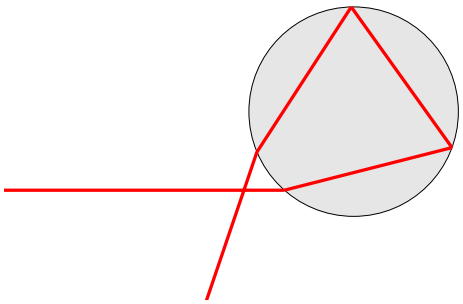
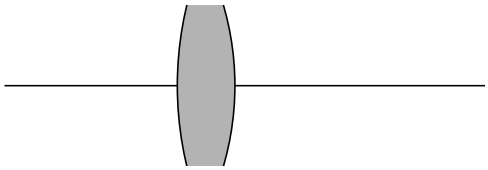
Mathematics 309 — Homework due Wednesday, March 5 — Part I

1. Write down a formula for the deflection angle in a double rainbow as a function of incidence angle. (Let n be the index of refraction concerned.) Graph it carefully in a PostScript picture. Find its derivative with respect to i. At what angle in the sky with respect to the horizon would you see a double rainbow at sunset?



2. Consider a lens system with one lens, roughly like this:



The surface at the left has a radius of r_1, that at the right r_2, and the distance between the two is d. How far from the right lens surface is the focal plane (where rays from ∞ at the left converge)? (This distance is called the focal distance.) The radius r_2 will be negative if the boundary curves the other way. What happens to the focal plane as r_2 gets larger and larger, passes through ∞, then becomes negative?

3. Objects at a distance ℓ_1 to the left of the left lens surface will focus at a distance ℓ_2 to the right. Find a formula for ℓ_2 in terms of ℓ_1. Find a formula for the magnification.

4. (a) Suppose two thin lenses with focal lengths f_1 and f_2 are placed a distance ℓ apart. Find the transfer matrix. Simplify it for the special case $\ell = f_1 + f_2$. In this system parallel rays coming in from ∞ are changed into parallel rays coming out at the right. Explain this.

(b) This is what telescopes do. Find a formula for the angular magnification of this system, or the effective magnification of the telescope.