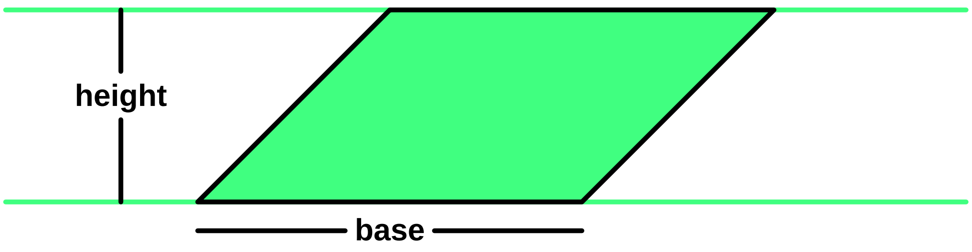
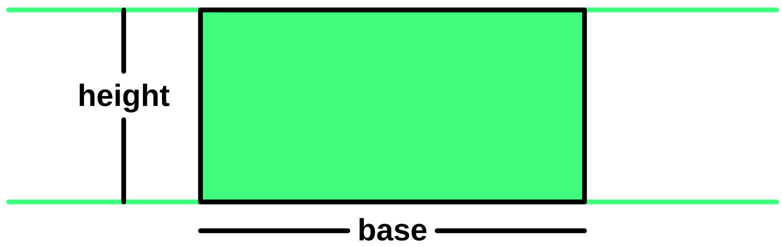
The area of a parallelogram

The area of a parallelogram is equal to the product of its base and its height.



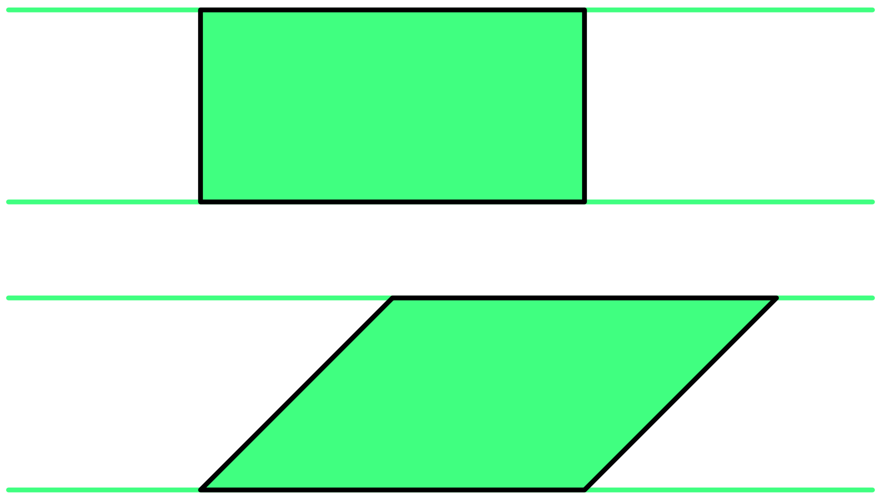
Why?

For a rectangle this amounts essentially to the basic meaning of area.

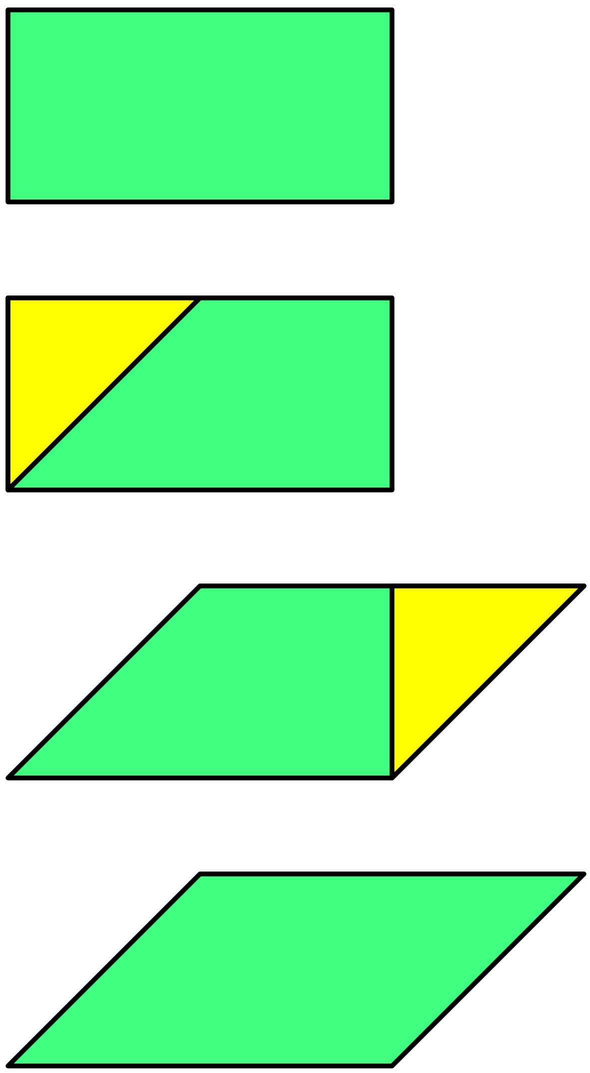


Theorem.

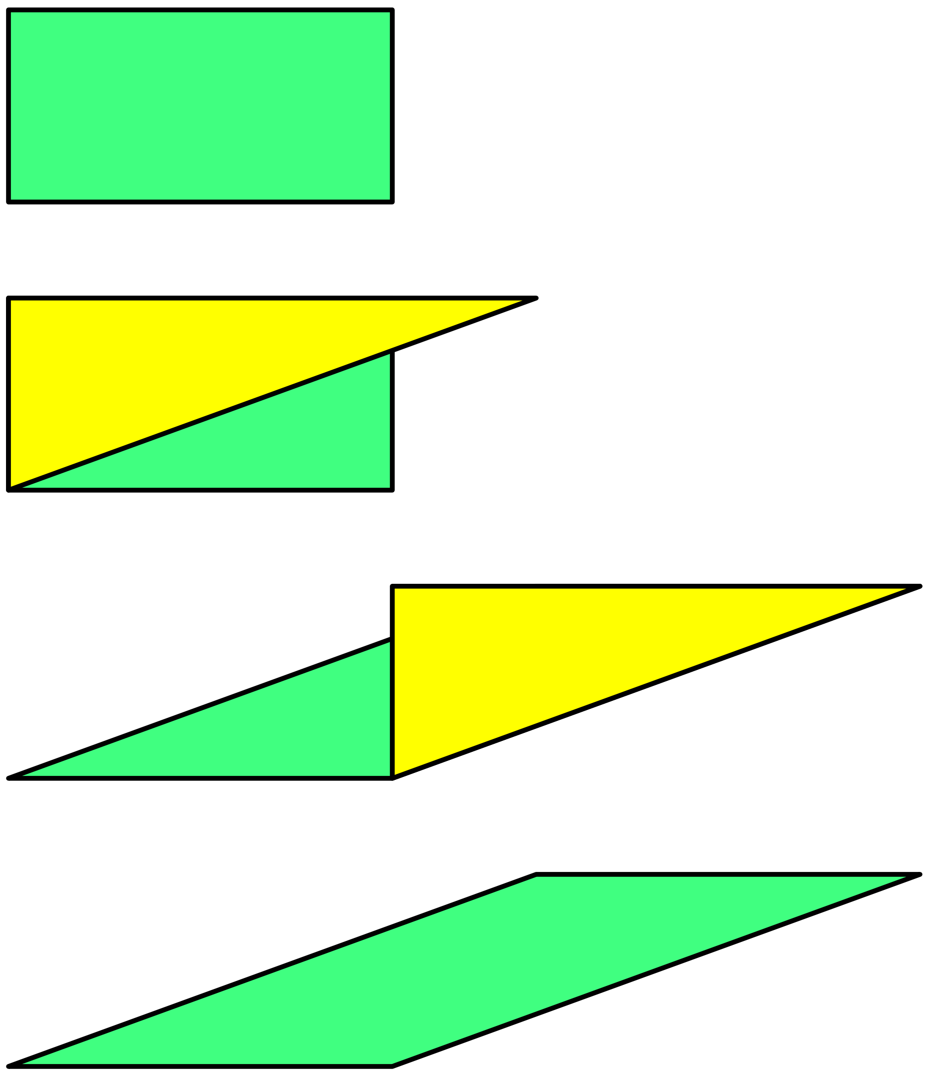
A rectangle and a parallelogram with the same base and height have the same area.



If the parallelogram is not badly skewed, this is easy to see.
Decompose the rectangle by chopping a triangle off one end.
Rearrange the pieces to make up the parallelogram.



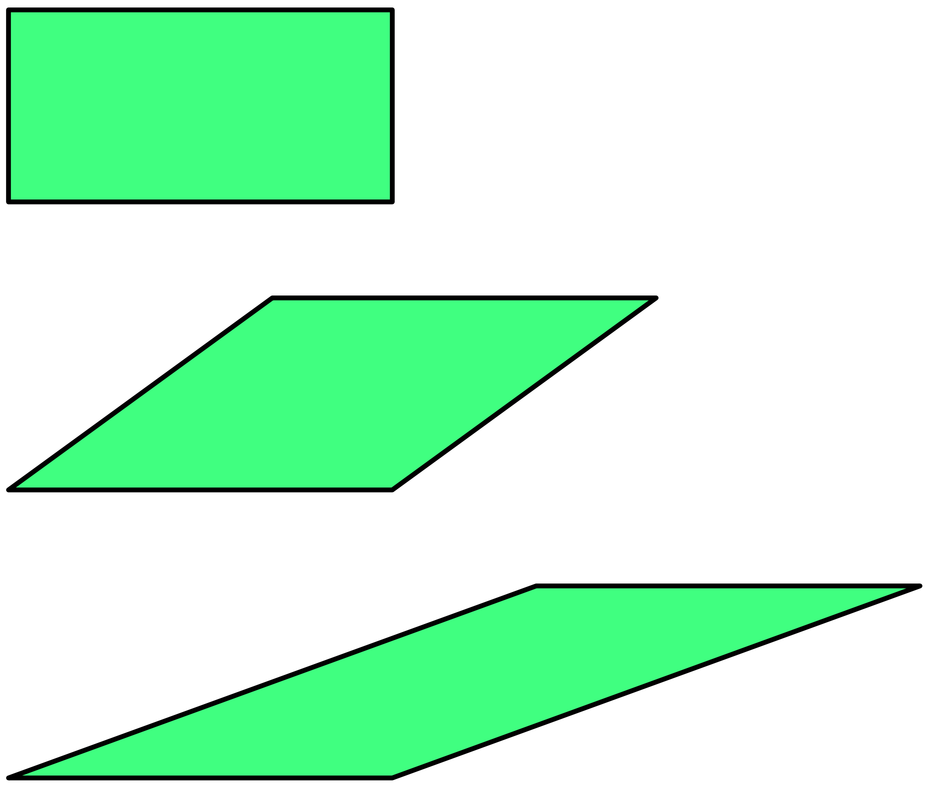
But if it is badly skewed, this reasoning fails.



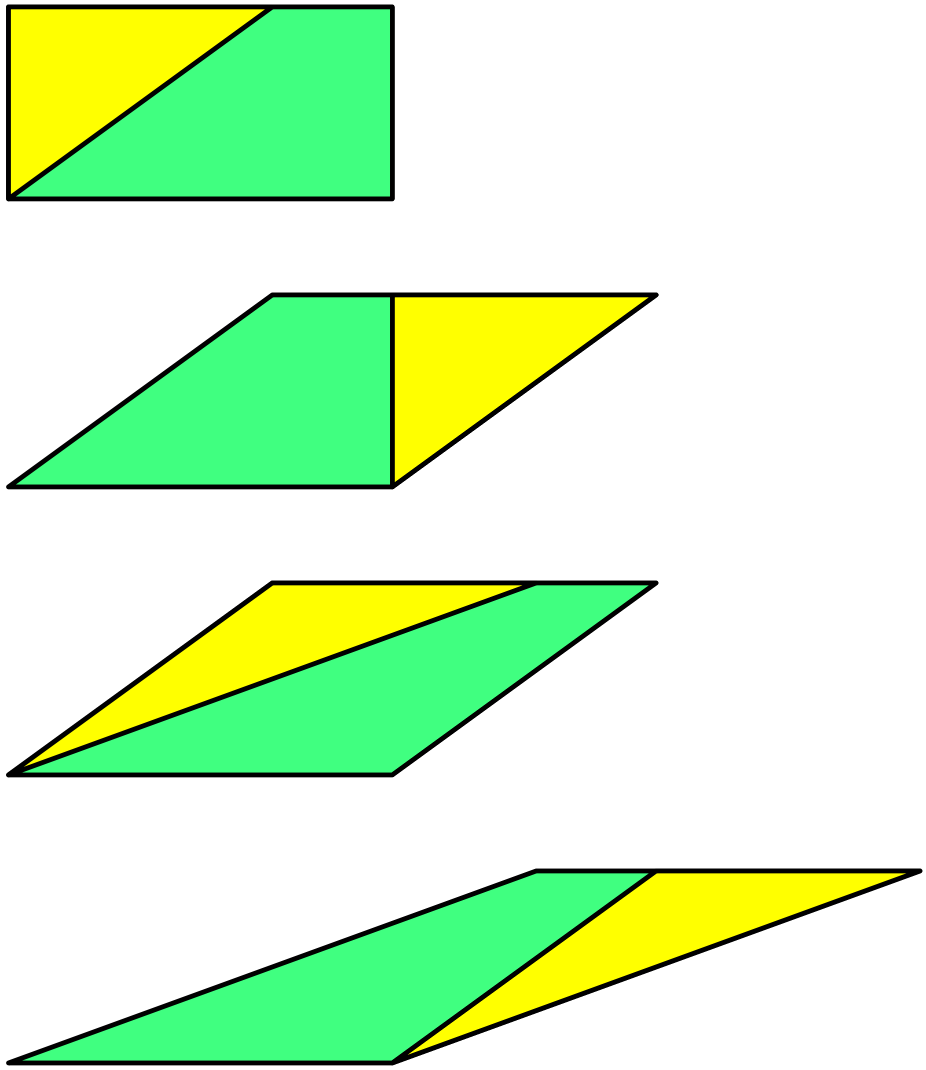
There are many different ways to modify the proof to remain valid no matter how badly the parallelogram is skewed. For example:

1. **Do it in several small steps**
2. **Allow subtraction (Euclid's proof)**
3. **Slices**

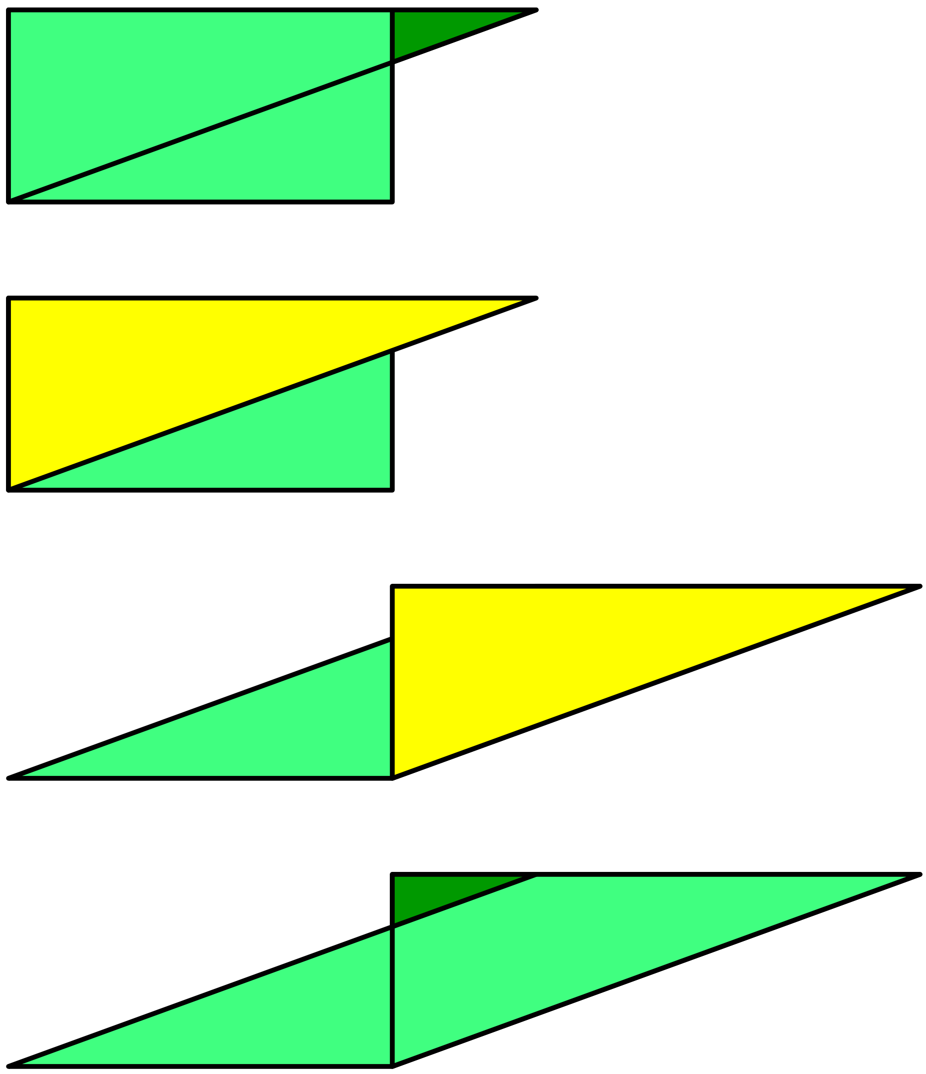
1. **Do it in several small steps**



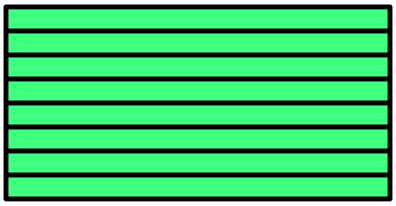
1. Do it in several small steps



2. **Allow subtraction (Euclid's proof)**



3. Slices



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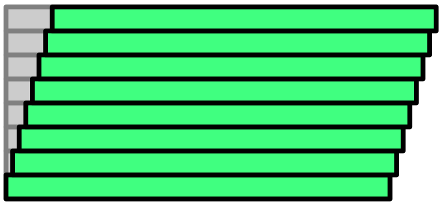

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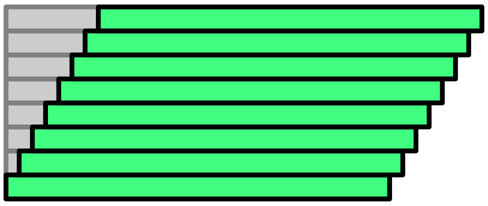

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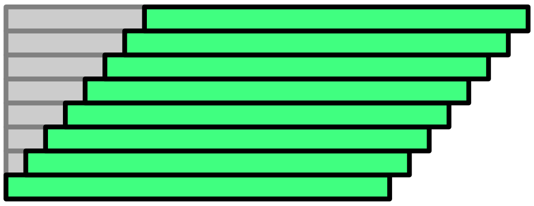

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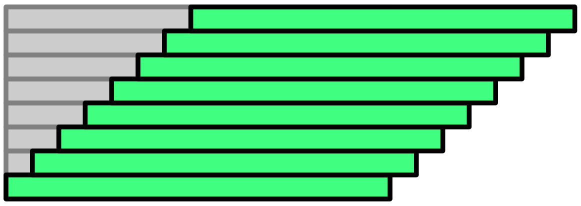

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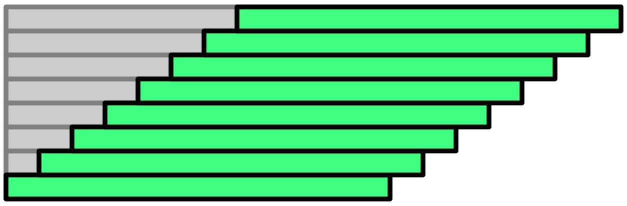

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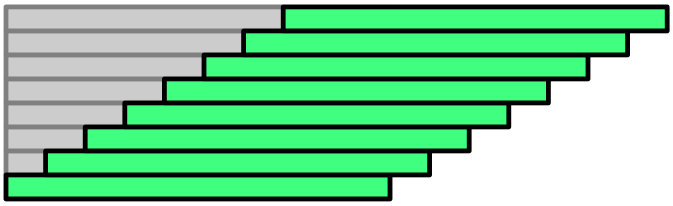

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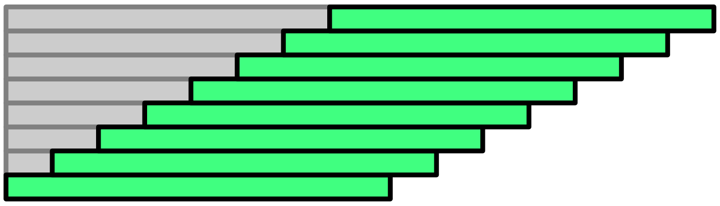

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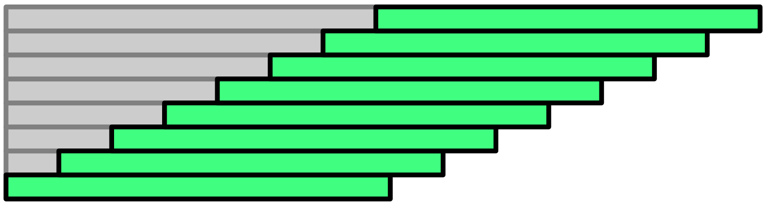

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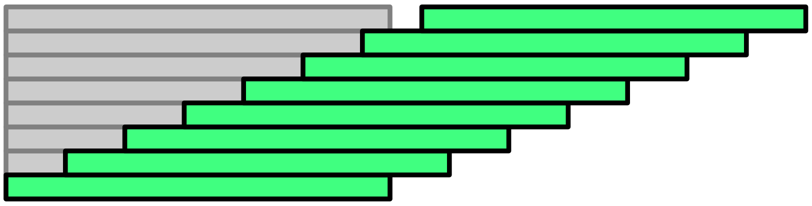

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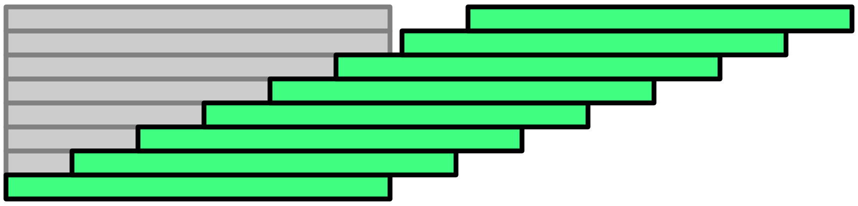

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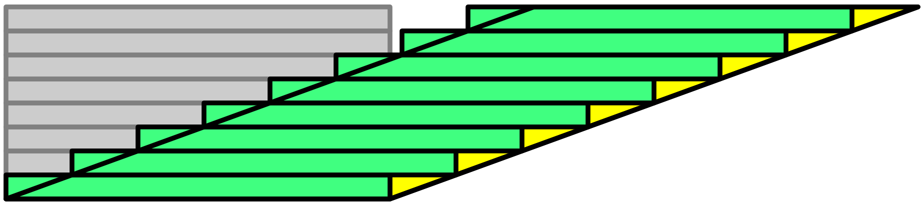

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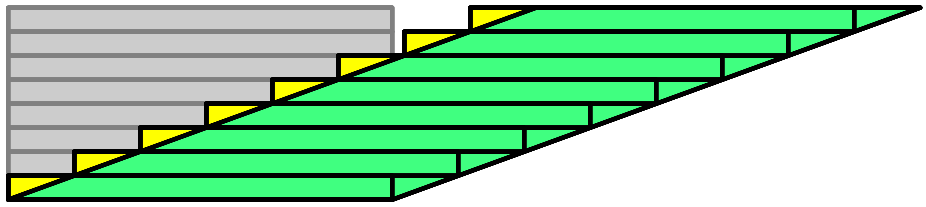
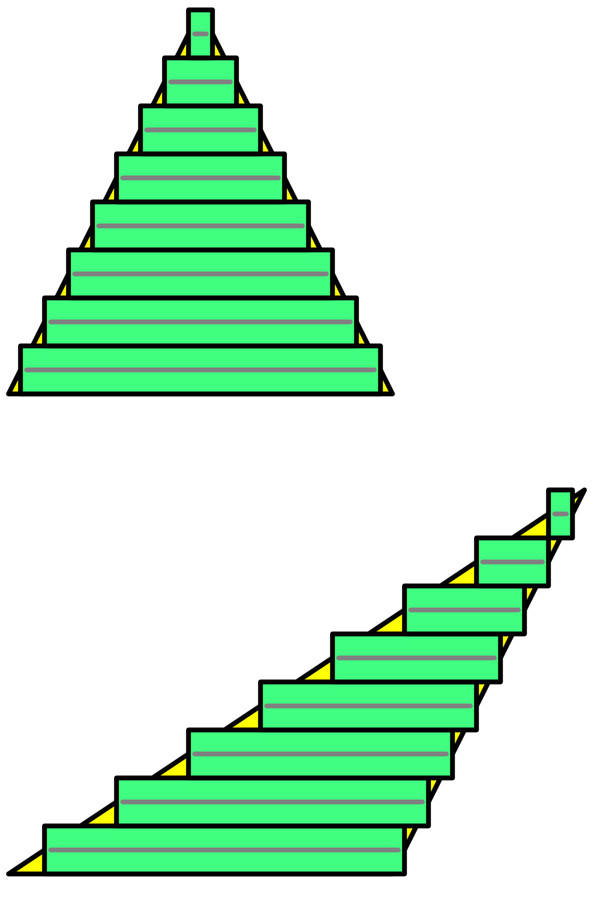


The same proof shows that two triangles with the same base and height have the same area.



Q. E. D.