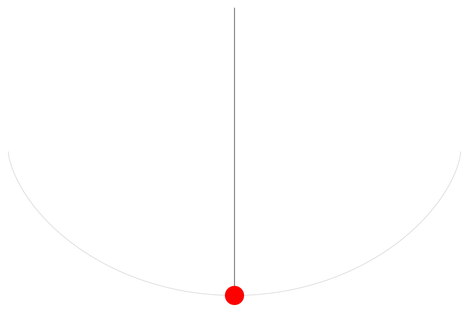
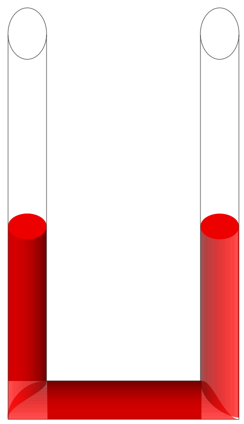
Proving Proposition XLIV. Theorem XXXV

of Book II of Sir Isaac Newton's Principia





A geometric interpretation by Ayda Mehrjou and Matt Steel

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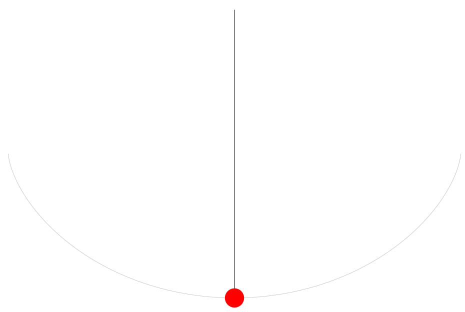
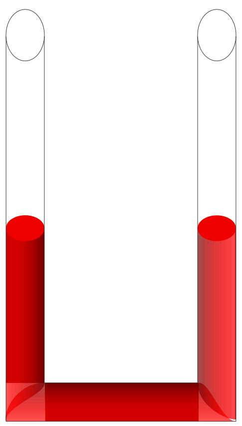
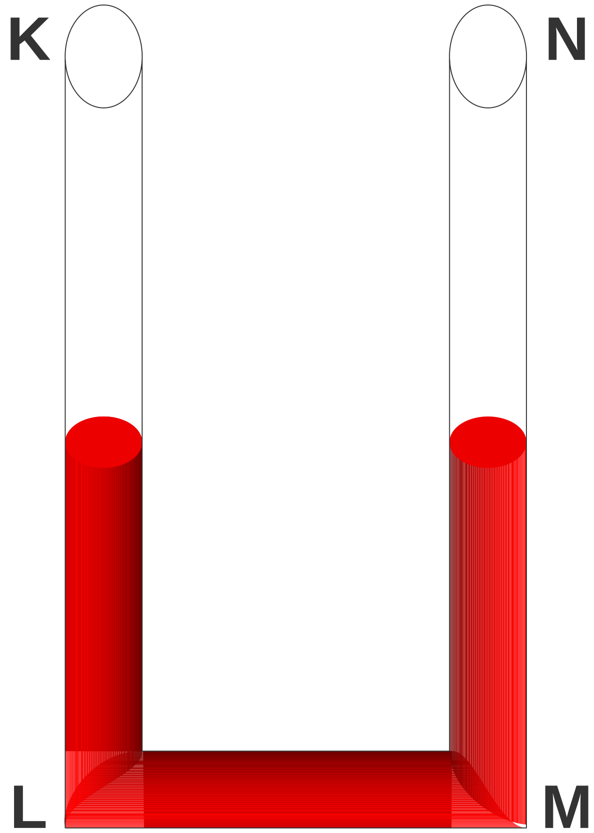
Statement of Theorem

If water ascend and descend alternately in the erected legs KL, MN of a canal or pipe, and a pendulum be constructed whose length between the point of suspension and the centre of oscillation is equal to half the length of the water in the canal: I say, that the water will ascend and descend in the same times in which the pendulum oscillates.

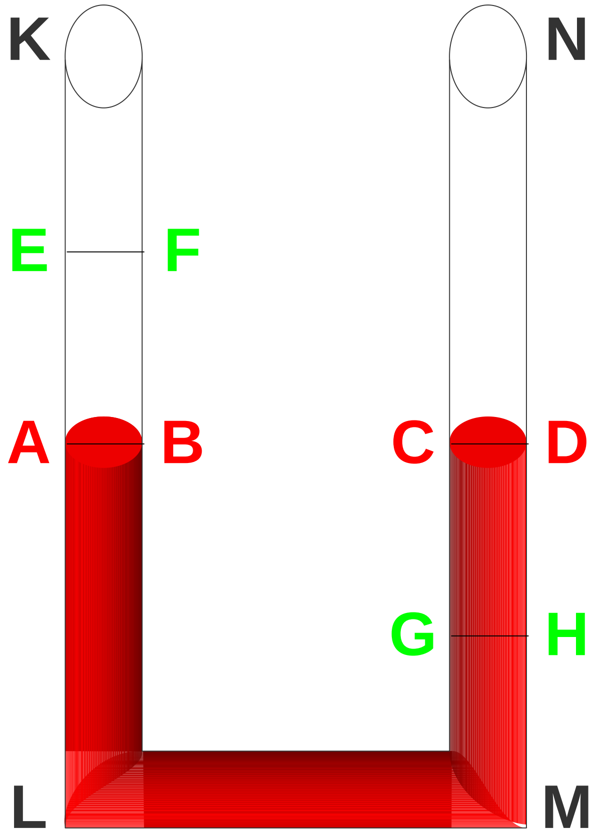




Water Cylinder Setup



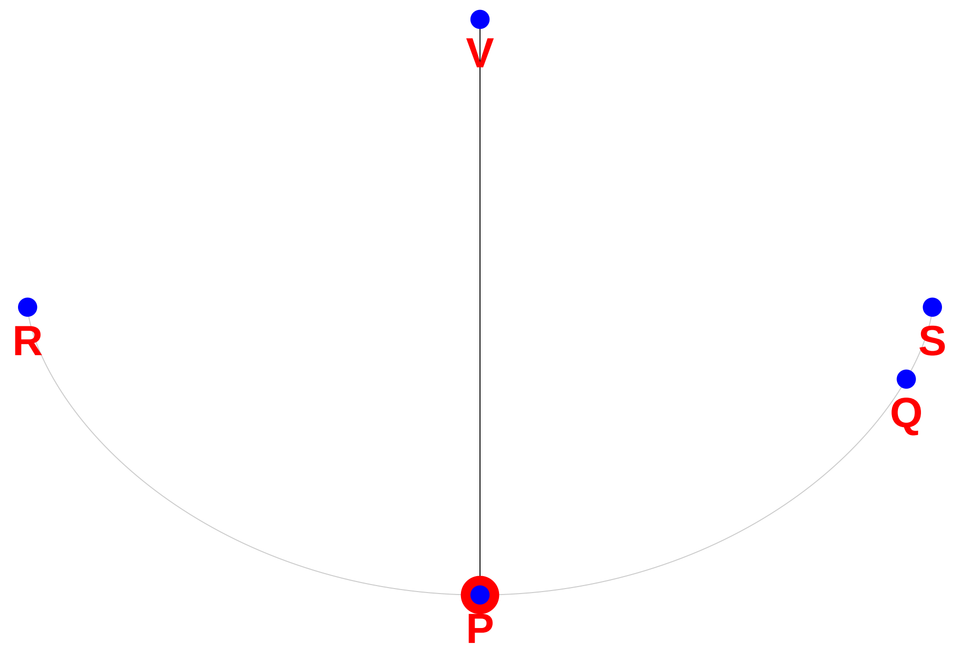
Water Levels



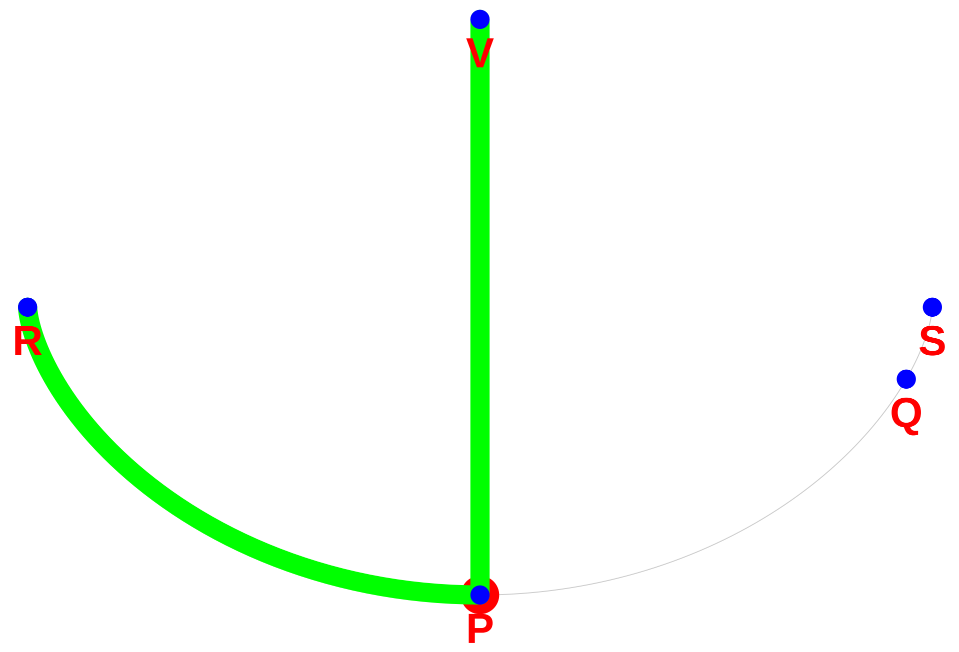
The Red heights are the mean water levels.

The Green heights are the levels the water will ascend or descend to

Pendulum Setup



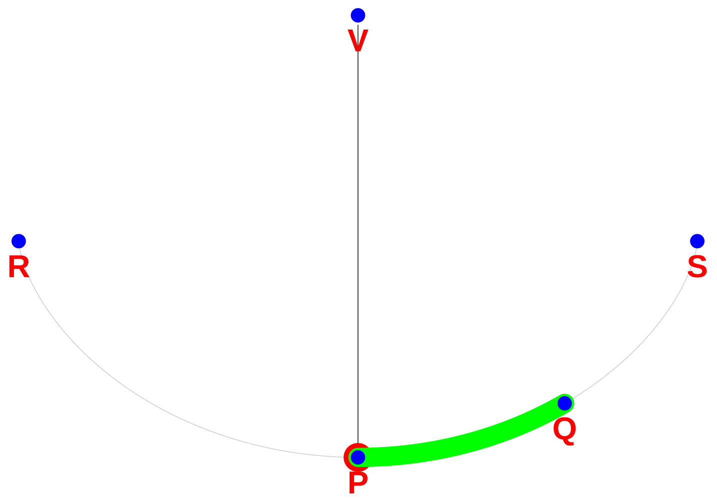
Pendulum Arc Lengths

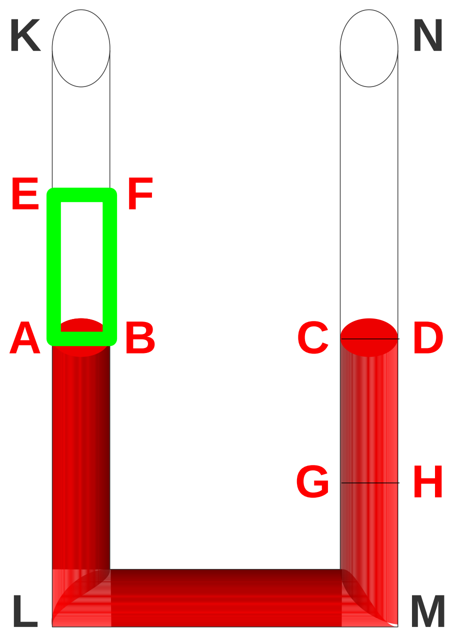


The Green paths are equal.

The pendulum path follows the arc of a cycloid.

Water and Pendulum Height Relation

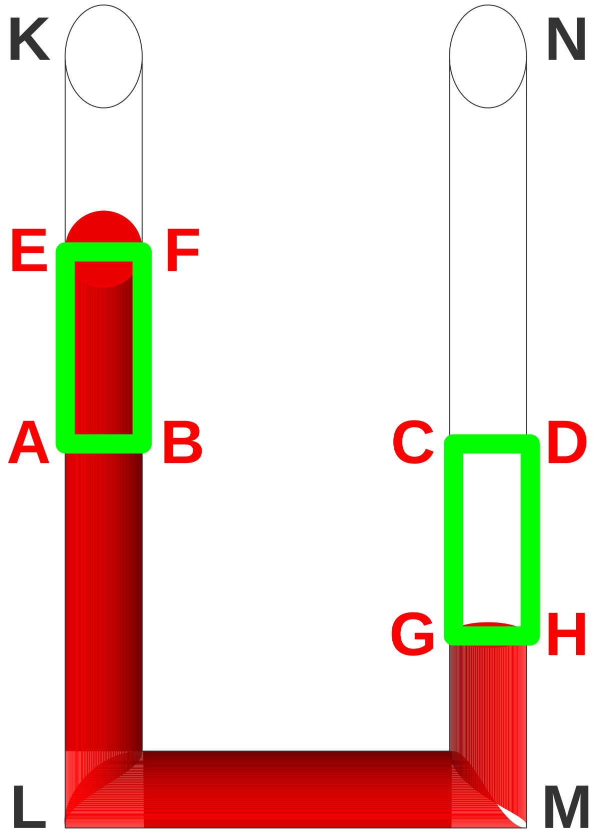




The lengths of the Green paths are equal.

AE = PQ

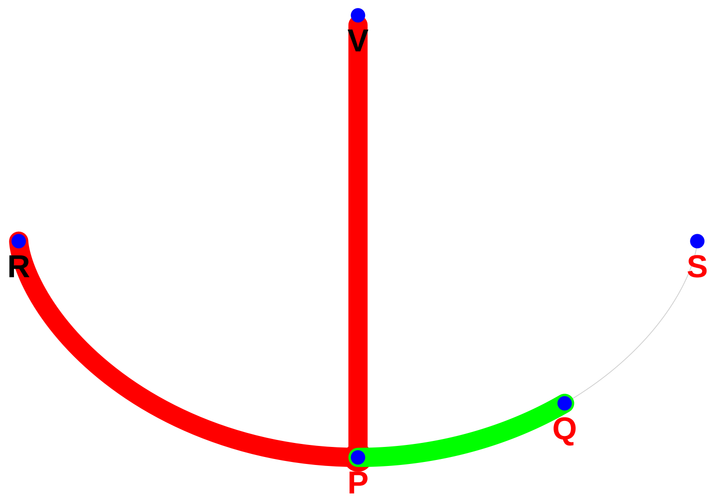
Accelerating Force on the Water

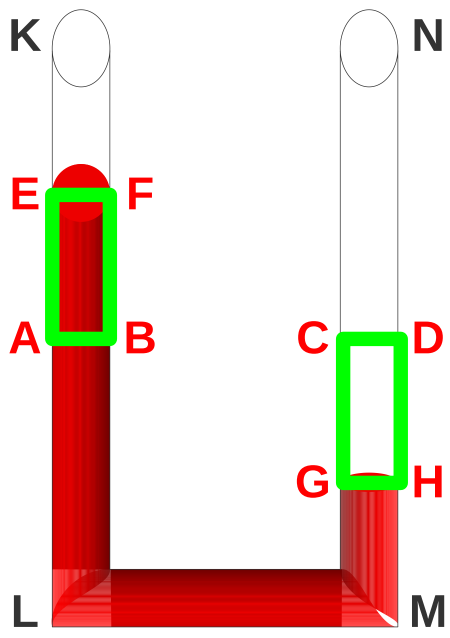


The two Green boxes add to make the force on the water.

Accelerating force on water = weight of EABF x 2

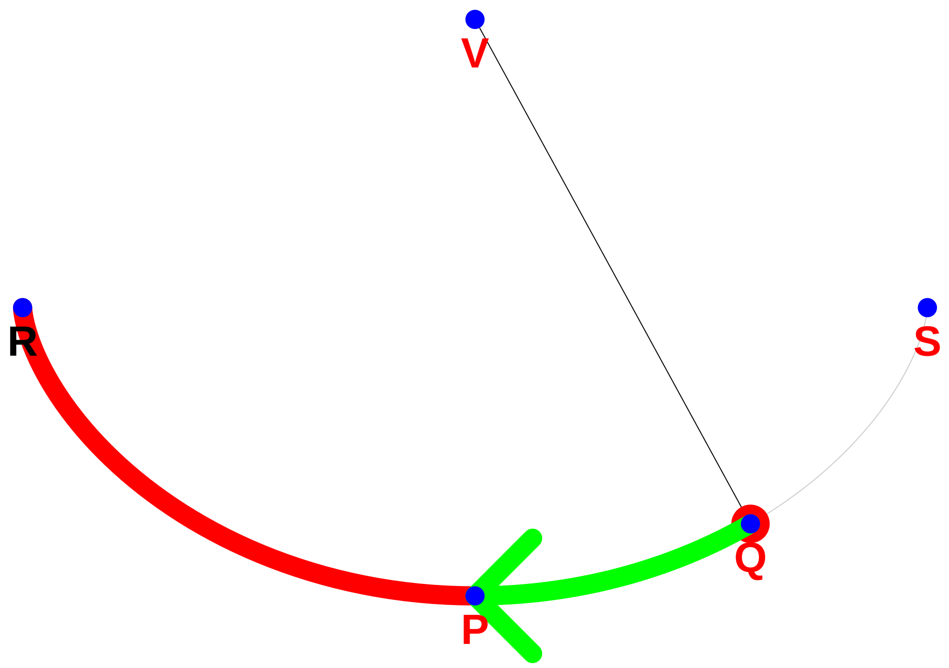
Ratio of Water Force to Water Height





The ratio of the water weight in Green to
the total water weight in Red, is the same as
the ratio of the length of any Green path to
the length of either red path on the pendulum
So, the force ratio = PQ over PR

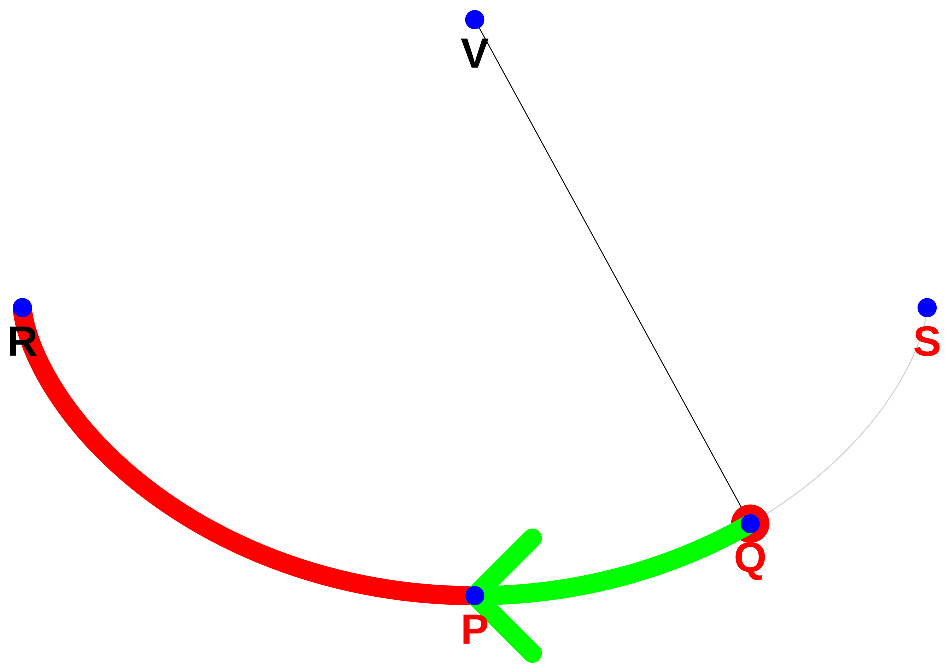
Accelerating Force on the Pendulum



The accelerating force of the Green arrow is propotional to its length.

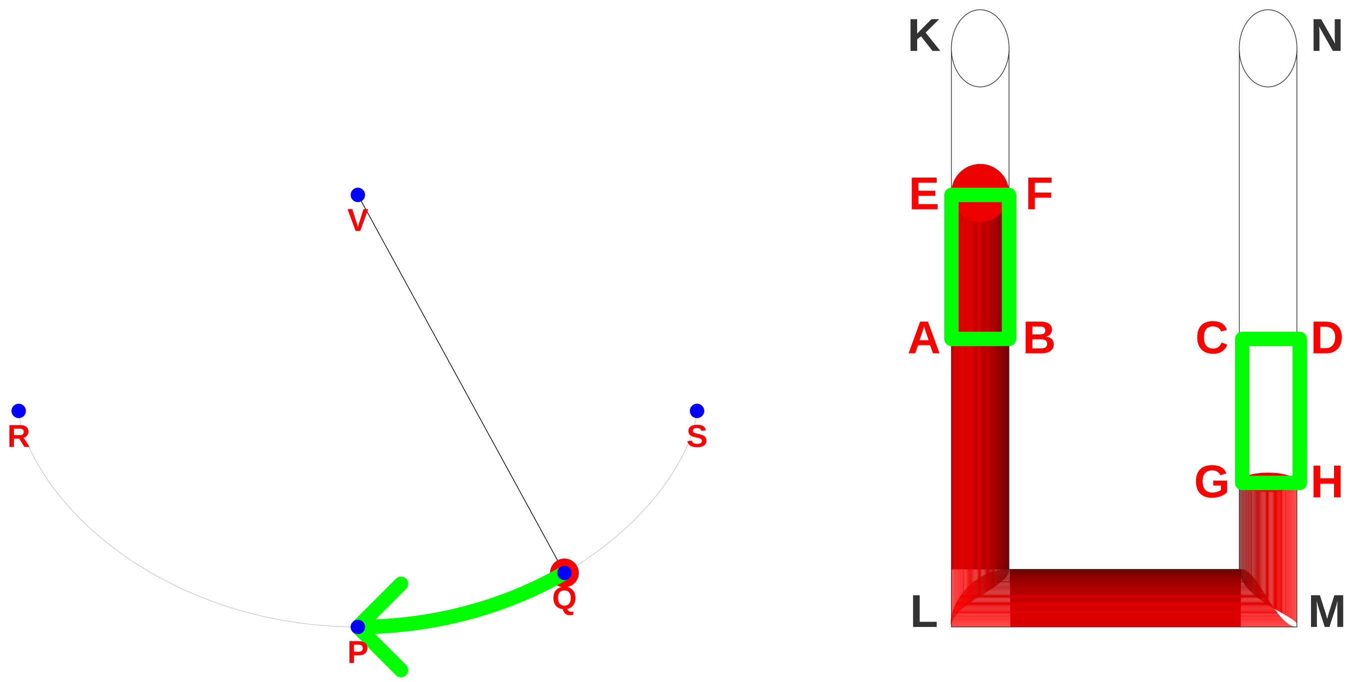
Accelerating force on pendulum ~ PQ over PR

Ratio of Pendulum Force and Pendulum Height



The ratio of the accelerating force of the Green arrow to
the total weight of the Red ball, is the same as
the ratio of the length of the Green arrow to
the length of the Red pendulum path.
So, the force ratio = PQ over PR

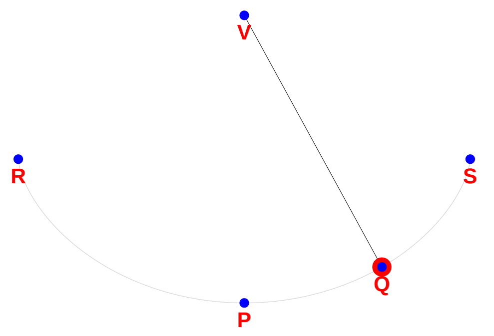
Forces on Water and Pendulum

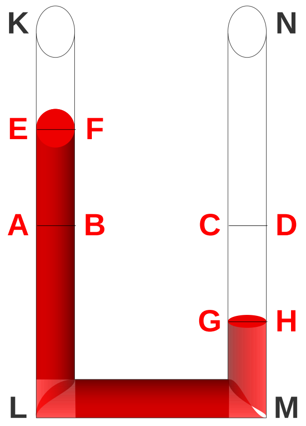


The Green forces on the water and the pendulum are dependent on equal distances

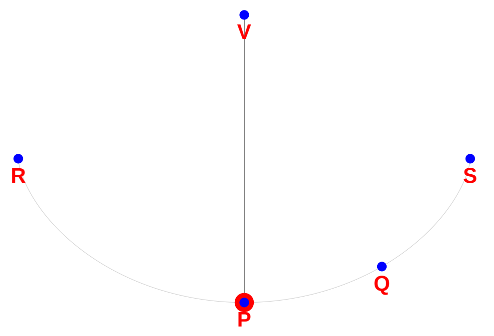
So, given that the initial distances AE and PQ are equal...

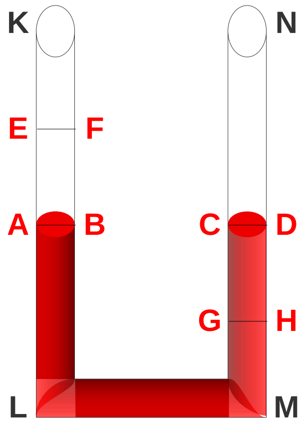
Forces act in Equal Time





time = 1

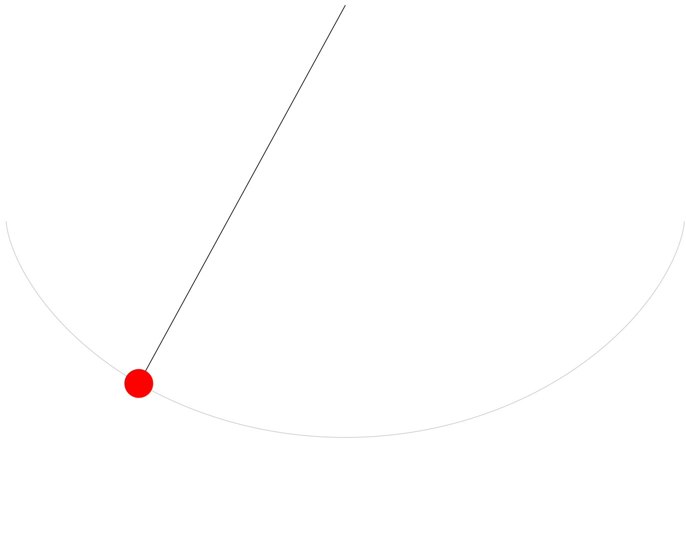


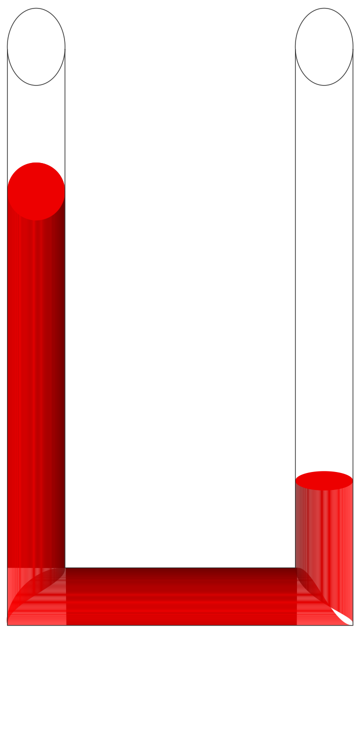


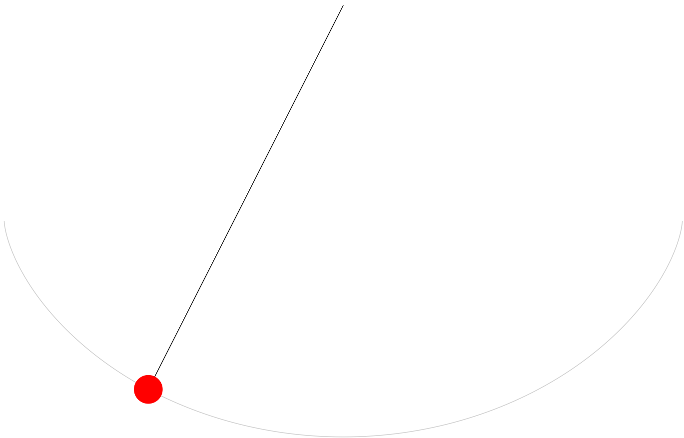
time = 2

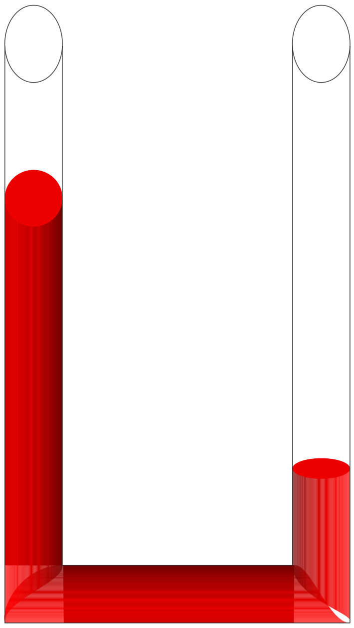
QED

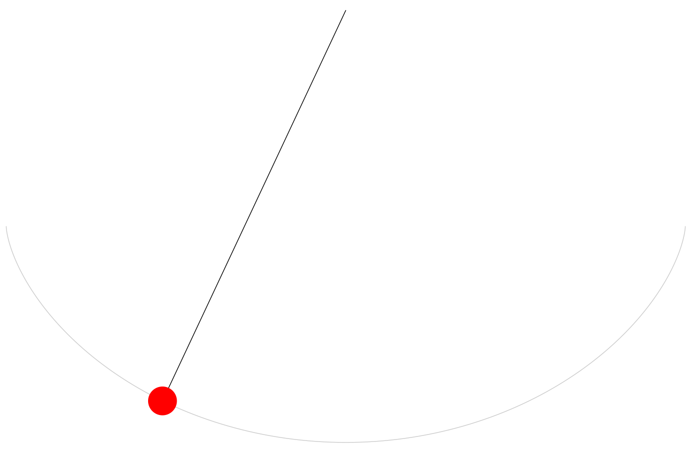
Animation of Theorem

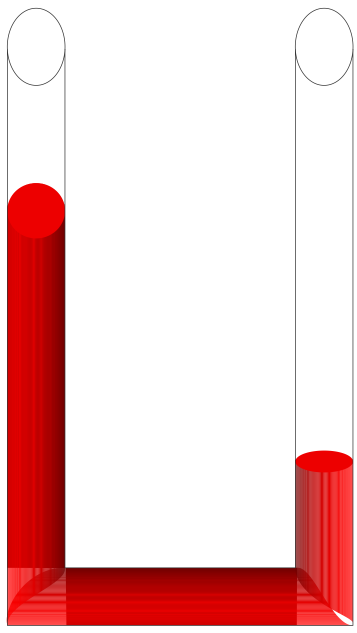


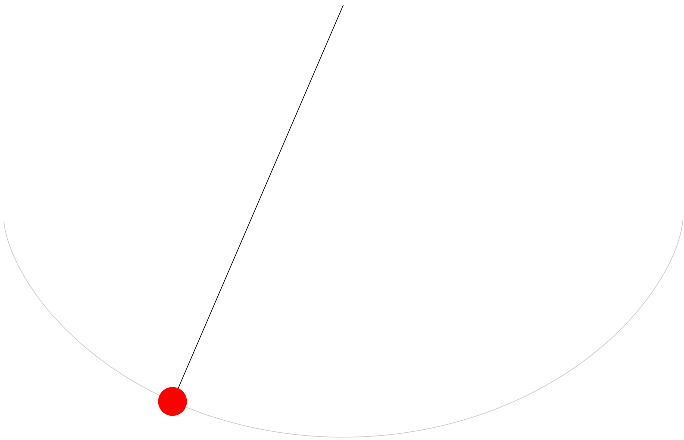


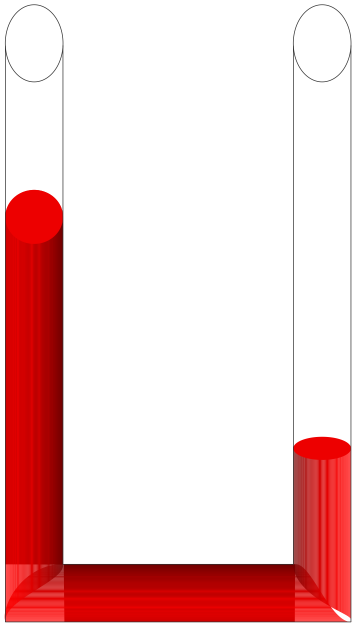


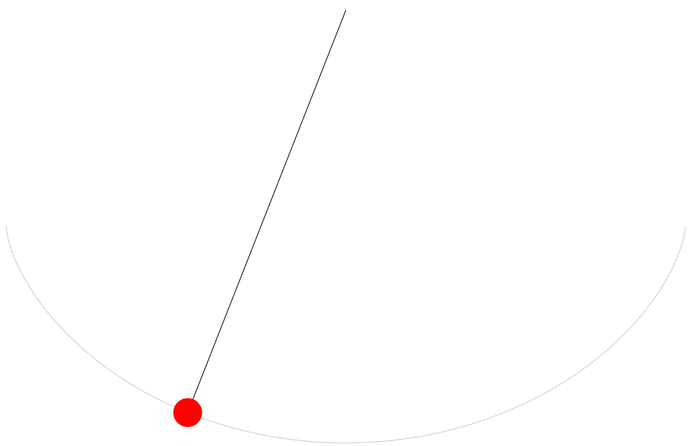


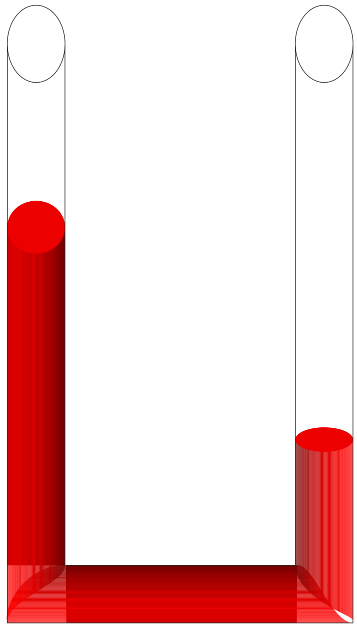


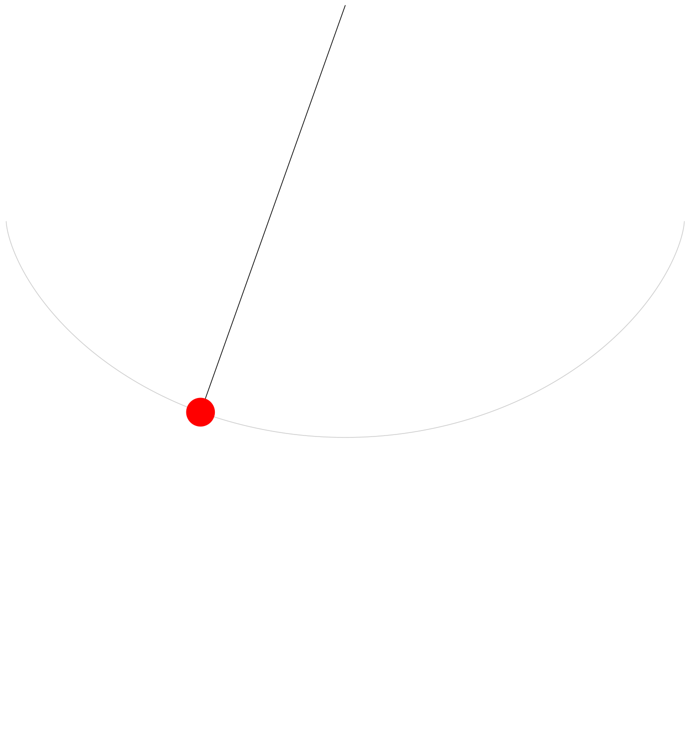


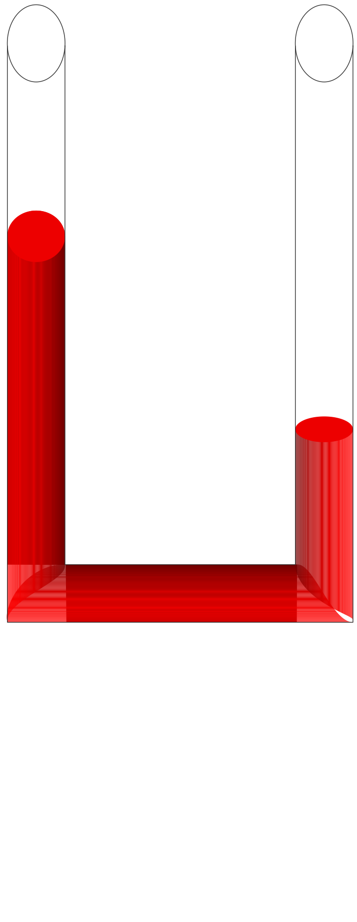


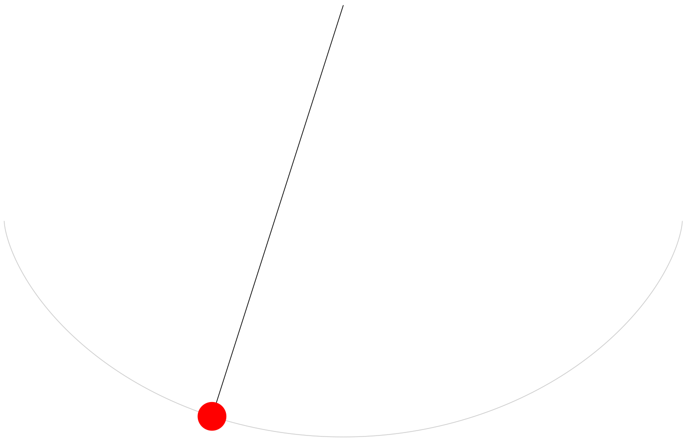


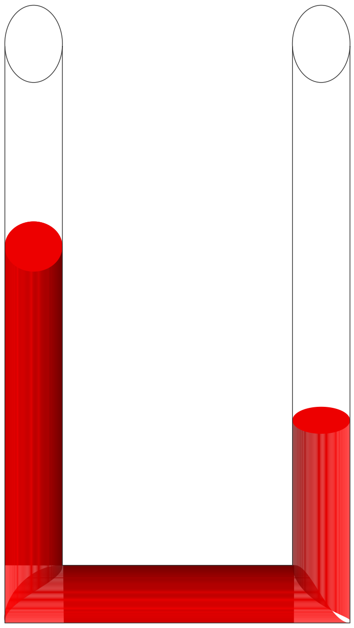


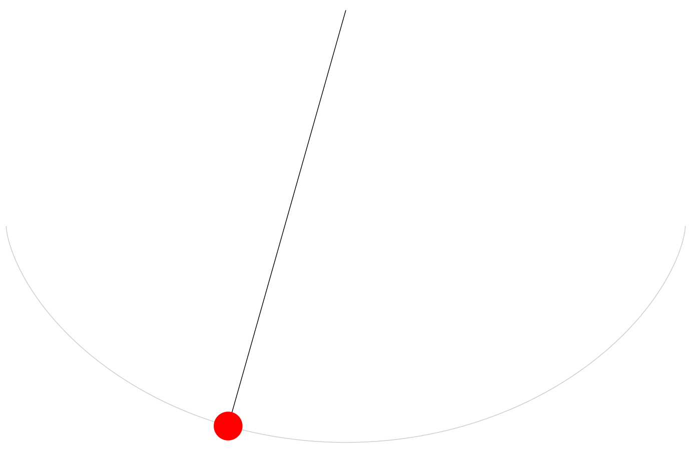


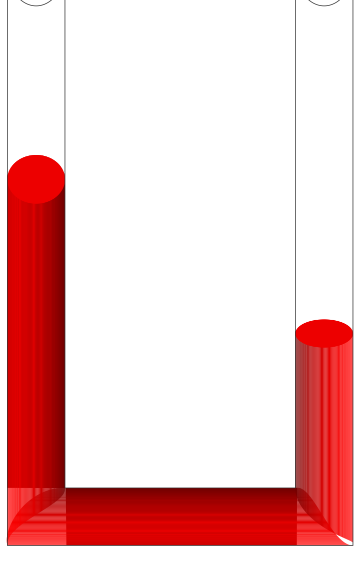


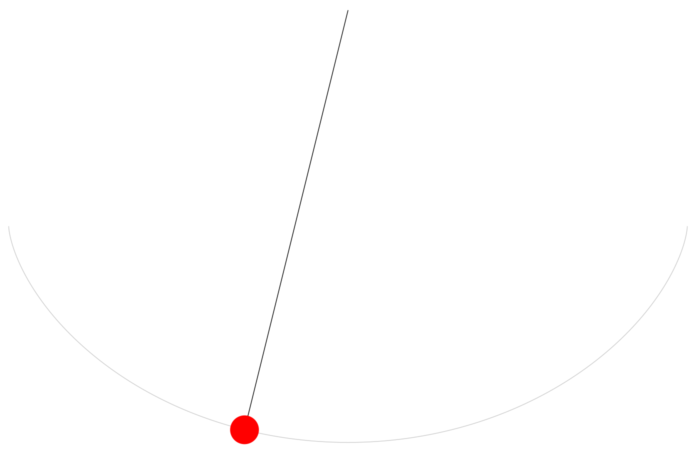


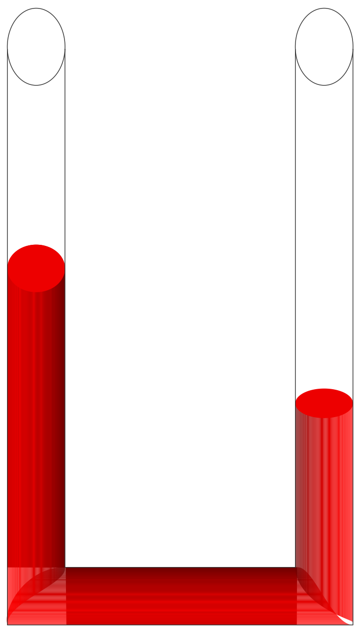


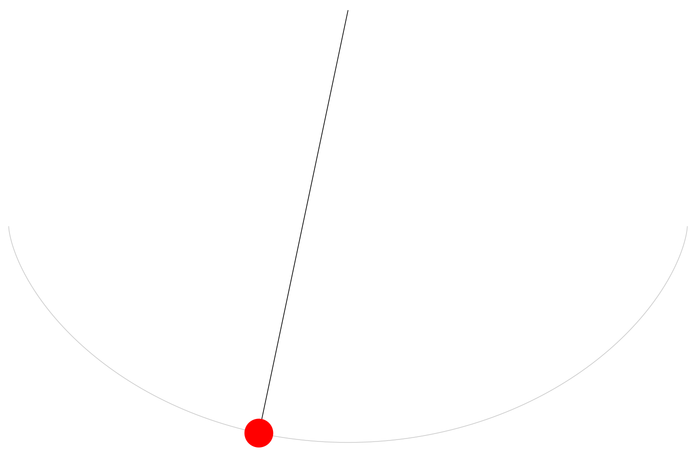


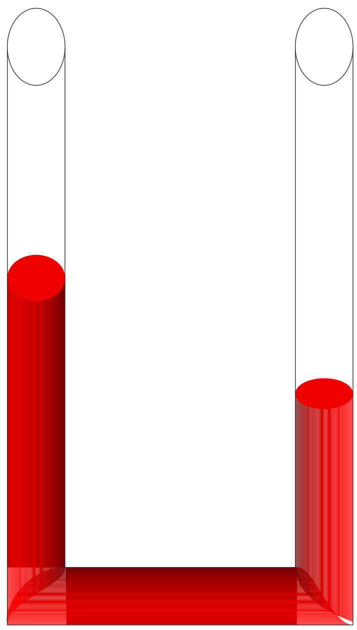


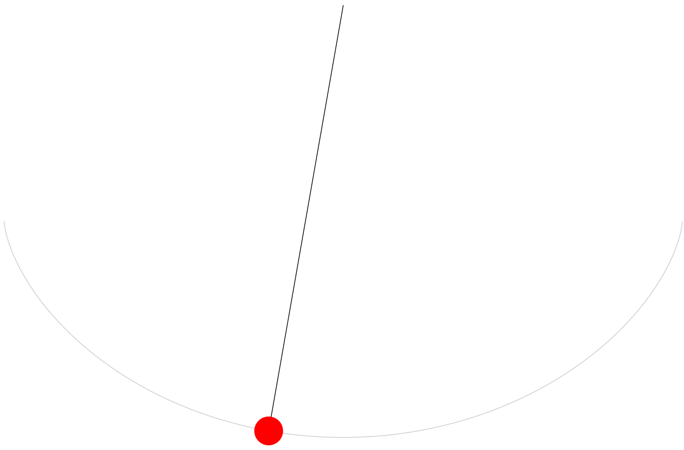


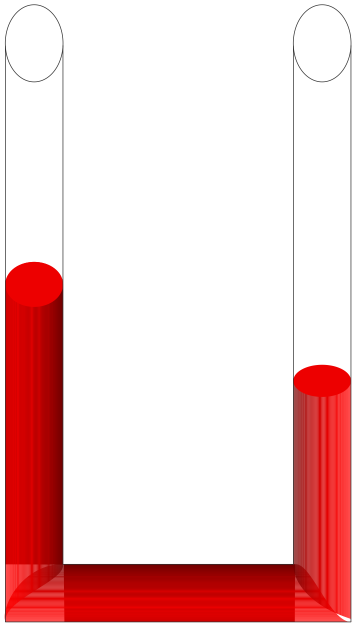


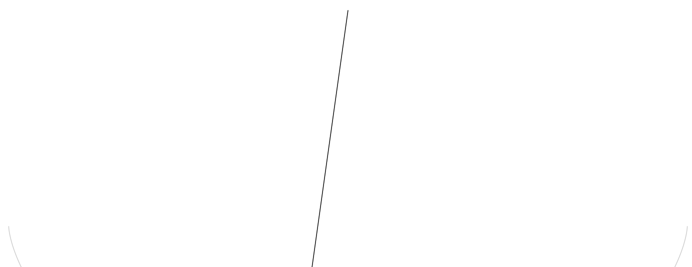


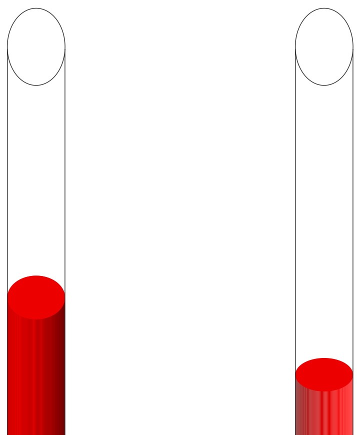


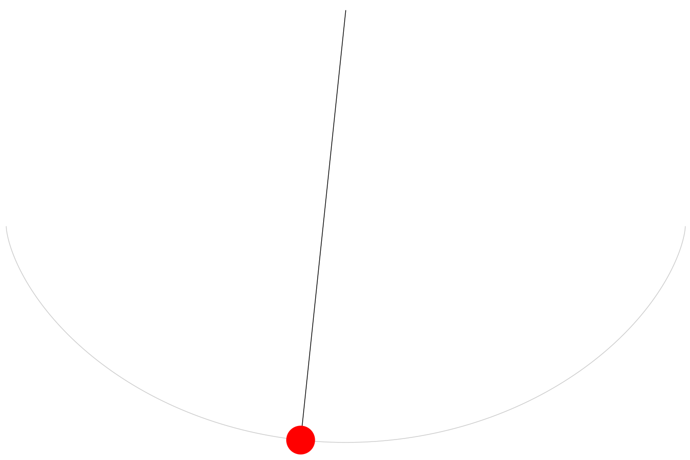


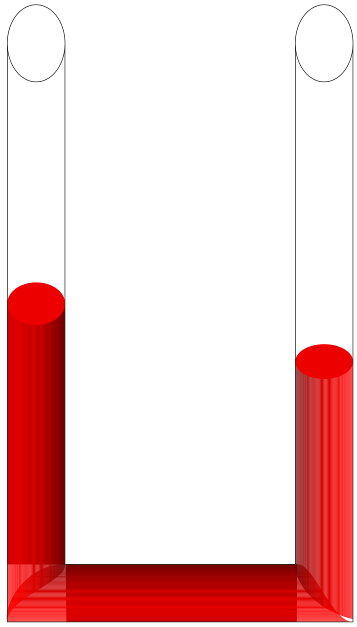


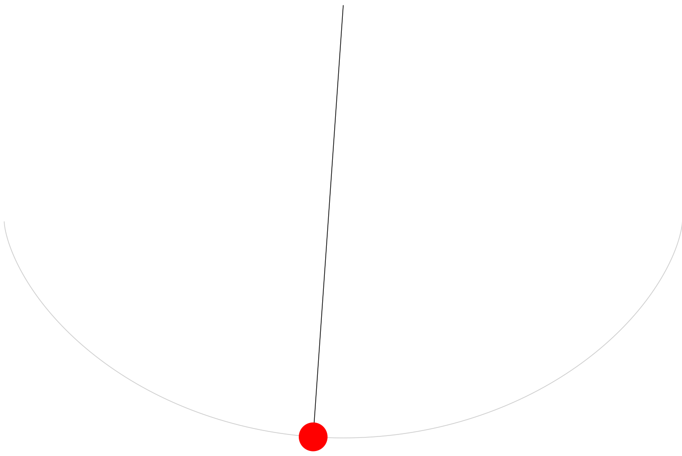


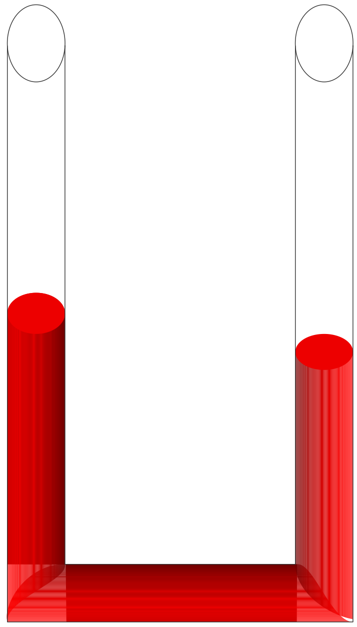


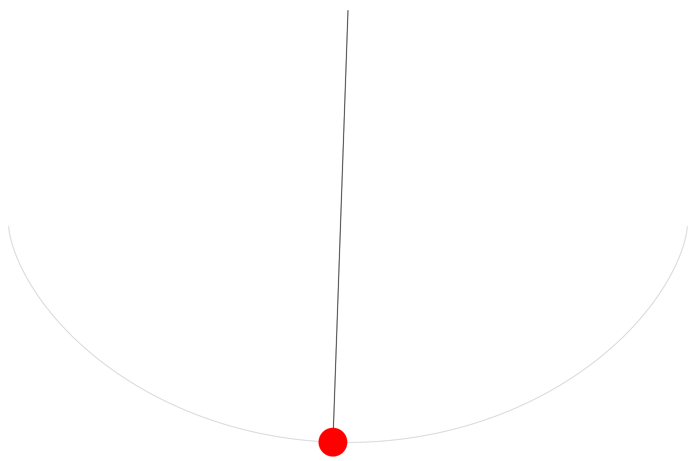
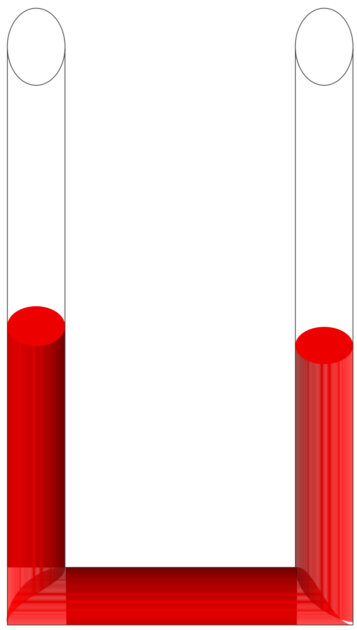


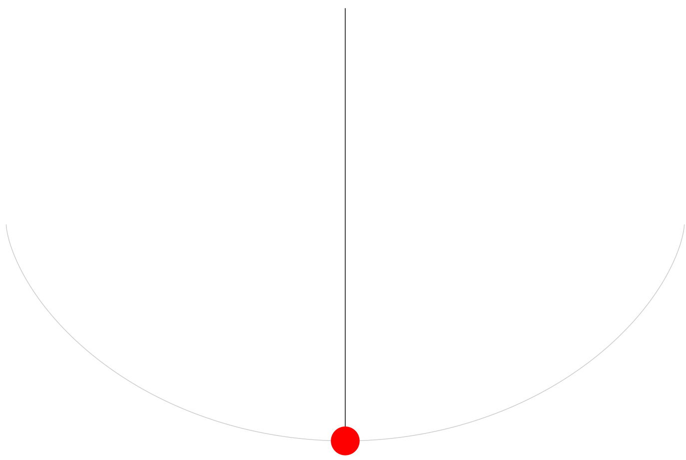


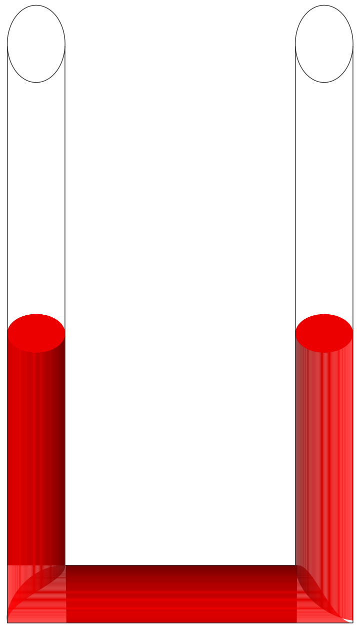


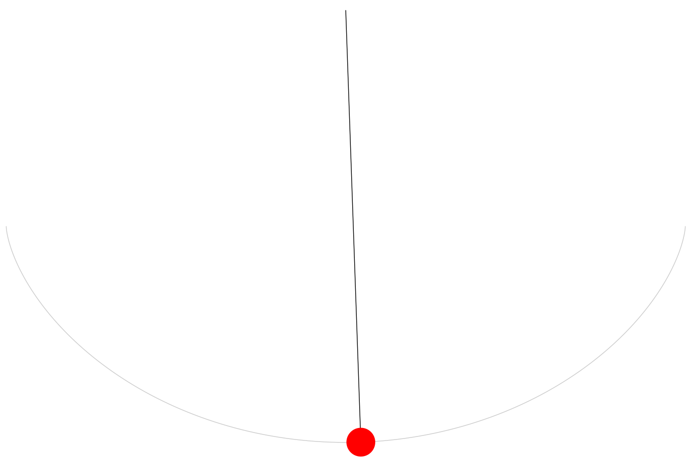


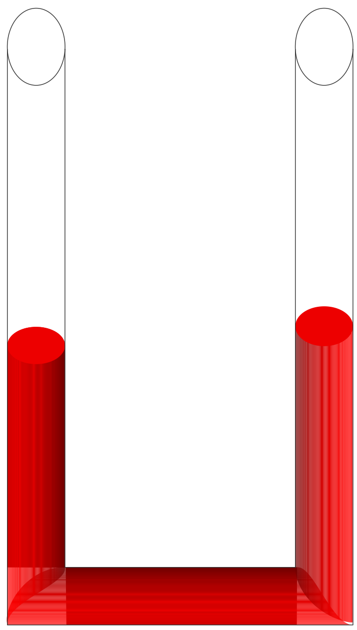


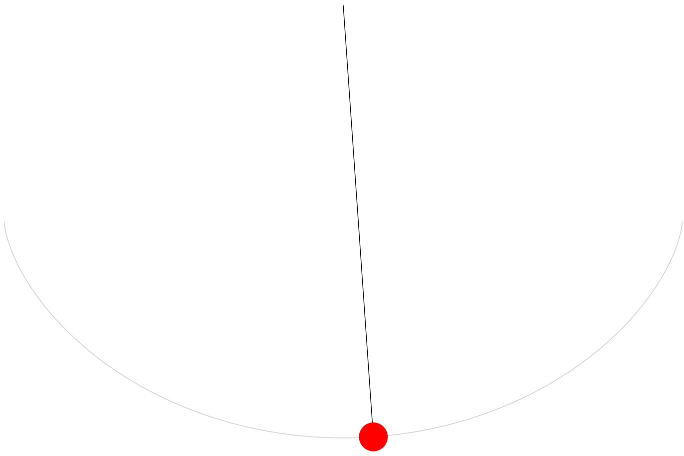



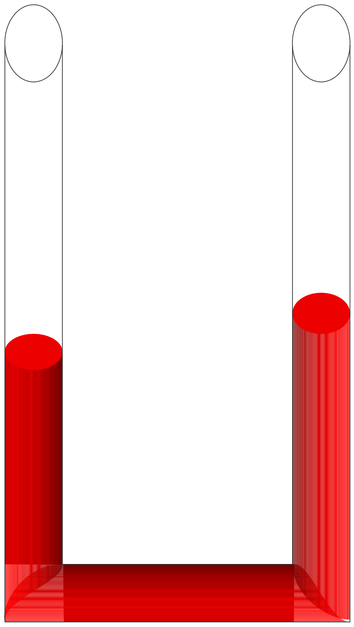


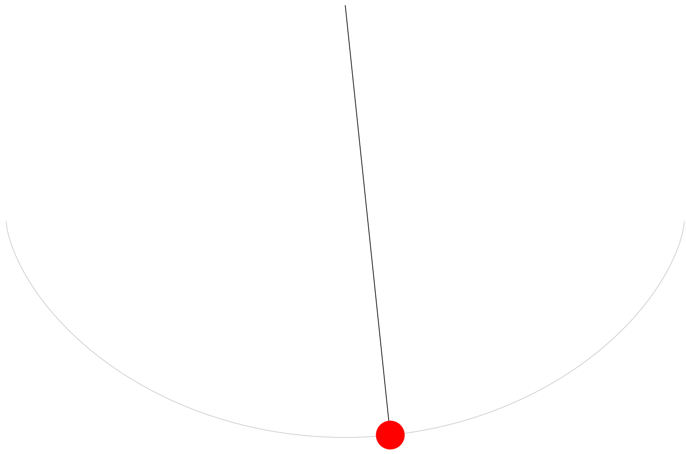


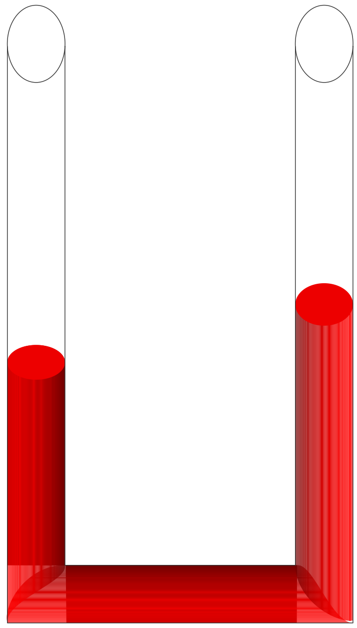


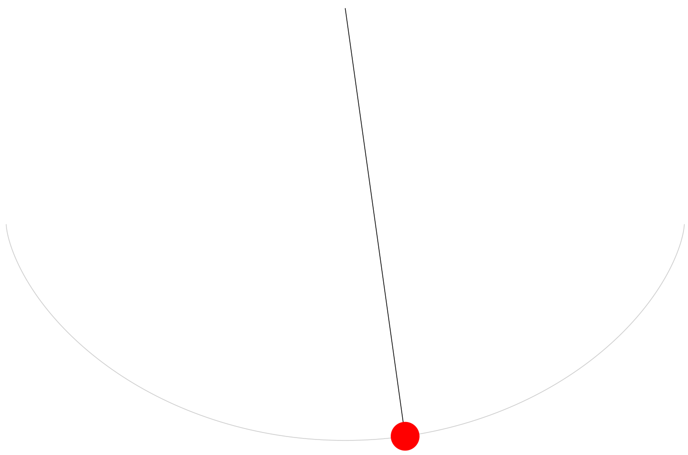


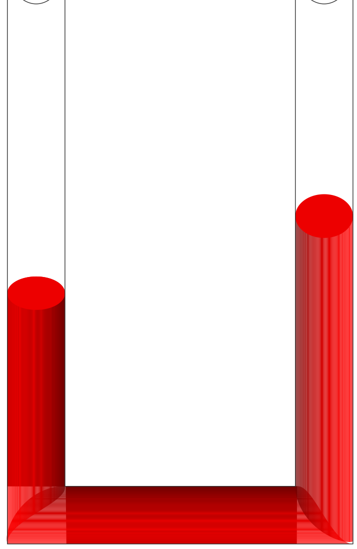


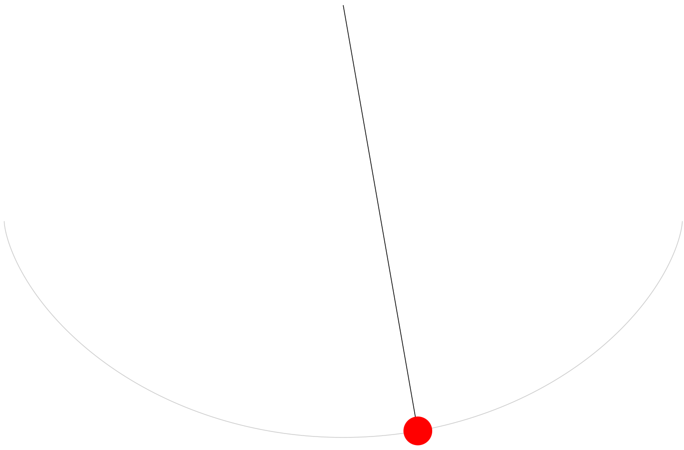
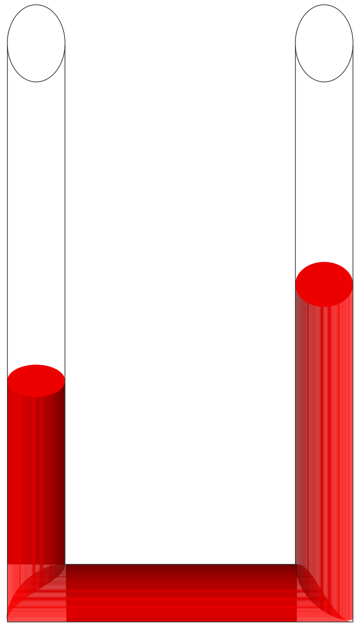


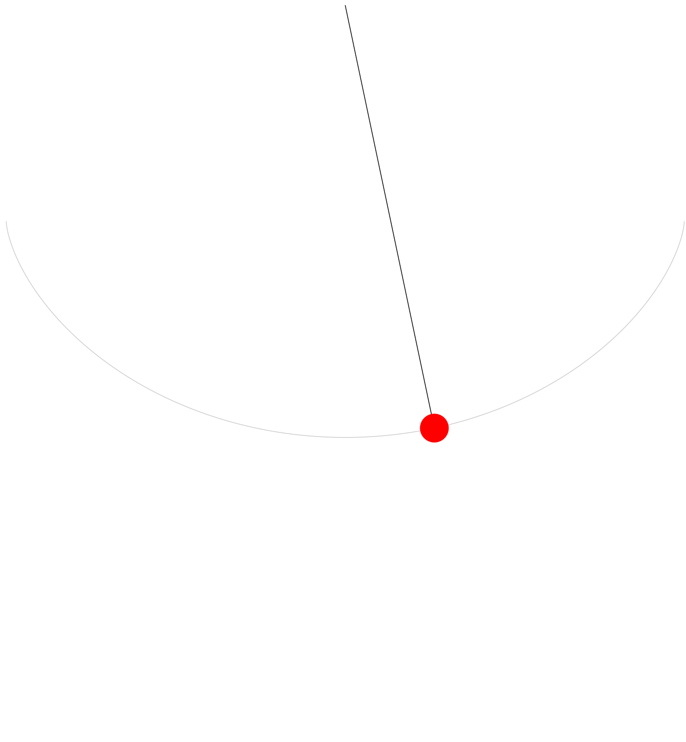


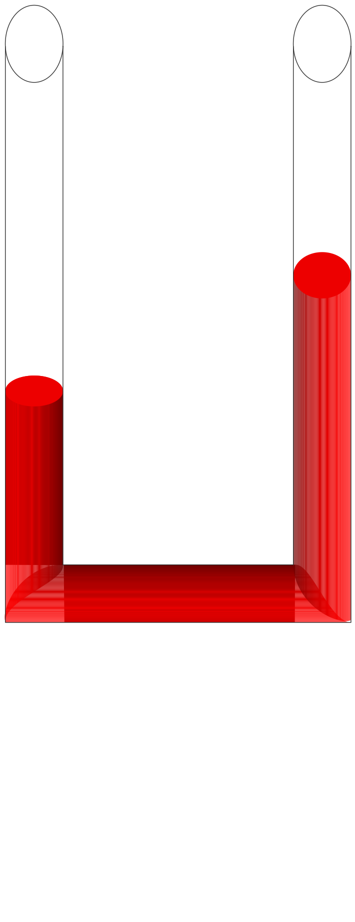


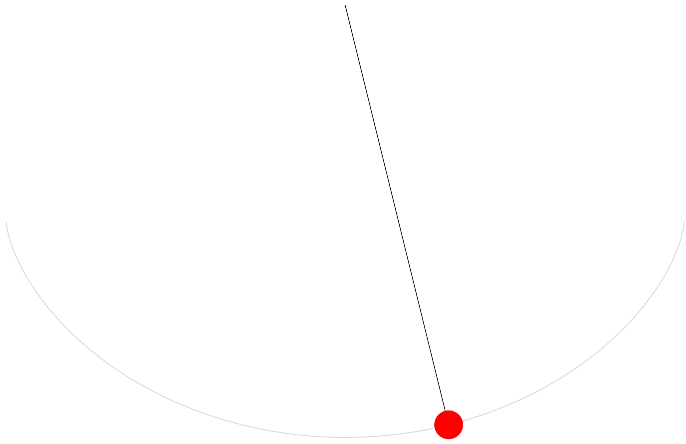


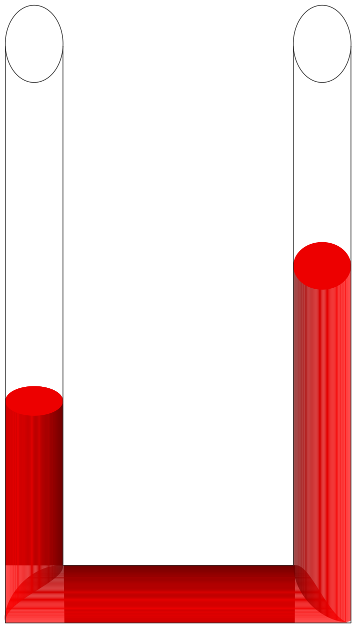


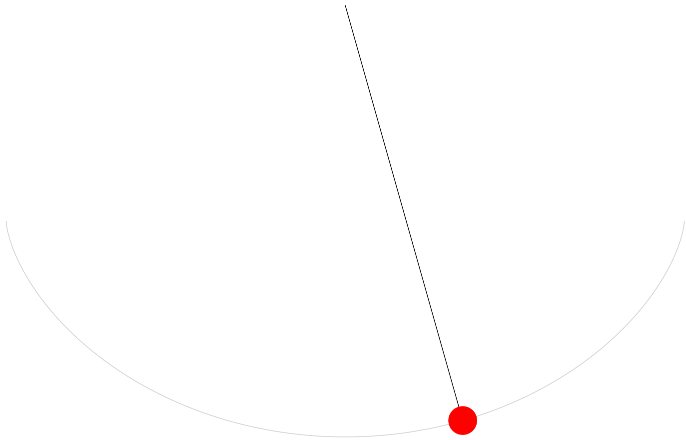
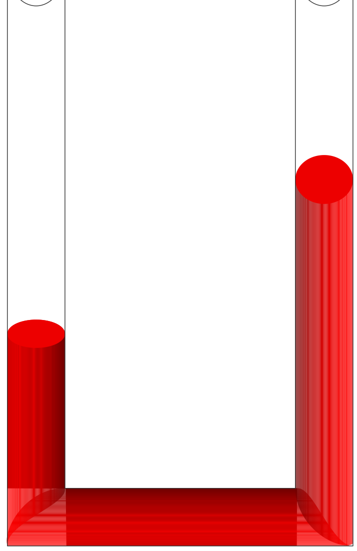



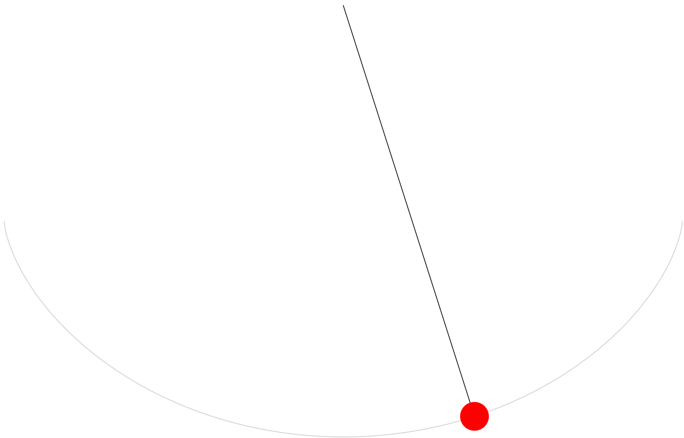


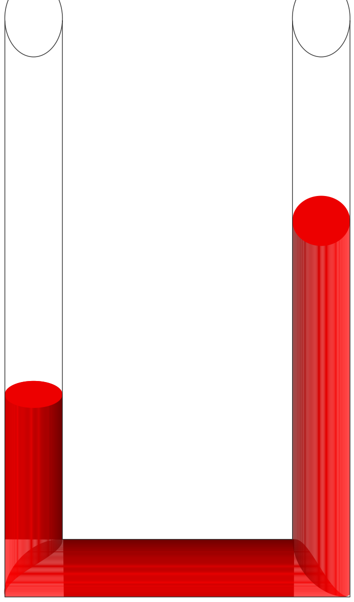


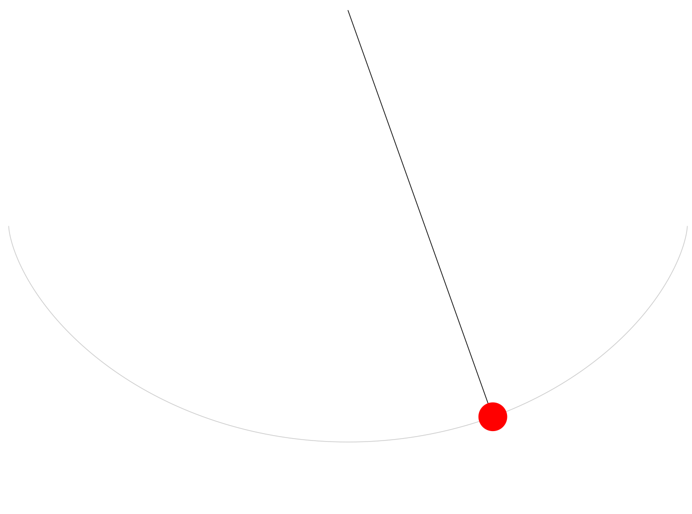


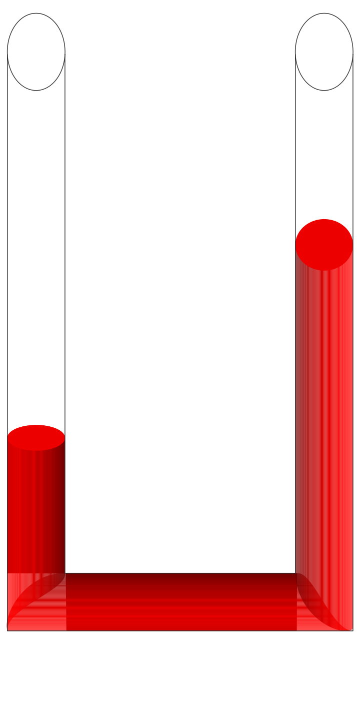


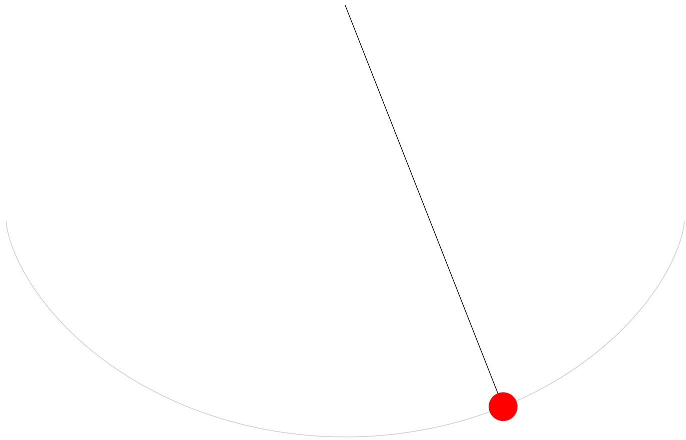
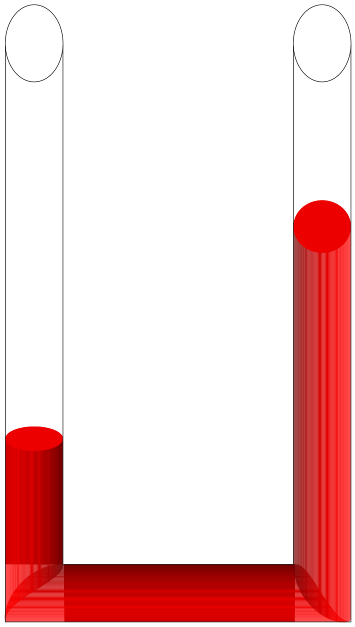



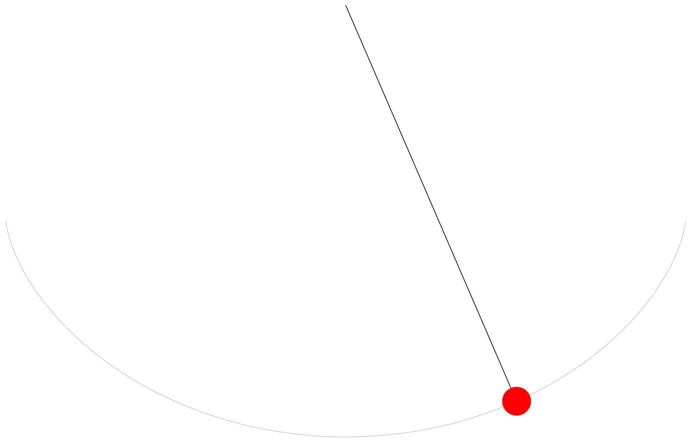


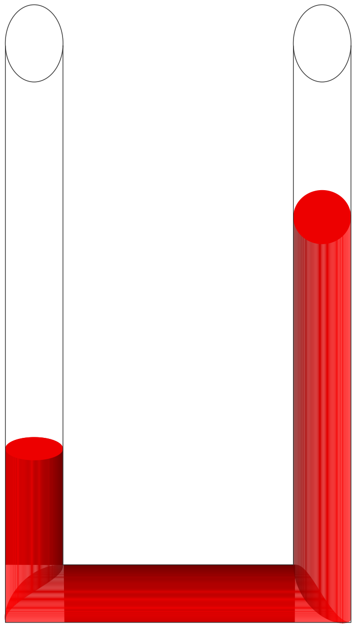


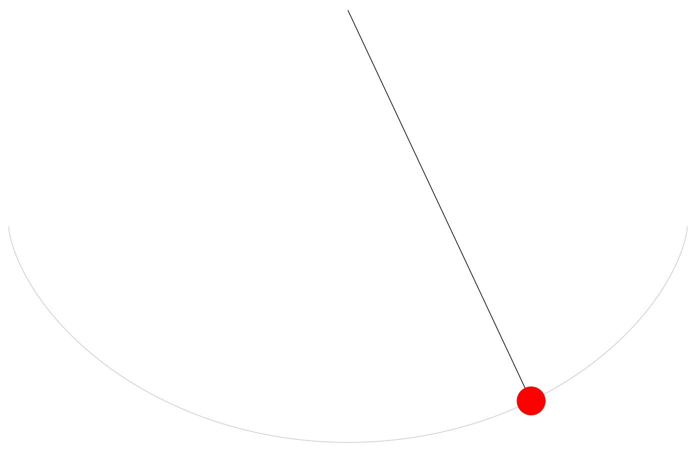


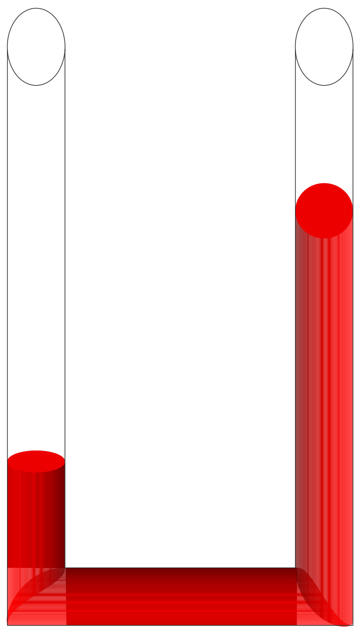


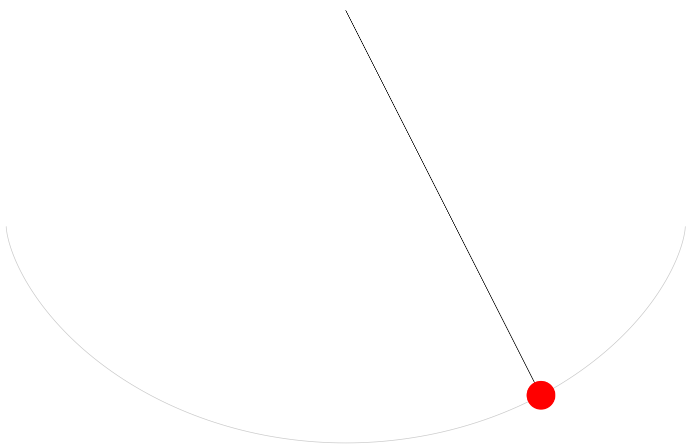



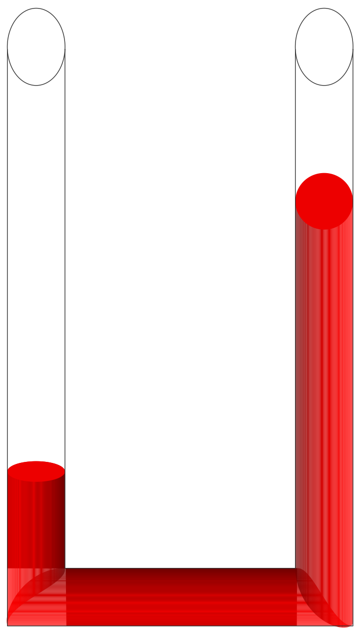


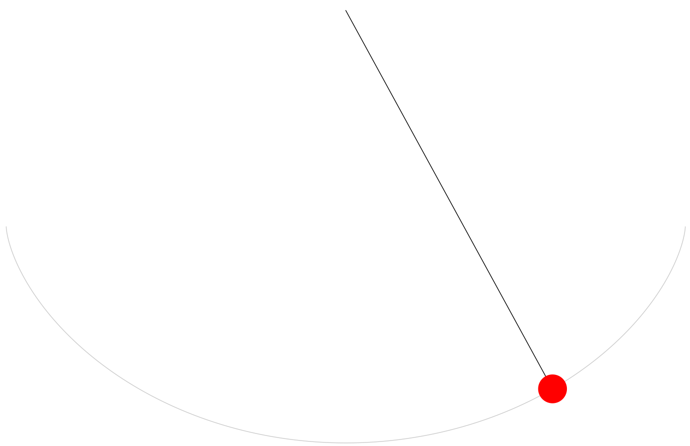
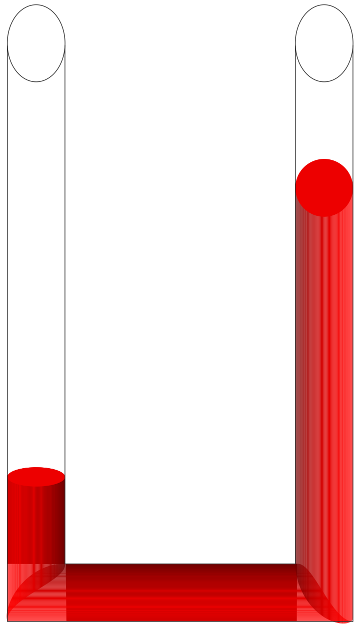


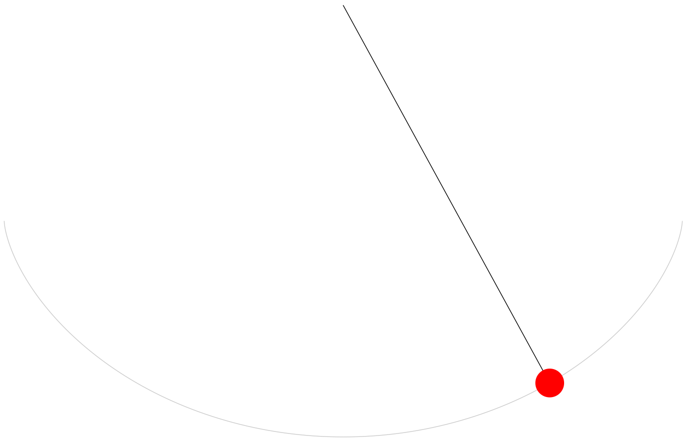


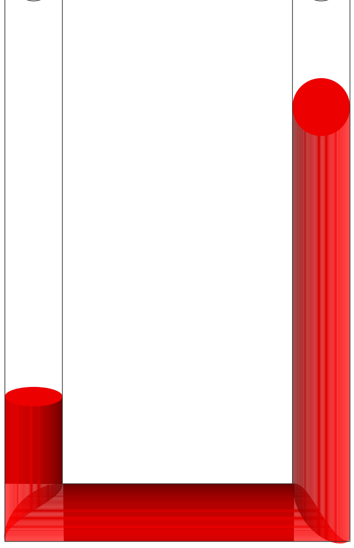


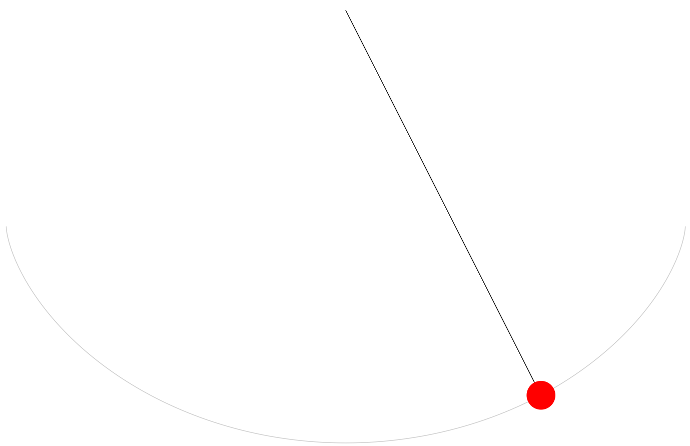


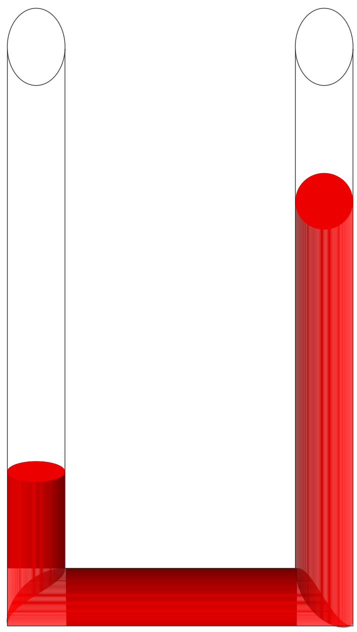


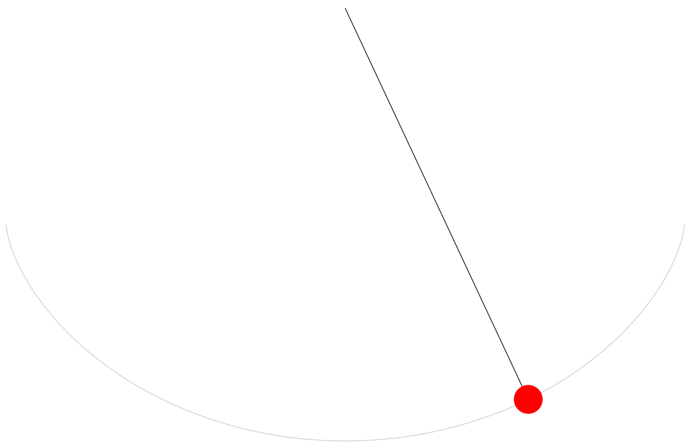
 

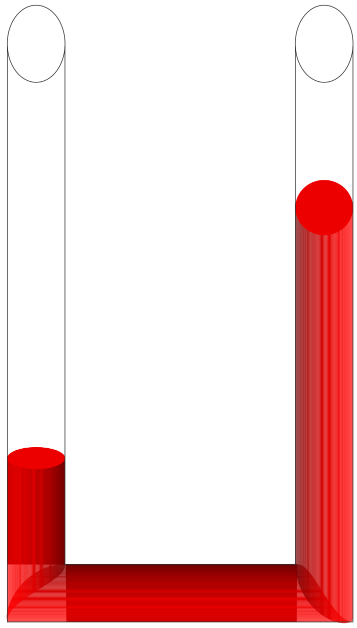


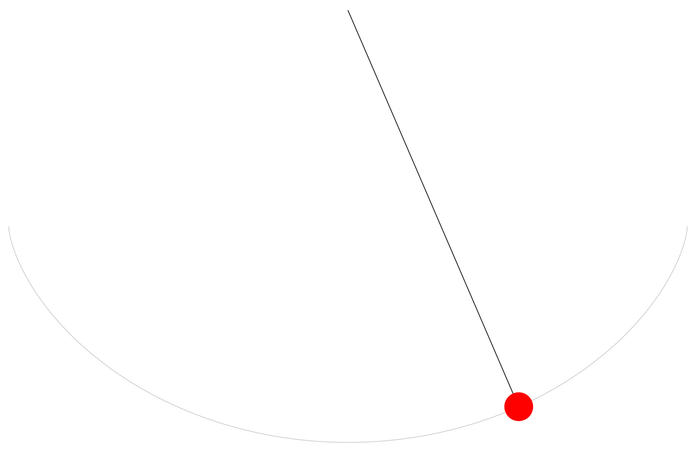


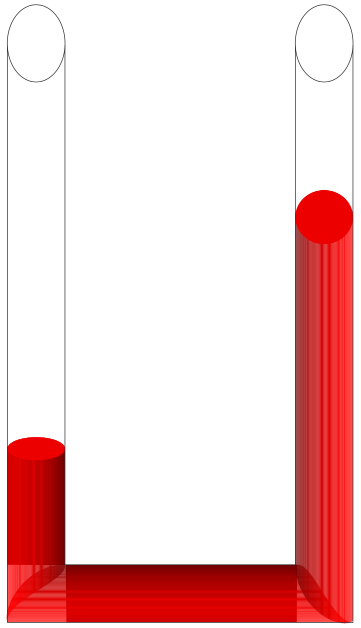


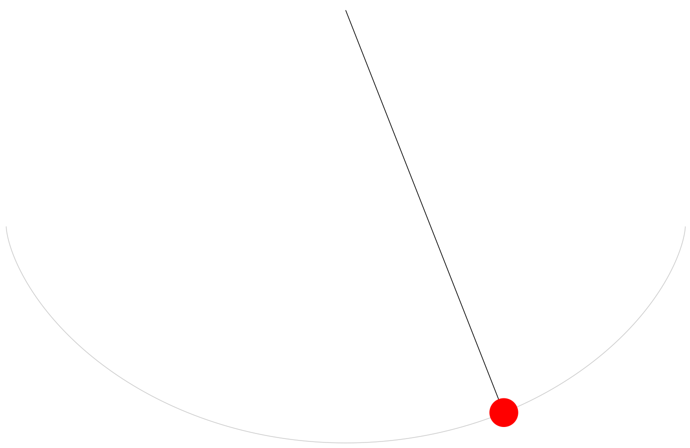


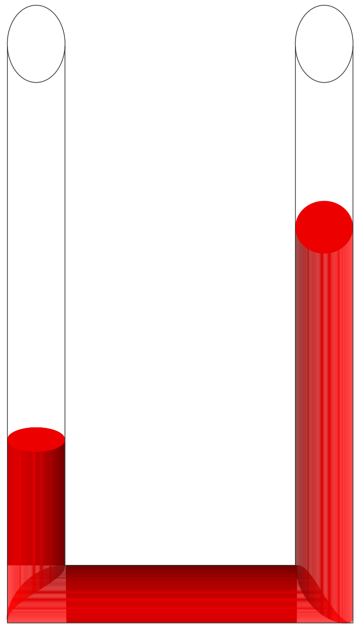


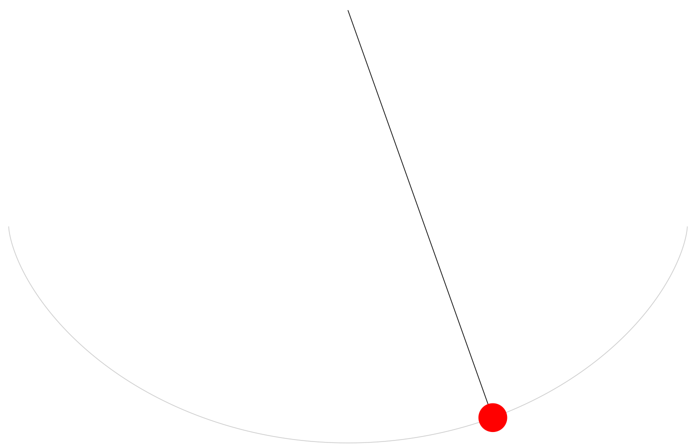


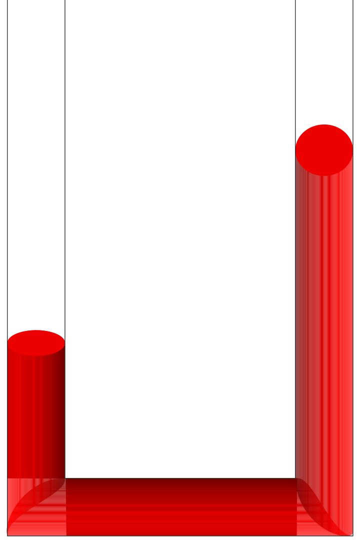


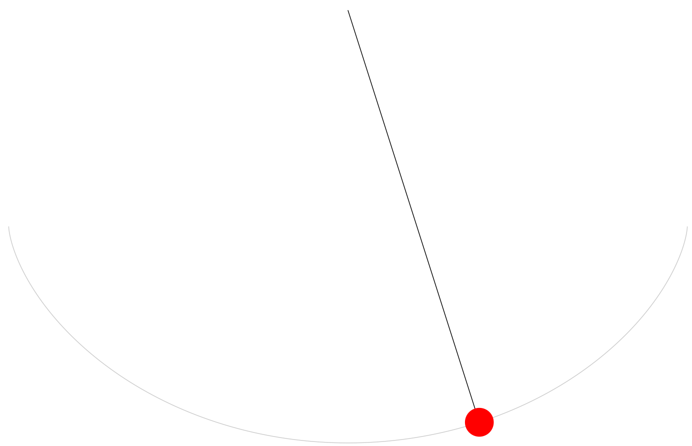


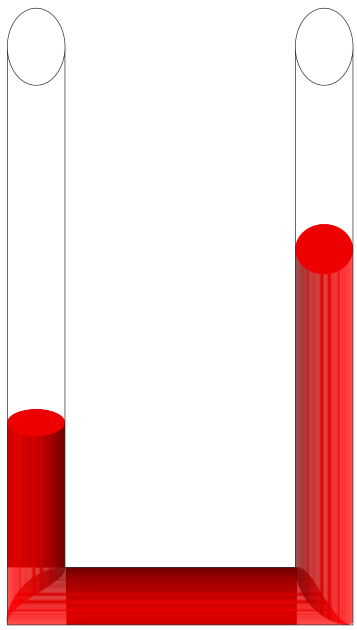


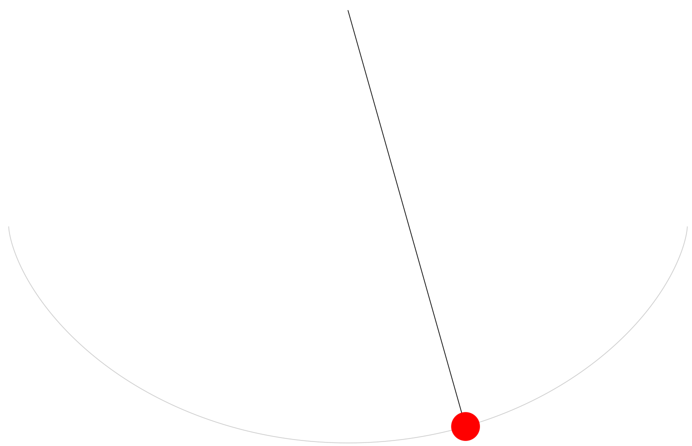


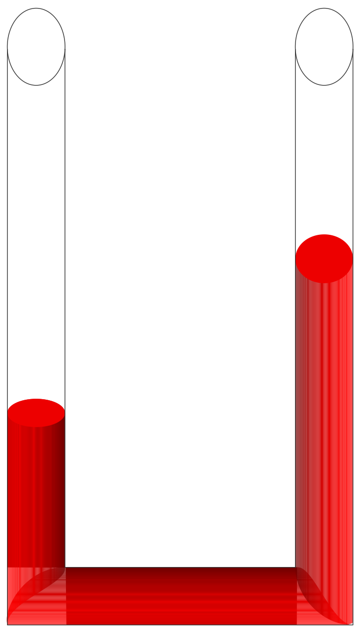


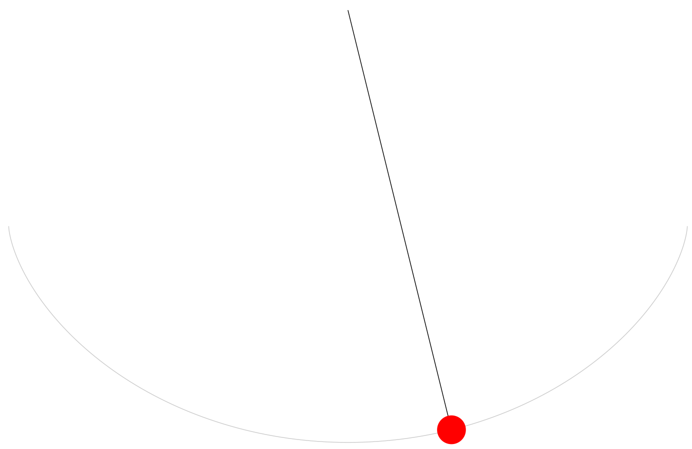


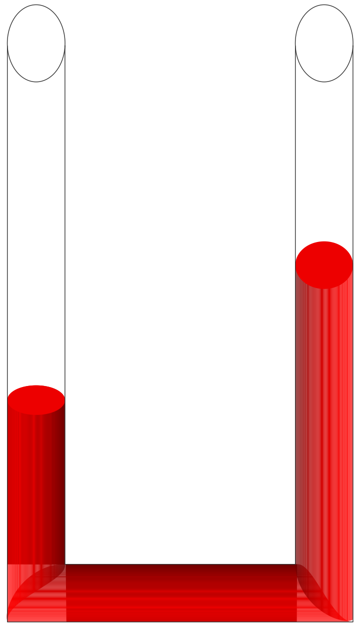


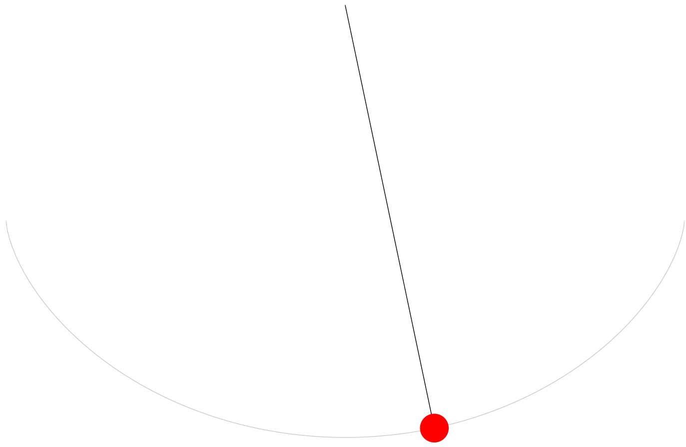


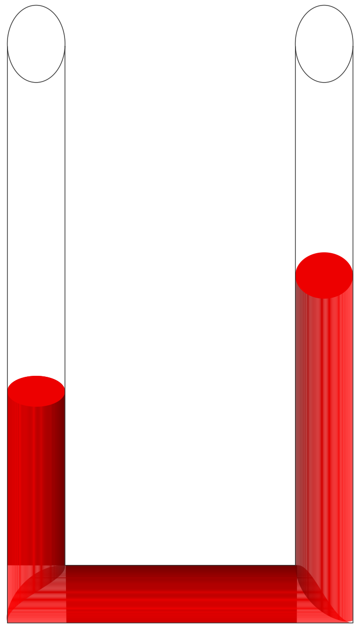


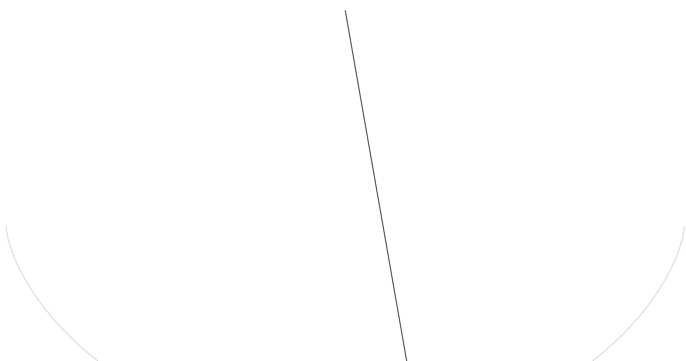


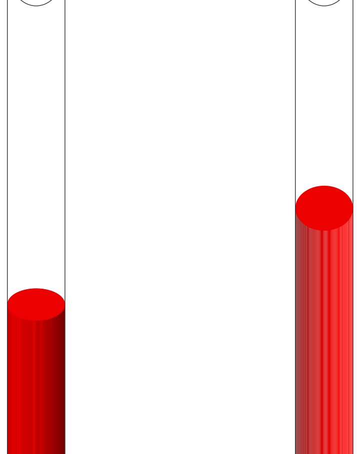


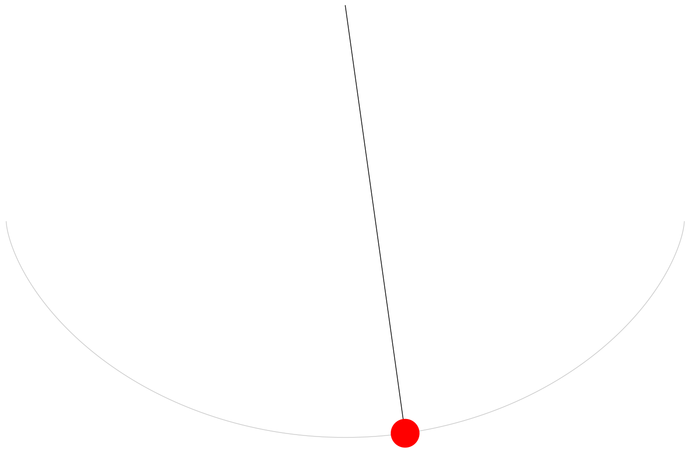
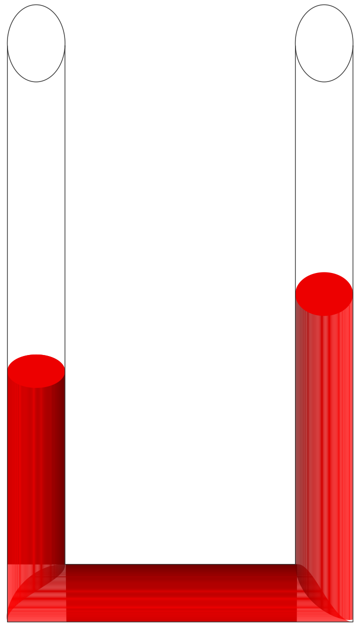


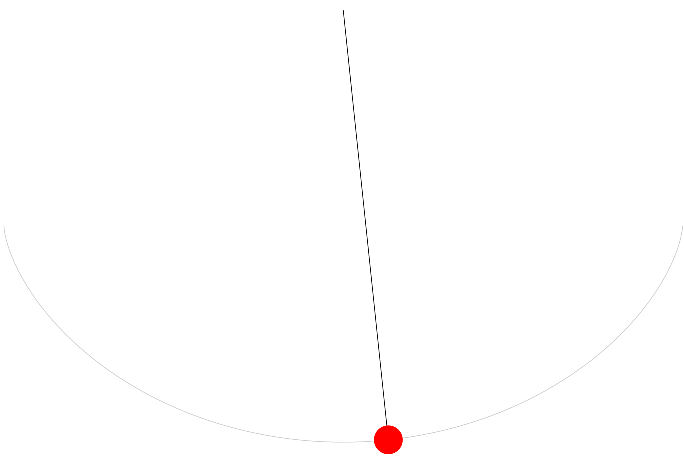


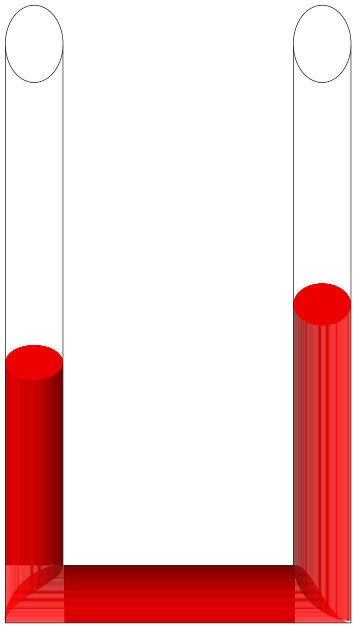


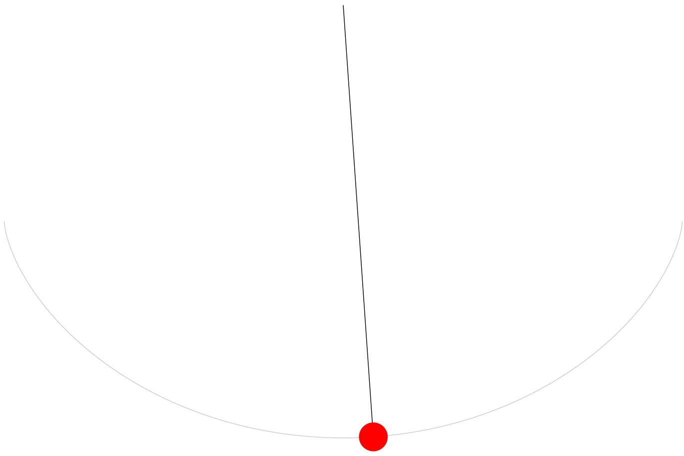


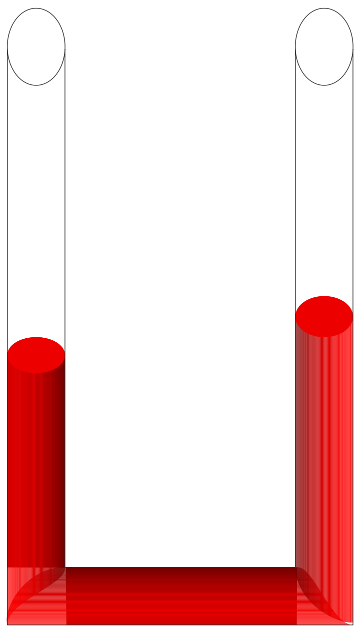


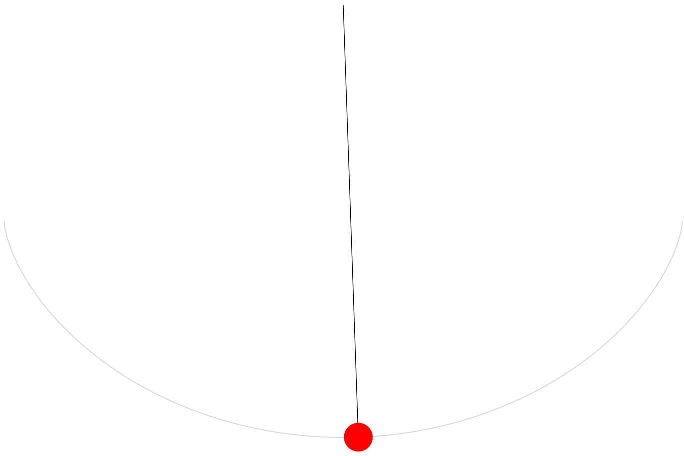



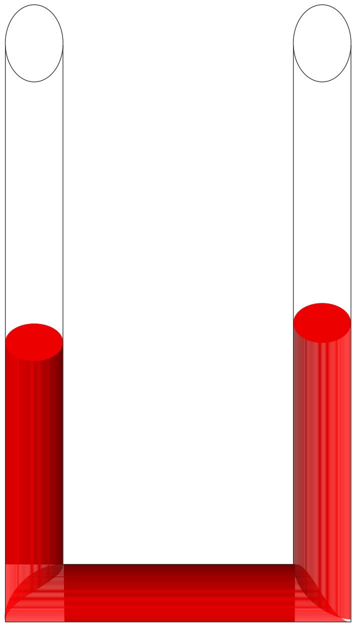


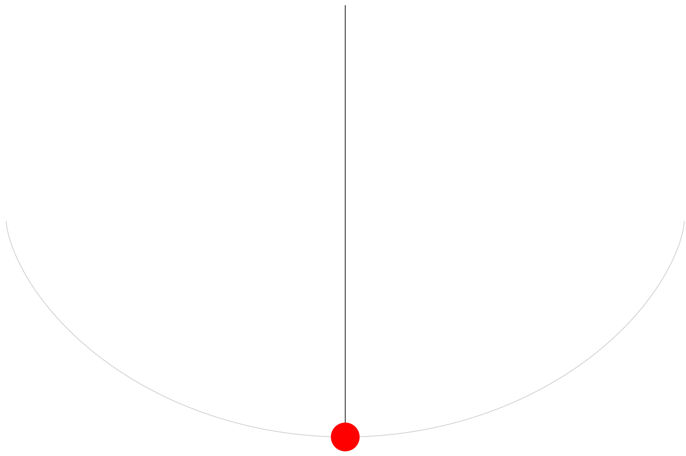


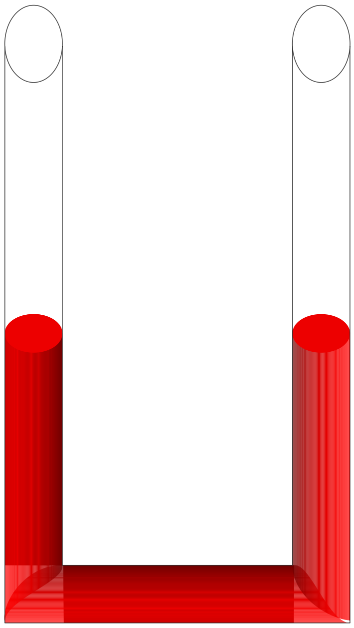


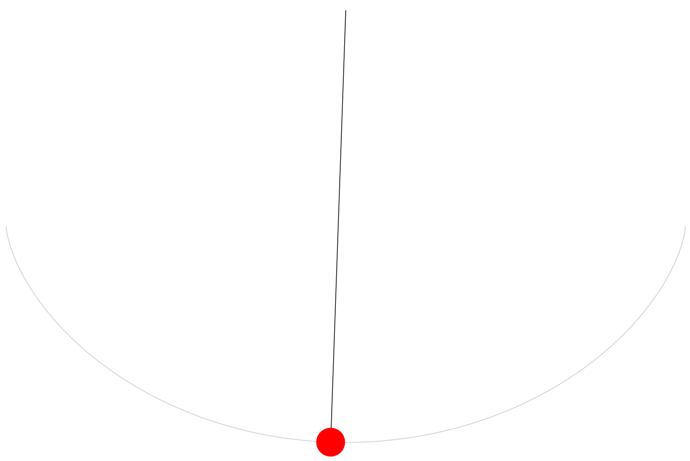


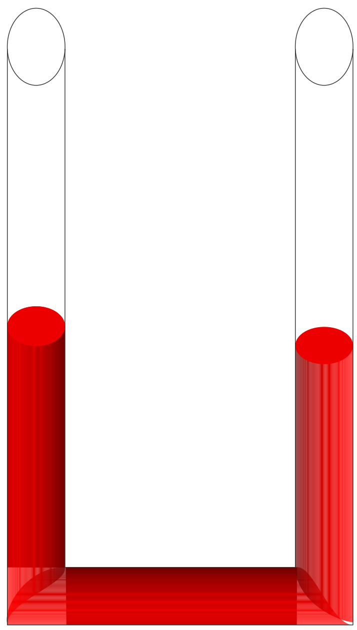


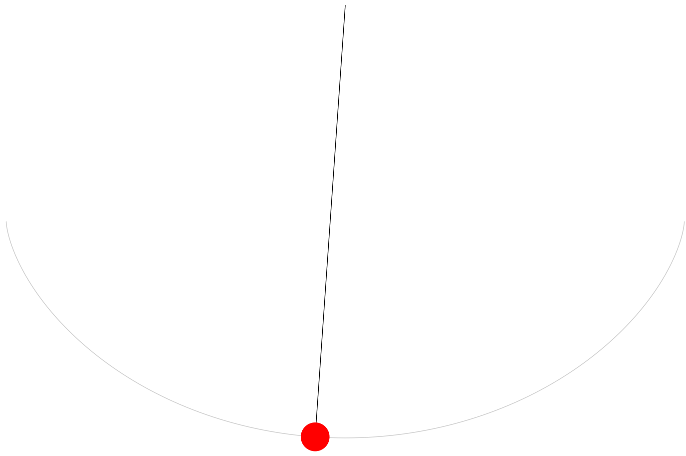
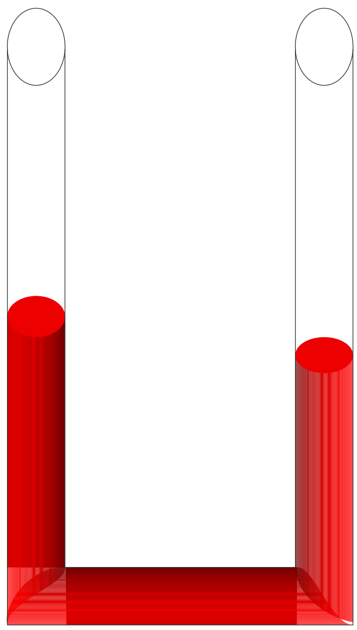


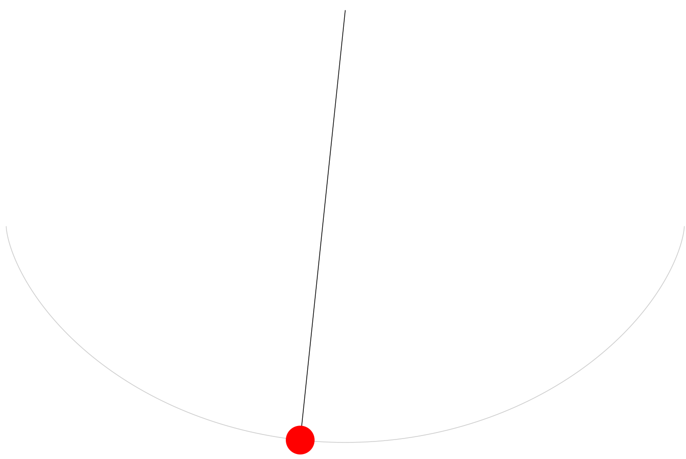


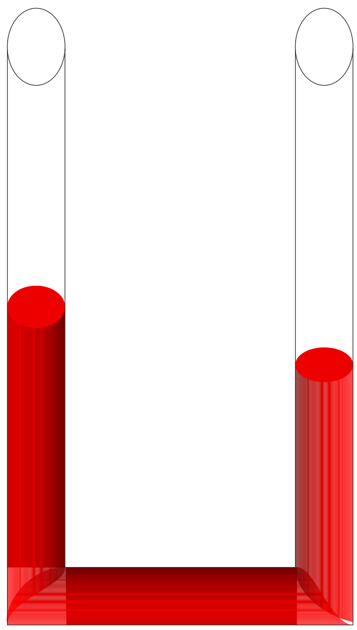


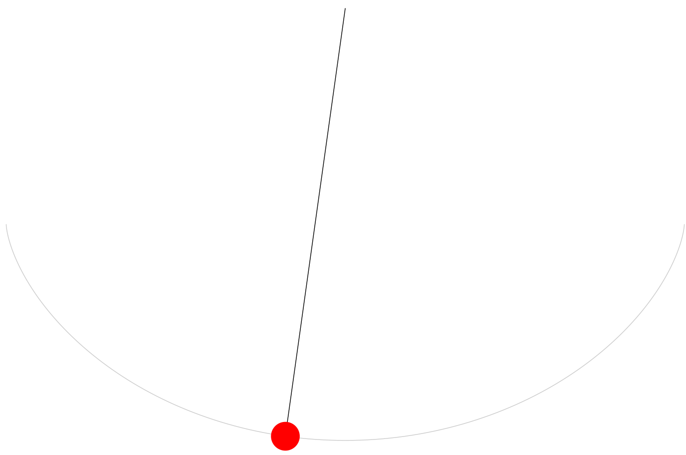


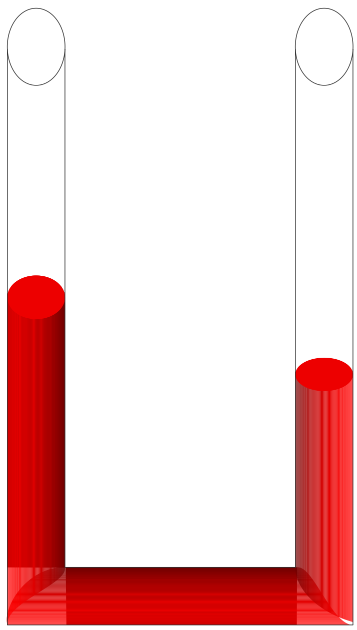


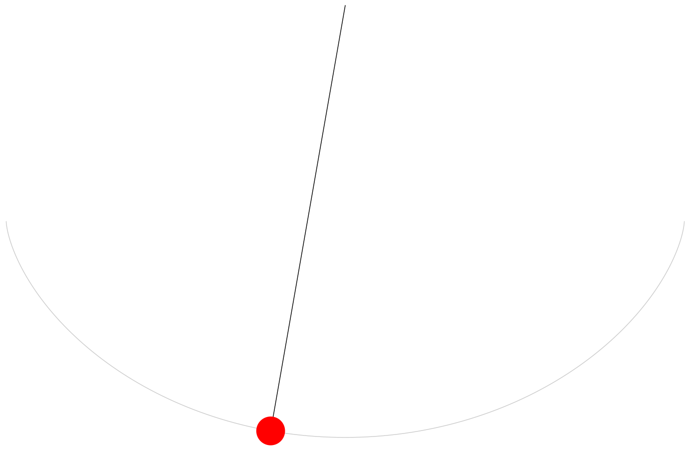



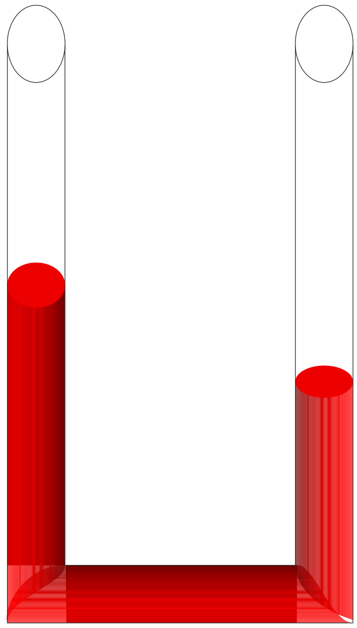


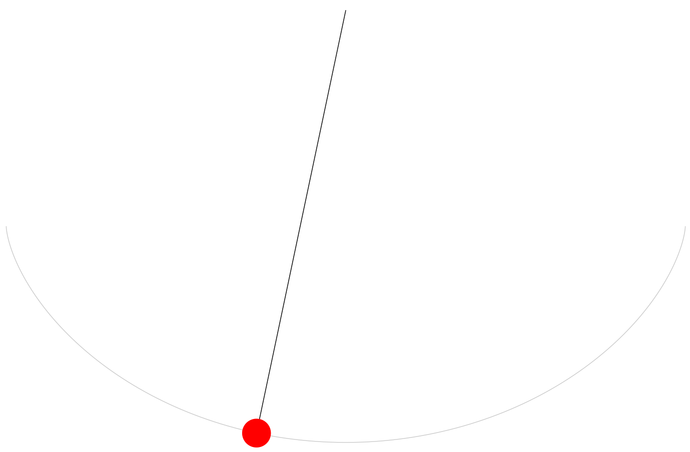


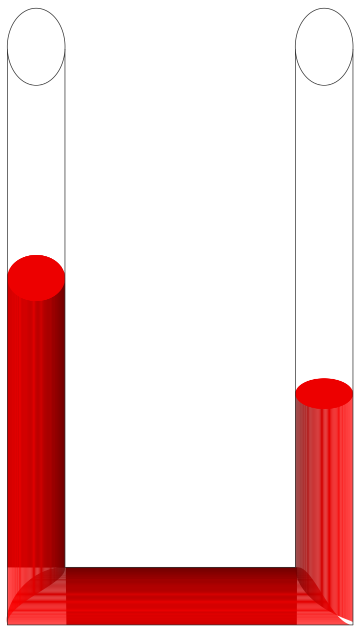


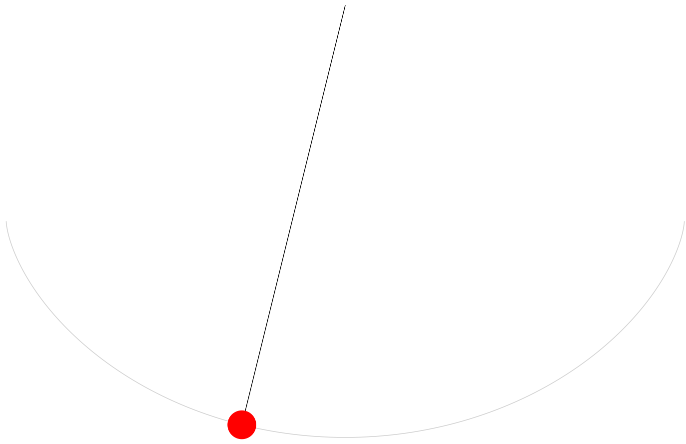


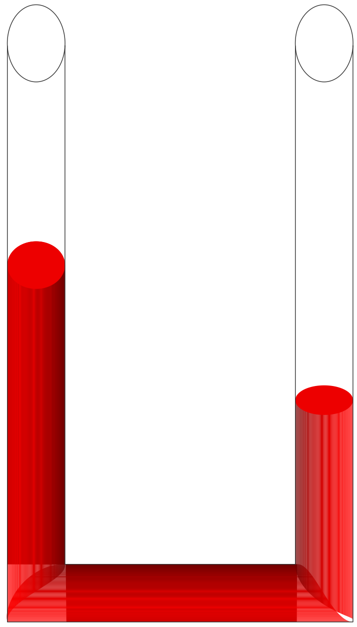


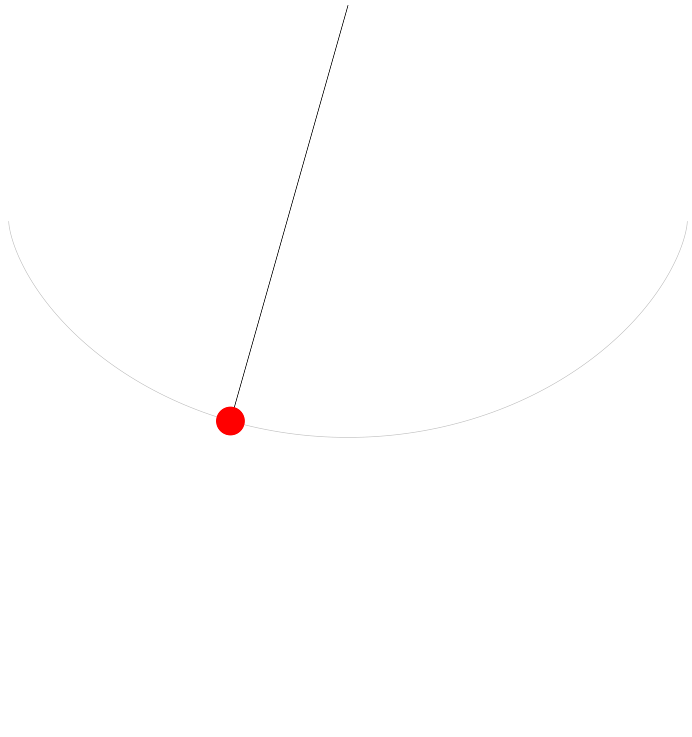


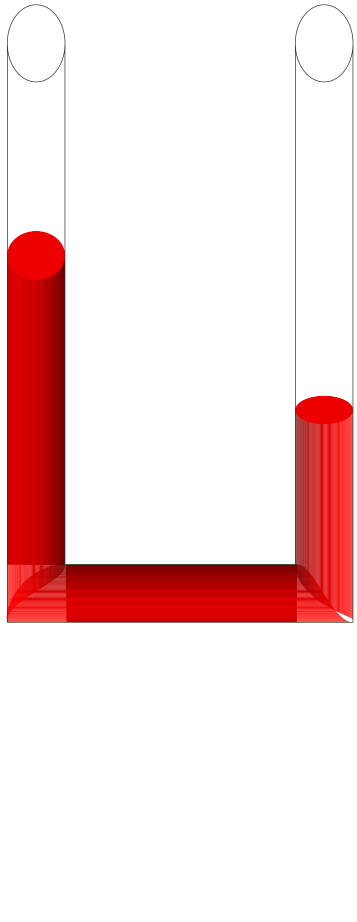


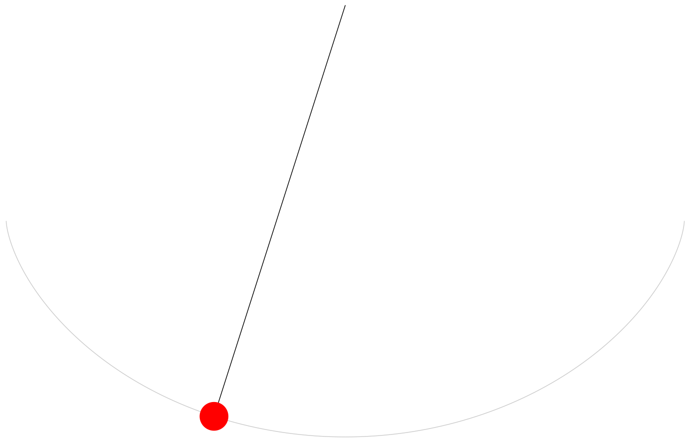


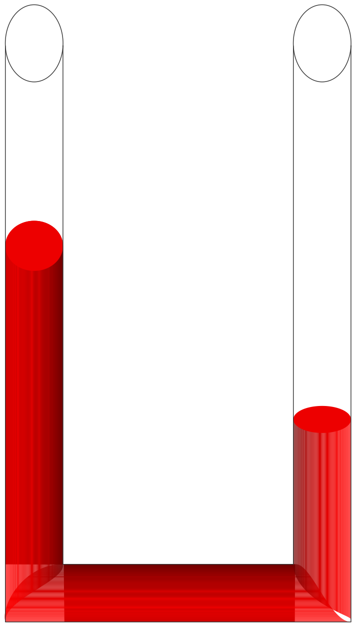


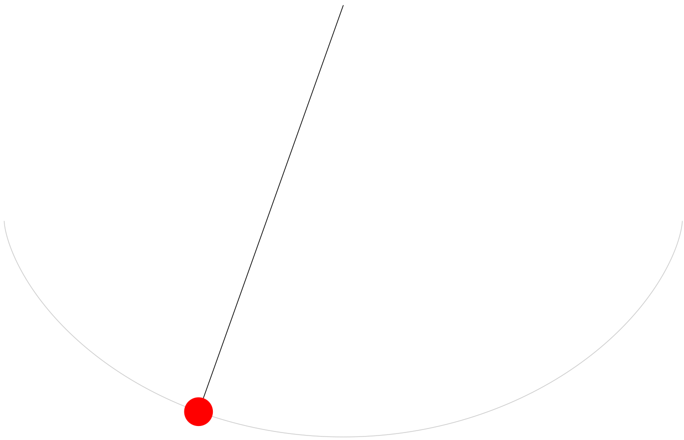


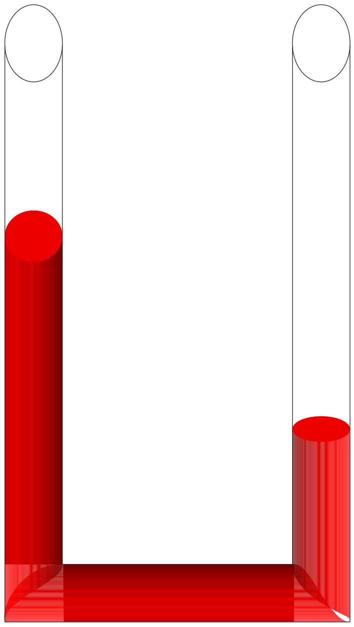


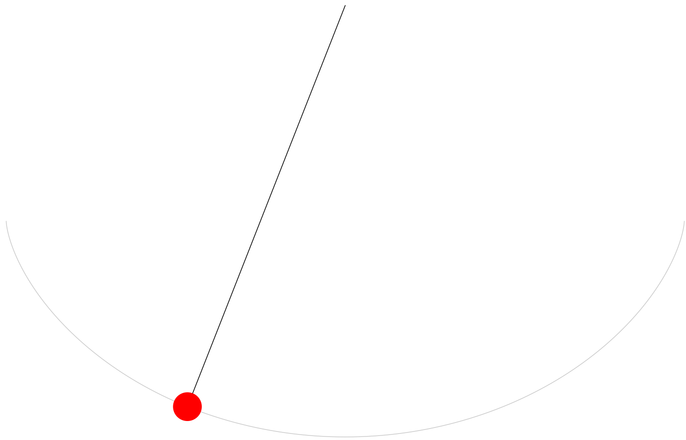


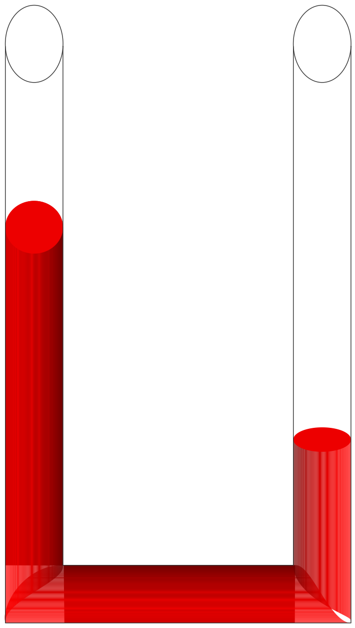


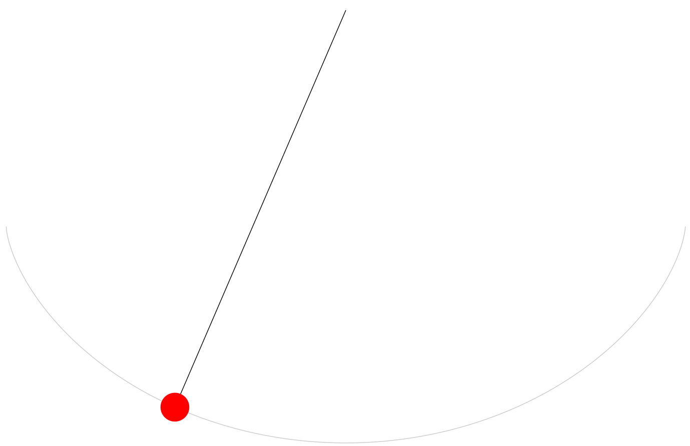


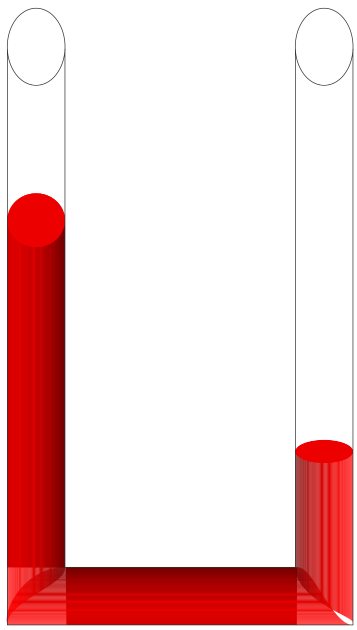


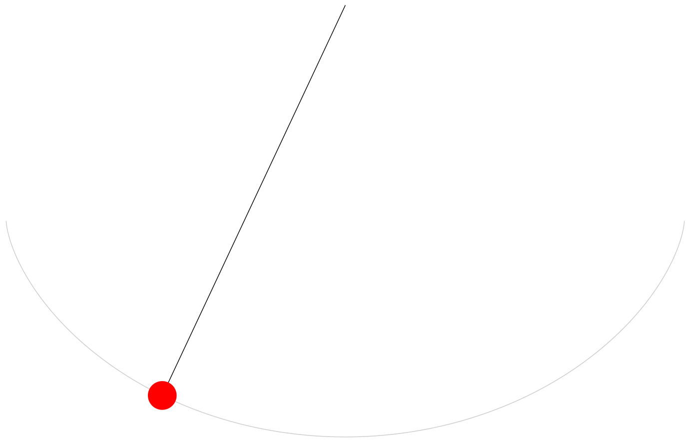


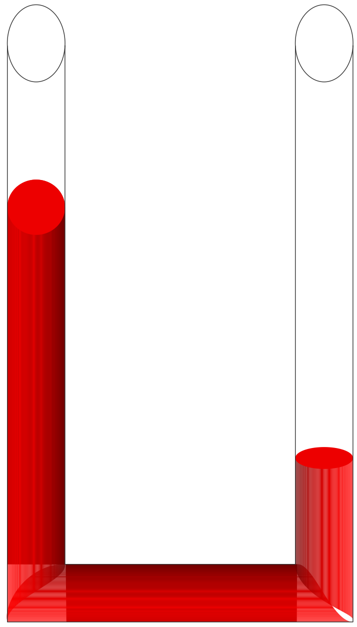


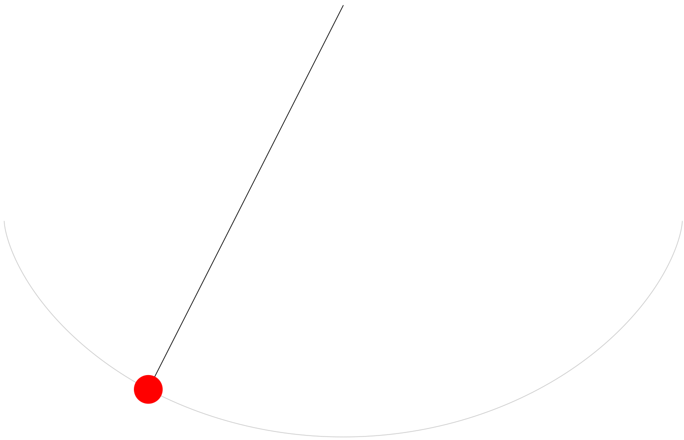
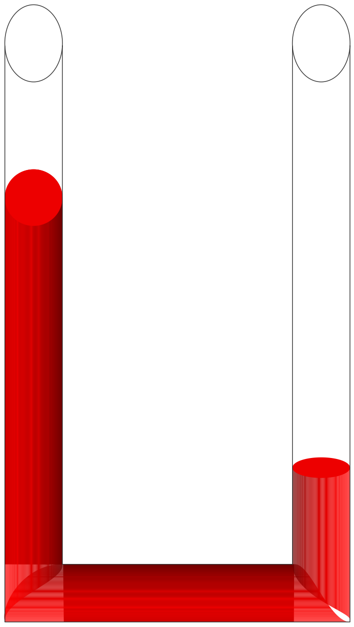


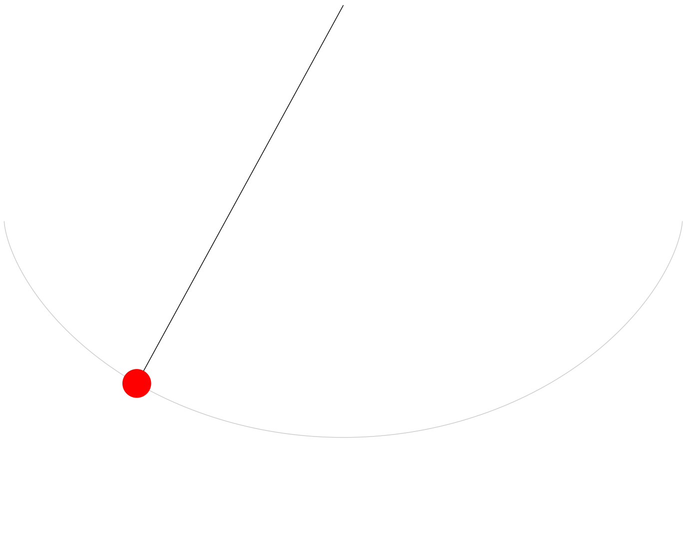


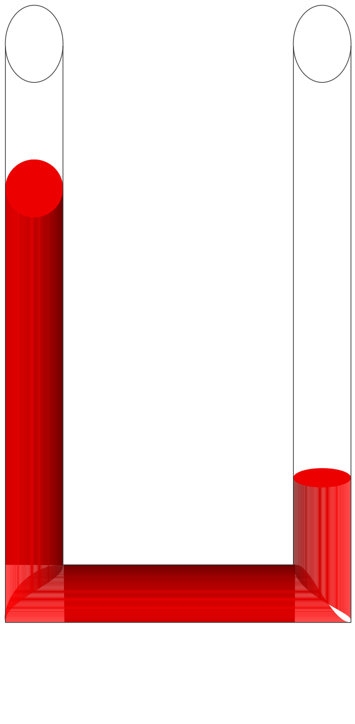




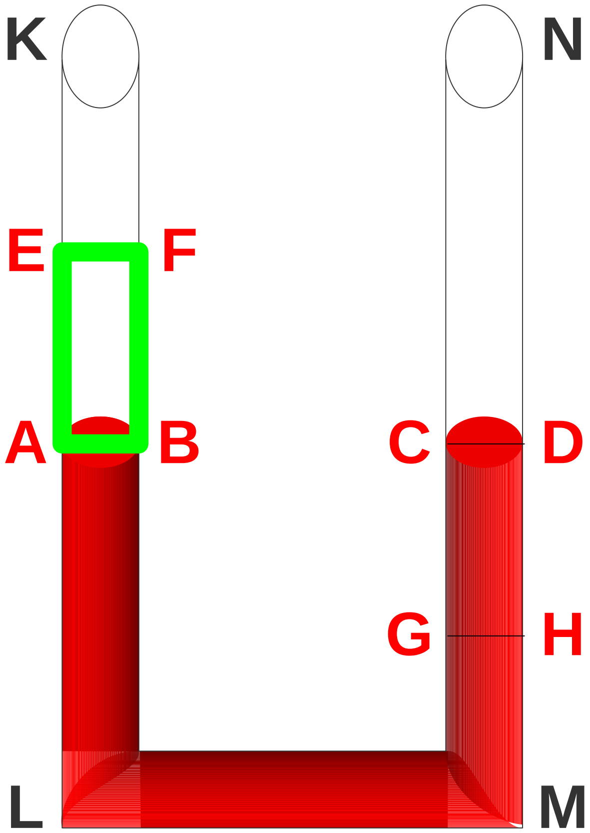






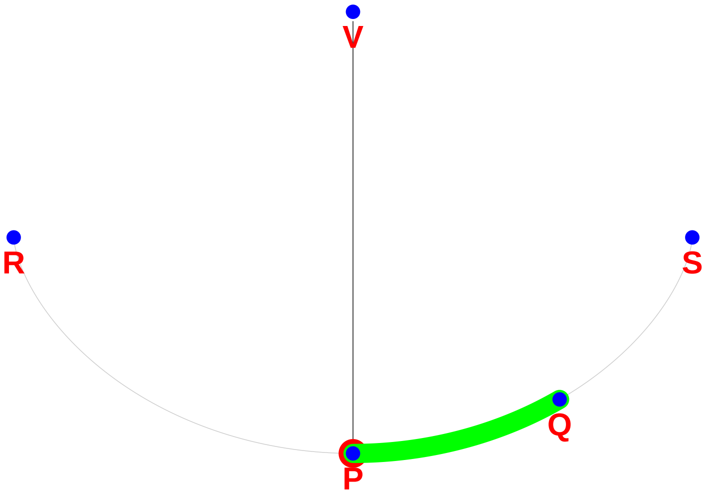
Statement of Corollary I

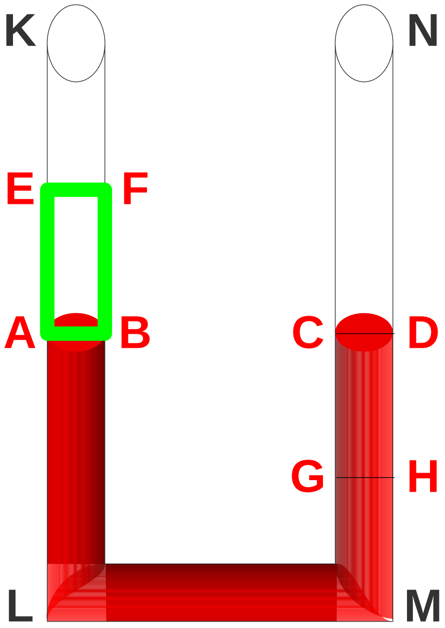


For any force applied to the water

time(AE)=time(CG)

Statement of Corollary II



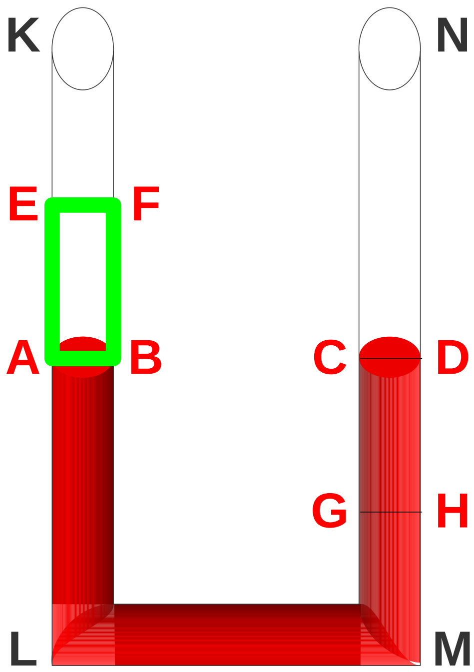


ABCD=6.5 feet of french measure

AE will take 1 second

PQ will take 1 second

Statement of Corollary III



ABCD is not equal to 6.5 feet of french measure

time(AE) is proportional to sqrt(ABCD)