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Term Project
Math 308

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I. Introduction

Some 300 years ago,

the following problem:

For a given triangle ABC,
how can we find a point
P in triangle ABC,
such that PA + PB + PC
is minimal?

was initially proposed by a famous mathematician named Pierre de Fermat. Ever since then, it has reappeared in many different variations with different solutions.

Jacob Steiner also proposed a solution to a variation of this problem in the 18th century. Therefore it became known as the Steiner problem.

Yet, history proves that the roots of this problem was proposed much earlier than the 18th century.

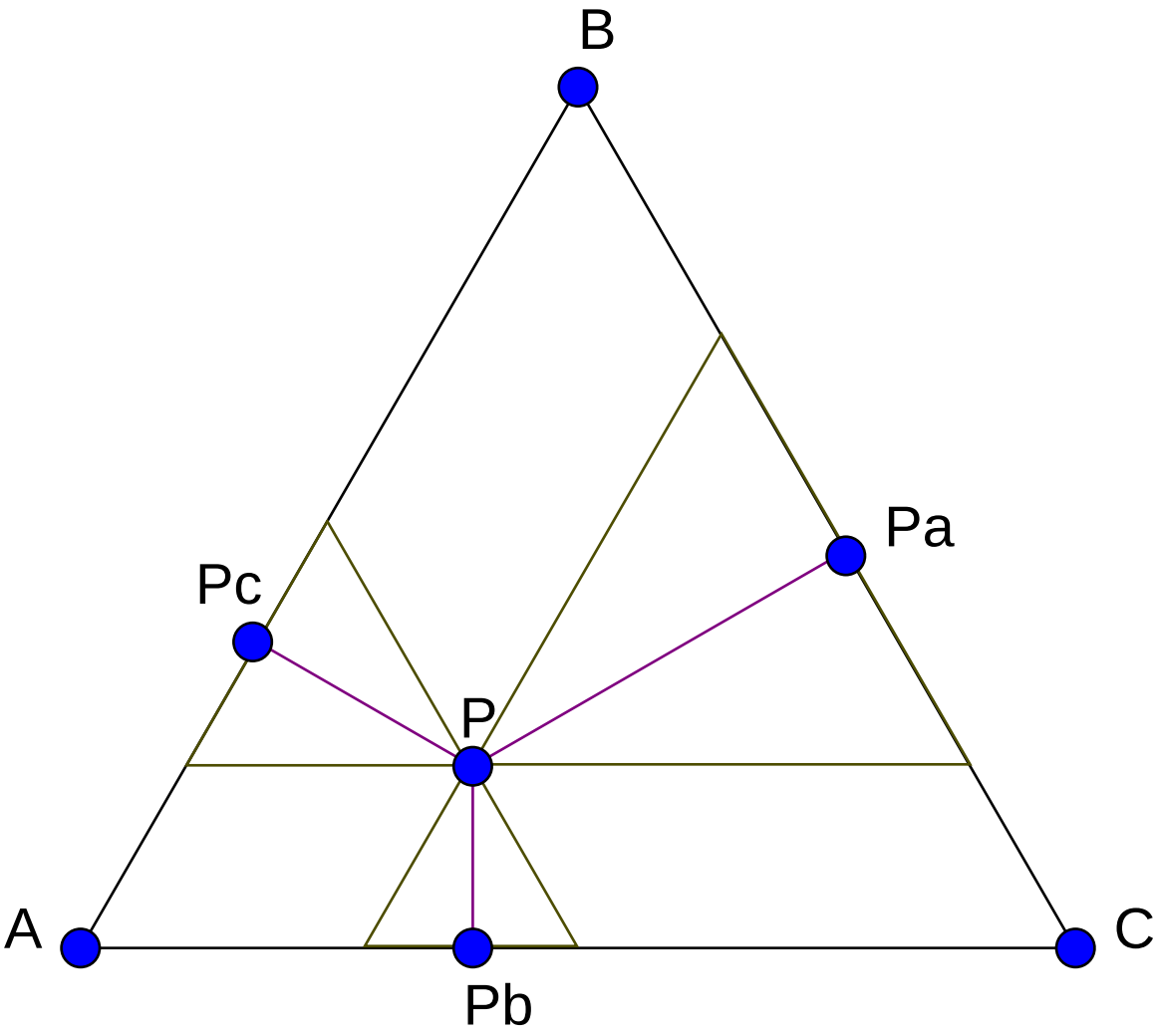
Fermat (1601-1665) was the first to propose this problem to Evangelista Torricelli. Torricelli solved this problem and passed it his student Vincenzo Viviani.

In 1659, Viviani published his own solution along with Torricelli's solution. Today, Torricelli's solution is called Viviani's Theorem.

II. Viviani’s Theorem

Viviani’s Theorem

For an equilateral triangle,
the sum of the distances of a
point to the side of the triangle
or a point on one of the triangle’s
side must equal the length of the
triangle’s altitude.



If Pa, Pb, and Pc are the projections of a point P on the lines BC, AC, and AB respectively on triangle ABC, then:

$$h = PPa + PPb + PPc$$

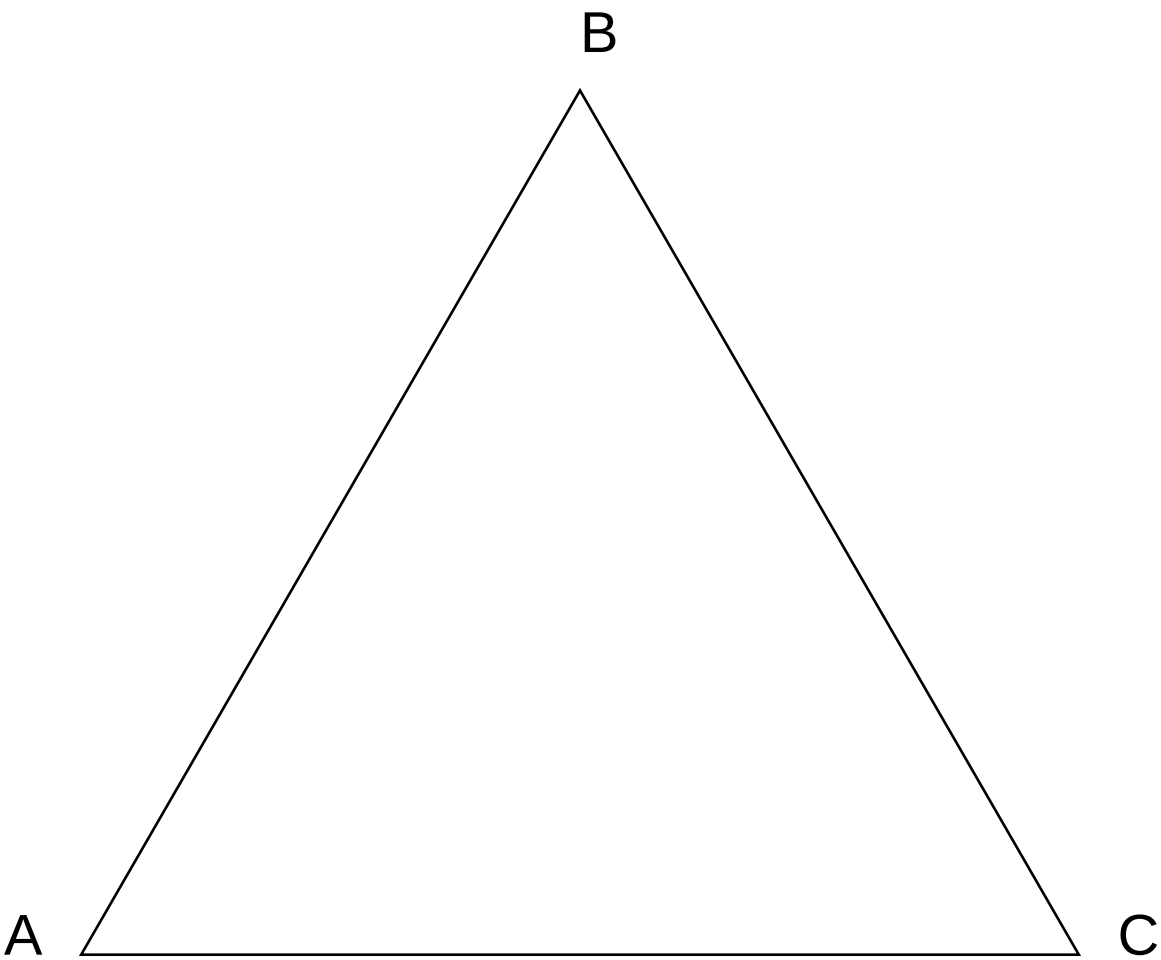
h is triangle ABC's altitude

III. Proof of Viviani's Theorem

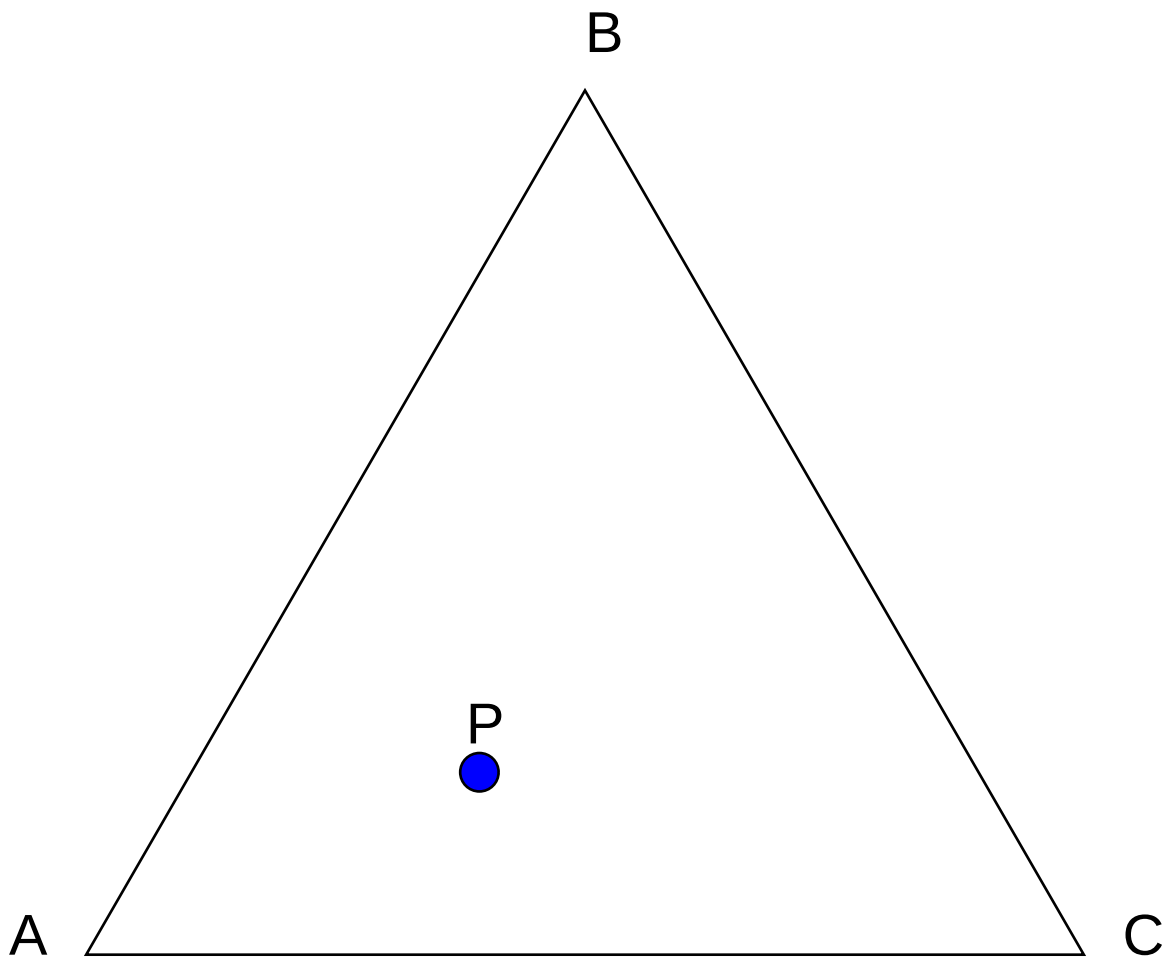
3 Methods to Prove

Viviani's Theorem

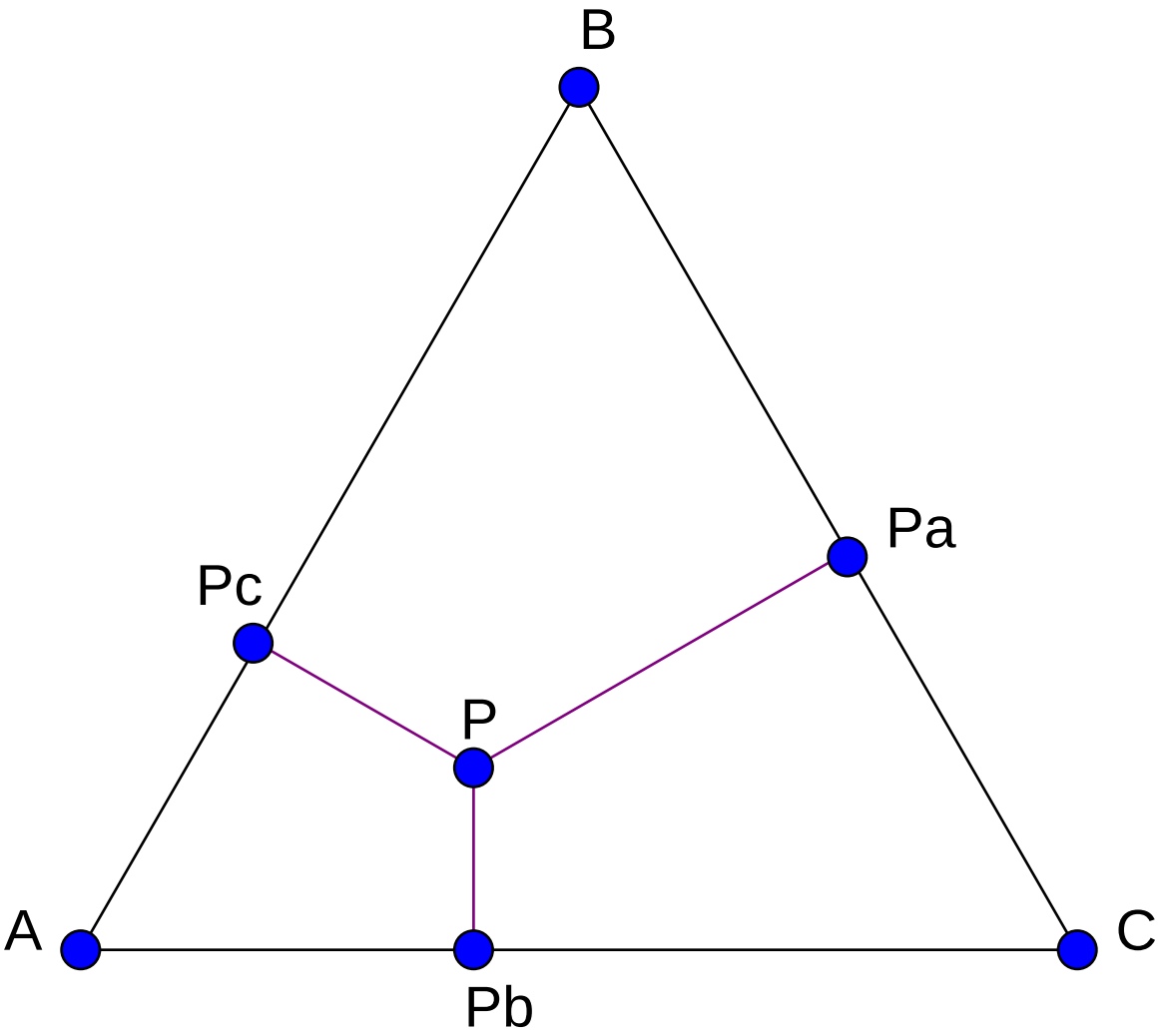
Method # 1



Given an equilateral triangle ABC



P is a point in triangle ABC

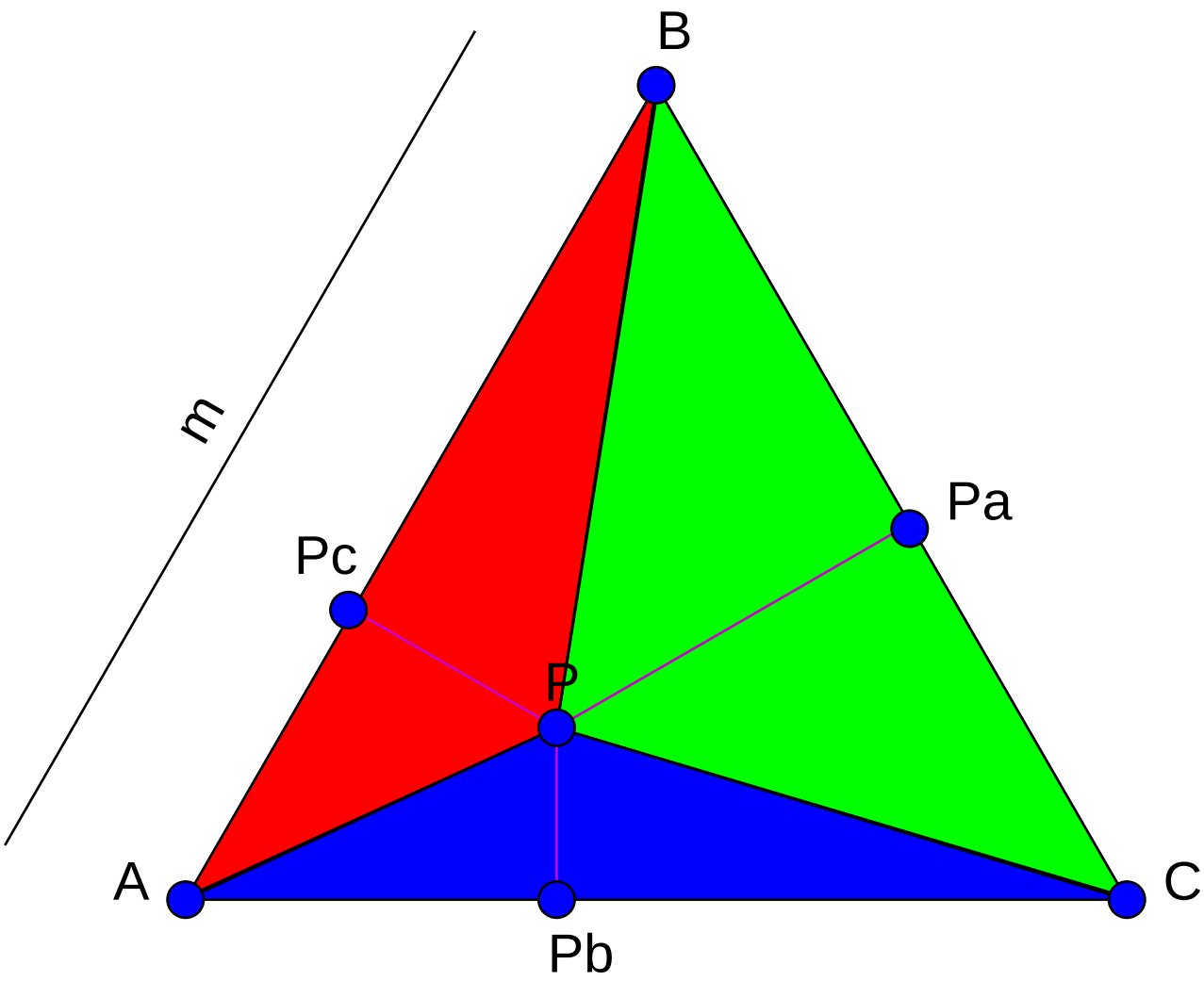


Pa, Pb, and Pc are projections of the point P on the side lines BC, AC and AB respectively.

PPa is perpendicular to BC

PPb is perpendicular to AC and

PPc is perpendicular to AB


B
m
Pa
Pc
P
A
C
Pb

(*) Area of triangle ABC = Area of triangle ABP + Area of triangle BCP + Area of triangle ACP

n is the length of a side of triangle ABC

Since triangle ABC is an equilateral triangle, the sides AB, BC and AC are the same length.

The area of a triangle is base * height/2

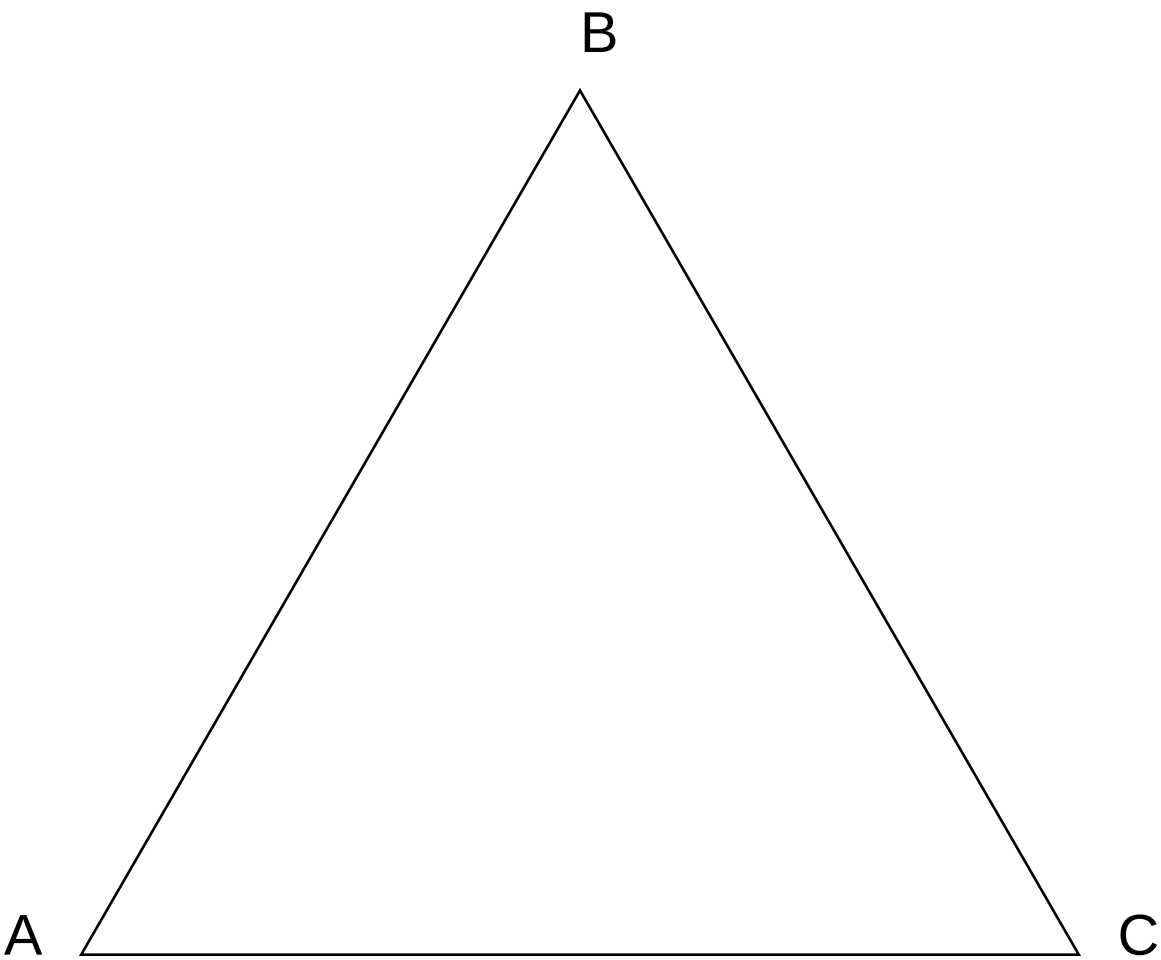
Then from (*) we can see that

$$m * h/2 = PPc * m/2 + PPa * m/2 + PPb * m/2$$
$$= m/2 (PPc + PPa + PPb)$$

Therefore,

$$h = PPa + PPb + PPc$$

Method # 2



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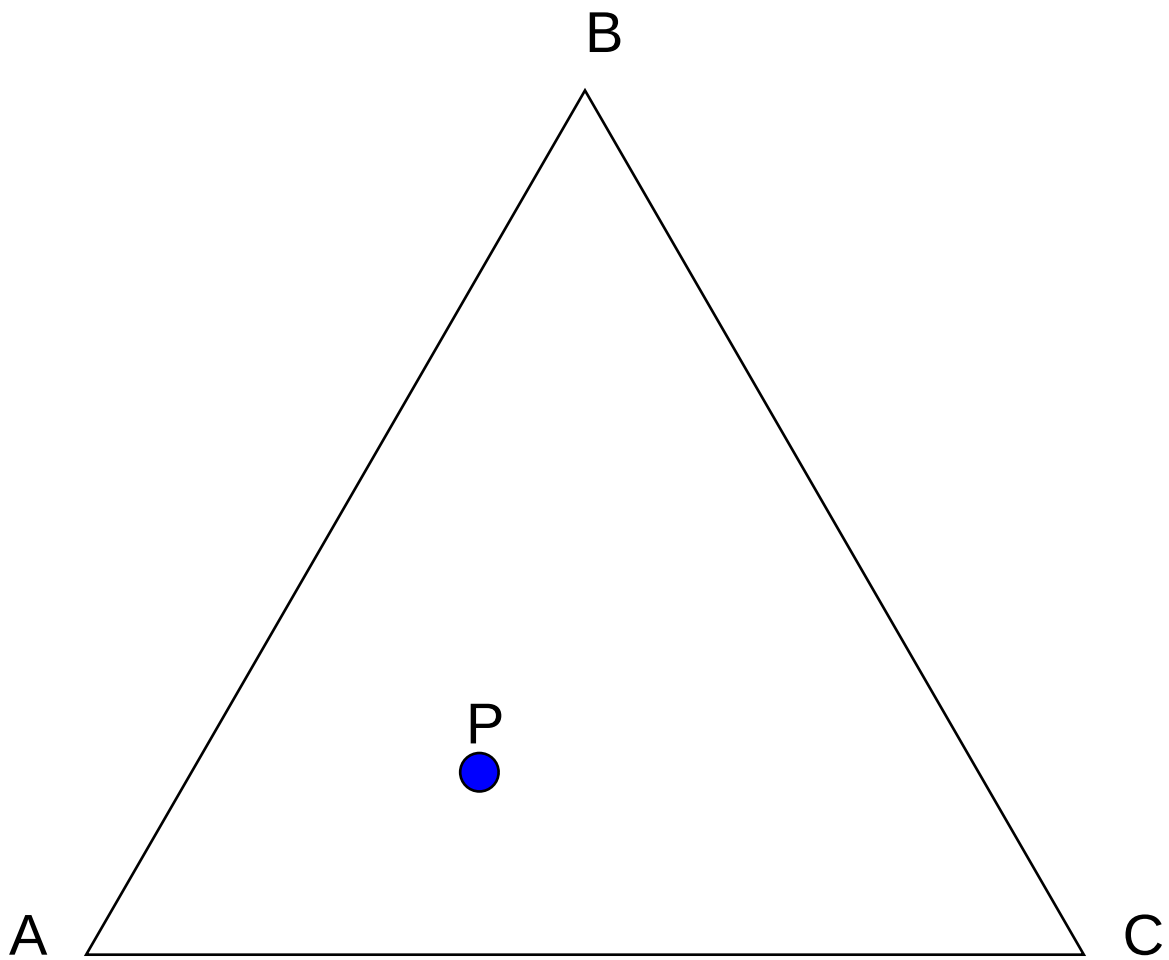

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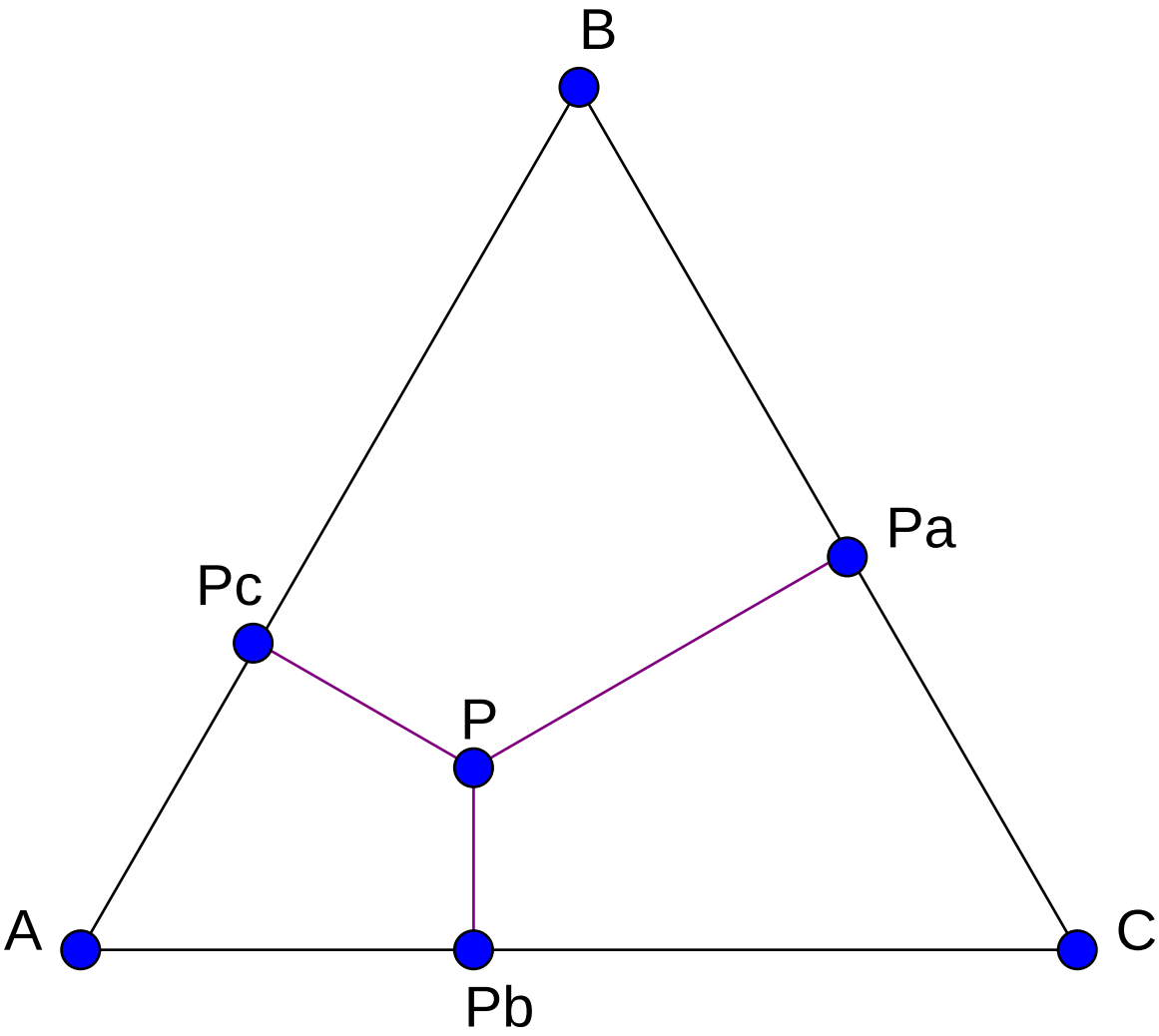

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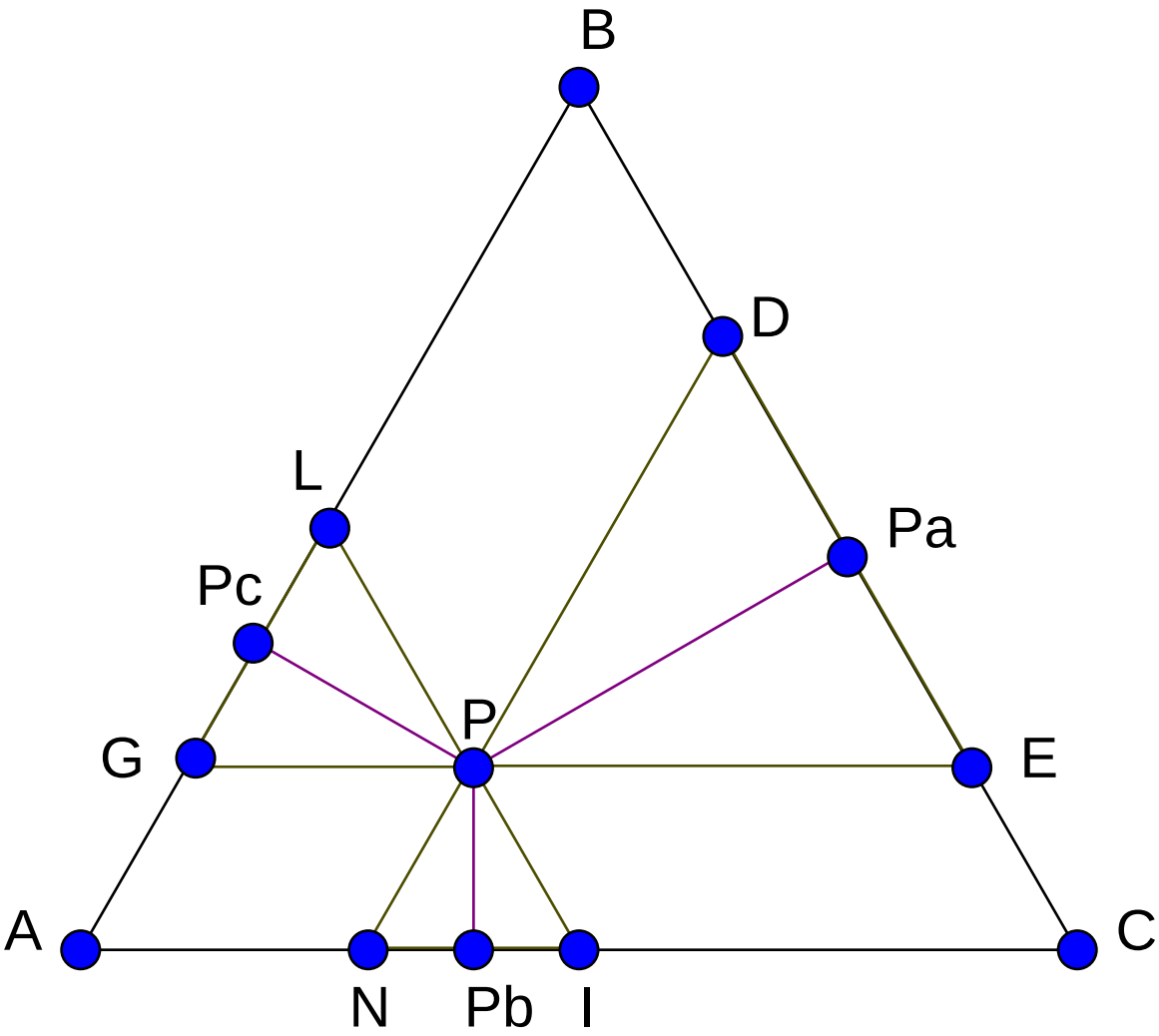
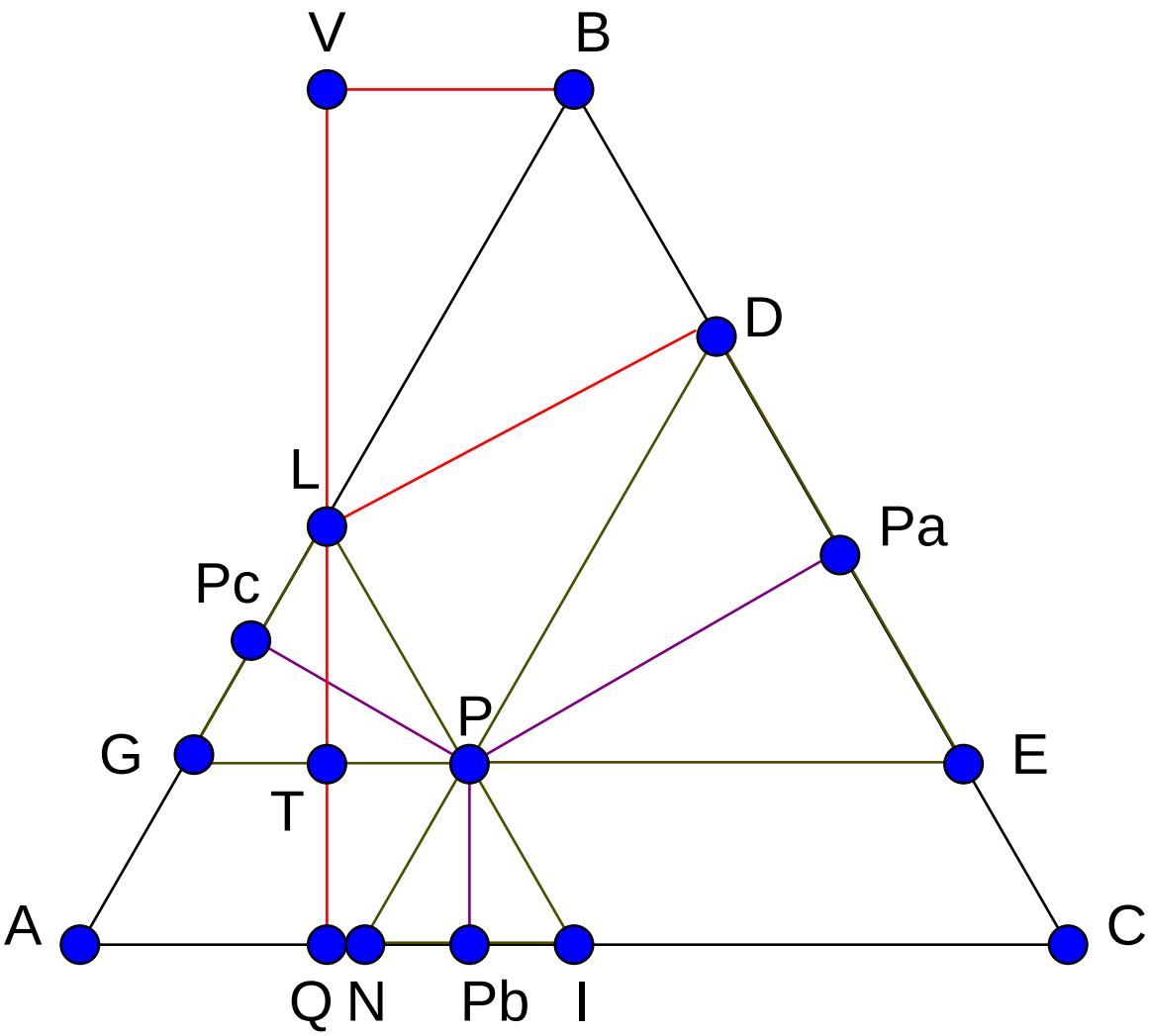


Triangles LGP, DEP and NIP are equilateral triangles with altitudes Pc, Pa and Pb respectively.



LU is parallel to PPa.

LU and PPa are the same length.

LV is the same length as LU.

LT is the same length as PPc.

TQ is the same length as PPb.