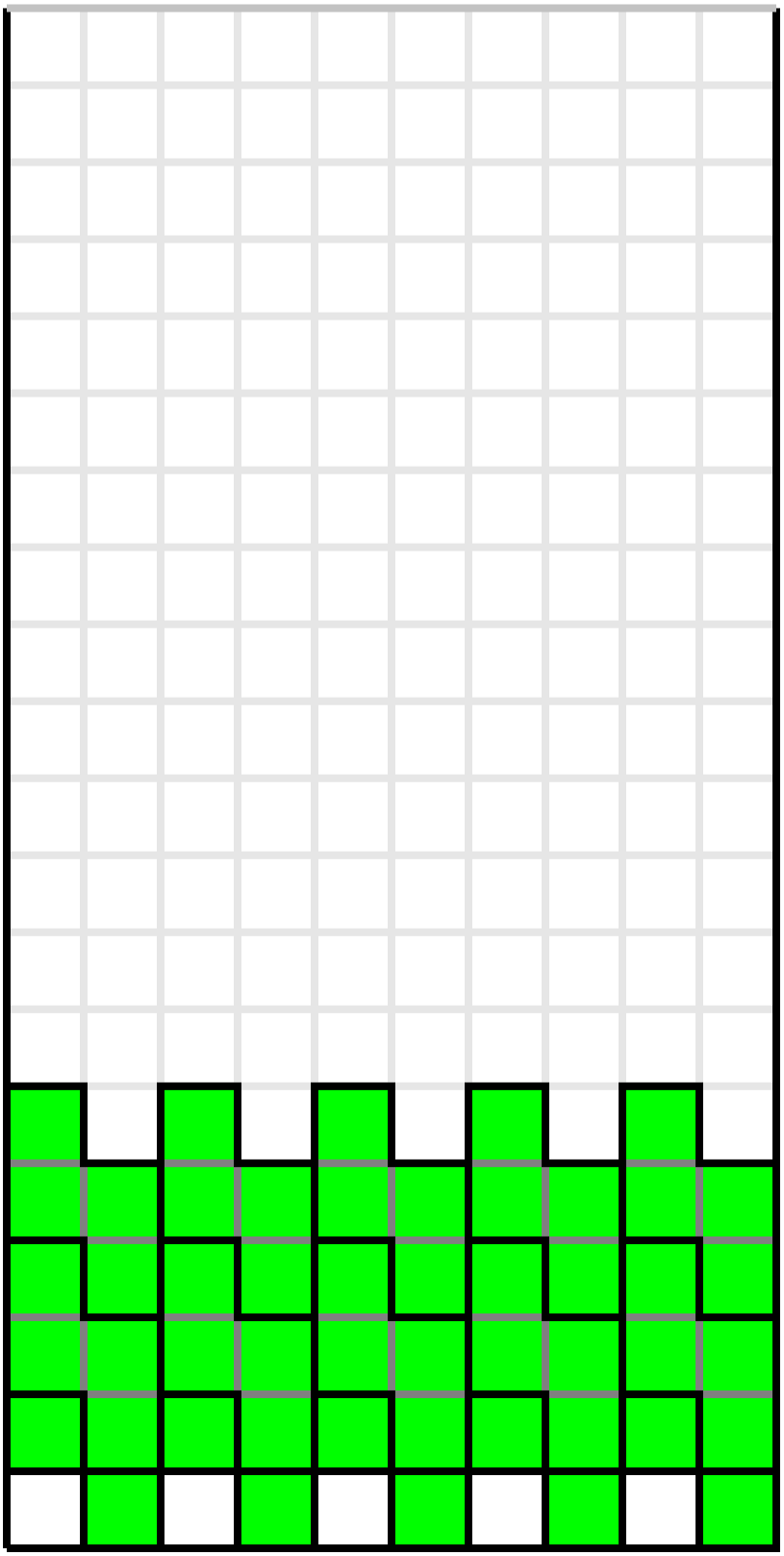
As we can see, these provide holes.
However, following the idea for "T"
we can continue forever with only "S"s.

Using only "S" pieces, the pattern is more simple than the "T" piece. We will use rotated "S"s.

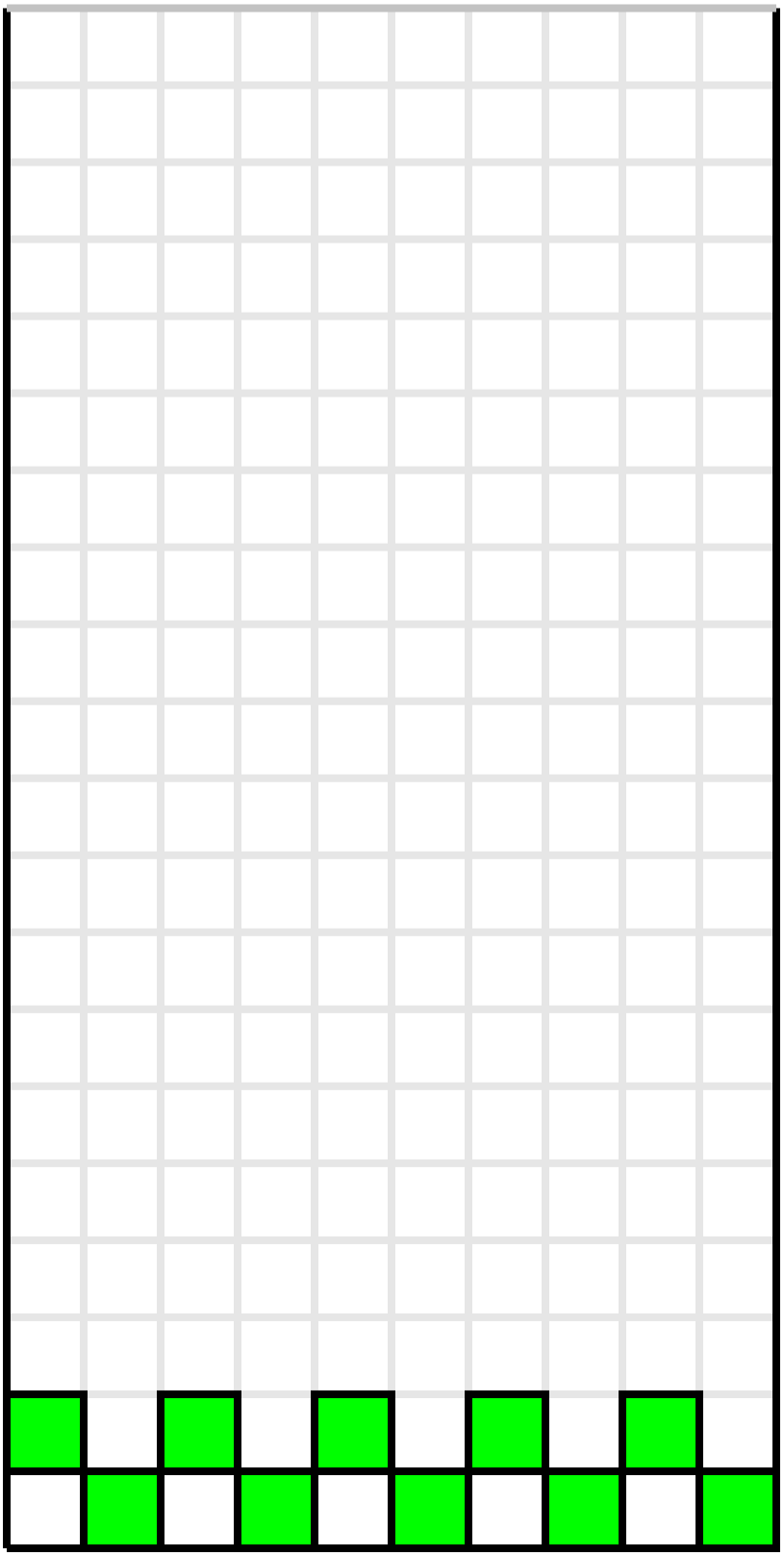
The 2nd row is removed and we end up with This pattern, which we call S*.



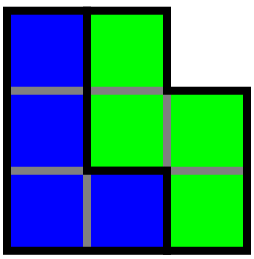
We can continue stacking the "S" and no matter how many rows we add, so long as it is evenly distributed amongst the rows, we will end up with S*.

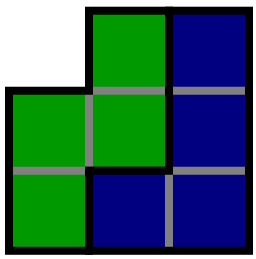


We can also see this will be the same for "Z".



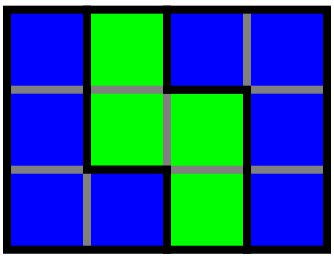
In order to remove the holes from the "S" and "Z" pieces, we must introduce more pieces.
We will try using a "L" with the "S" and
a "J" with the "Z".



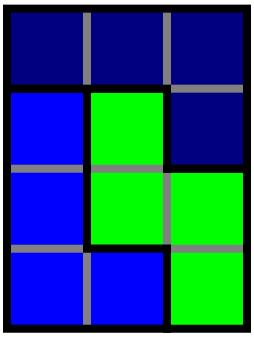


These both give us the opportunity to create a 3x4 or 4x3 set by adding another "L" or "J"

Looking at the "L" and "S", we can add another "L" to get LS’L" or we can add a J on top to get J’LS’.



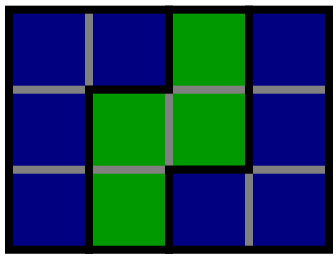
LS’L"



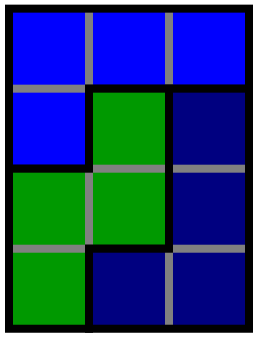
J’LS’

These both give us a 3x4 or 4x3 set.

Similarily we can add a "J" or an "L" to the Z’J to get J"Z’J or L"’Z’J



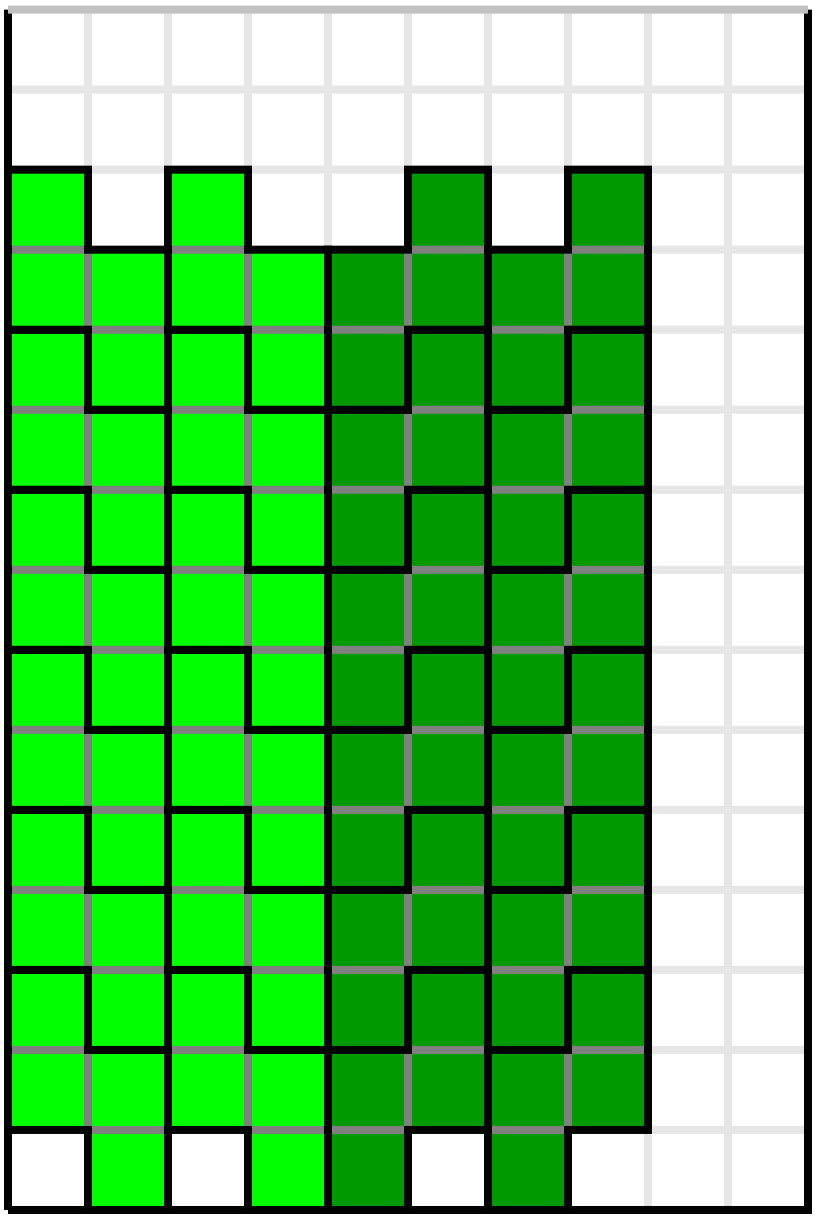
J"Z’J



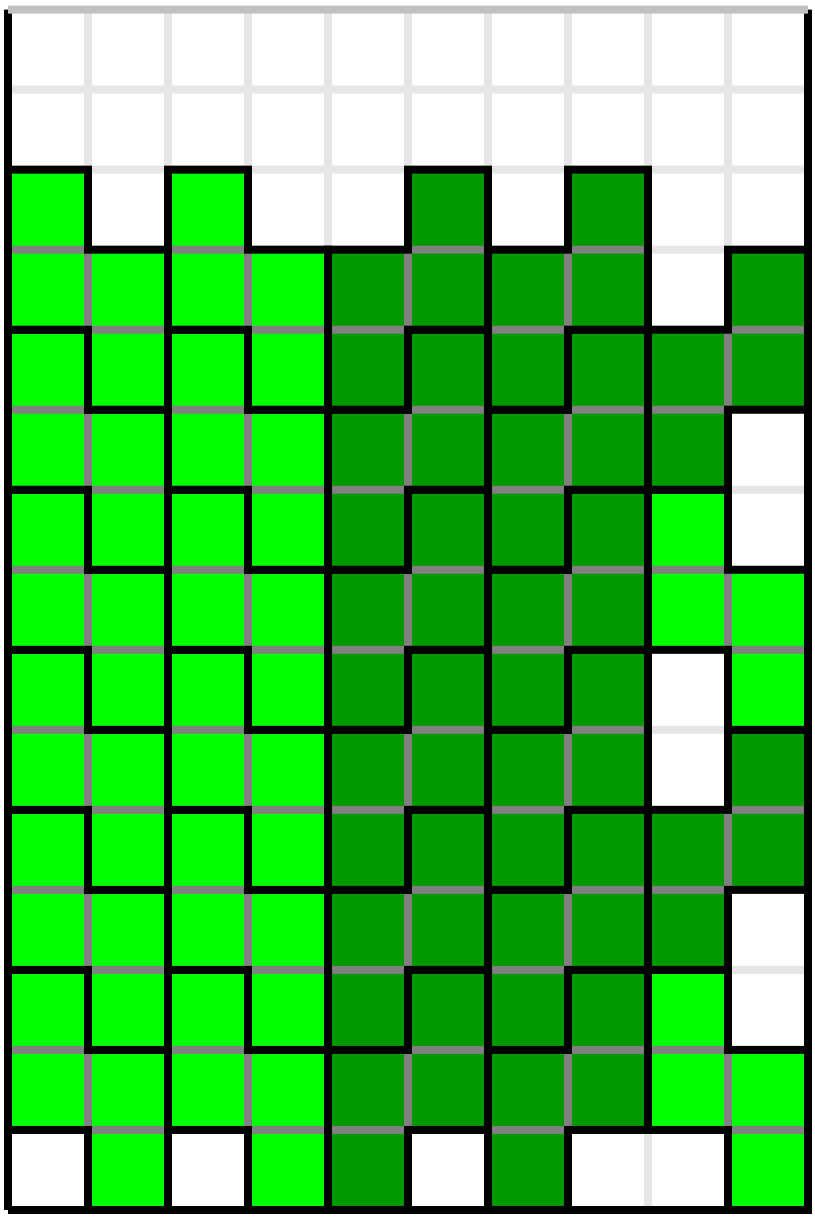
L"’Z’J

These also give a 3x4 or 4x3 set

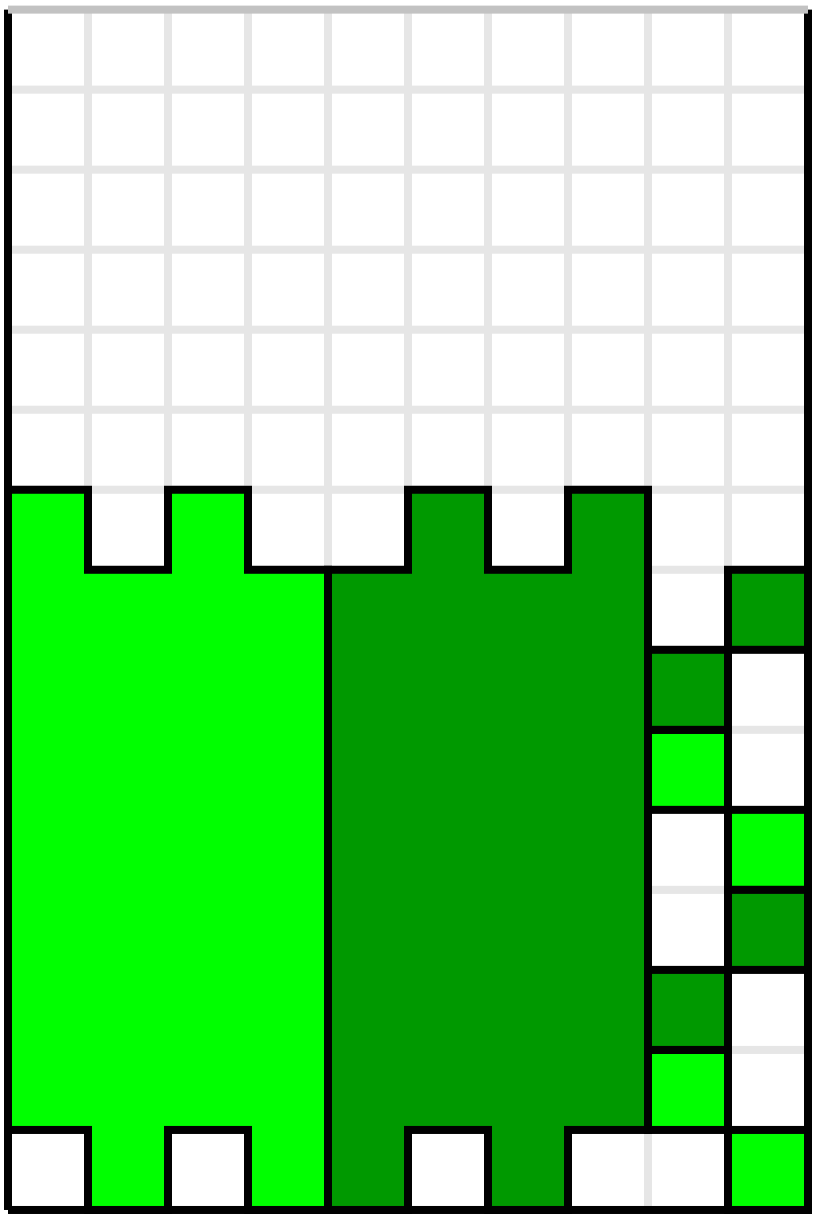
It is difficult to predict the length of a game.
To show that the game is NP-Complete, we need to see what particular part of Tetris makes it impossible to finish. We will use only two pieces: the "S" and the "Z" in an alternating fashion.
Using a similar pattern as previously shown before we will put all "S"'s in the 1st four rows, and the "Z"'s in the 5th to 8th row.



We are left with alternating "S" and "Z" pieces. Assuming we started with an "S" we can see that not only do we end up with holes, the holes gradually build ontop of each other.



We've seen before that if we continue this trend, the game will end. We've also that if we tried to put the next "S" piece, in any other place, we would end up with a hole elsewhere.



The reason why Tetris will be impossible to finish is based on the fact that if we have alternating "S" and "Z" pieces AND the fact that the only ways to remove the holes are to use a combination of 2 "L" and/or "J" pieces.

Because the pieces are randomly chosen, over a long period of too many "S" and "Z" pieces and few "L" or "J" pieces, we will eventually end up with too many holes and the game will end, making Tetris NP-complete.

The biggest factor is not the variability, but rather the base of the Grid. If the base was a number 4n we can easily show that we can alternate the "S" and "Z" pieces so they can continue forever.

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