

# Atmospheric Refraction

And Related Phenomena

# Overview

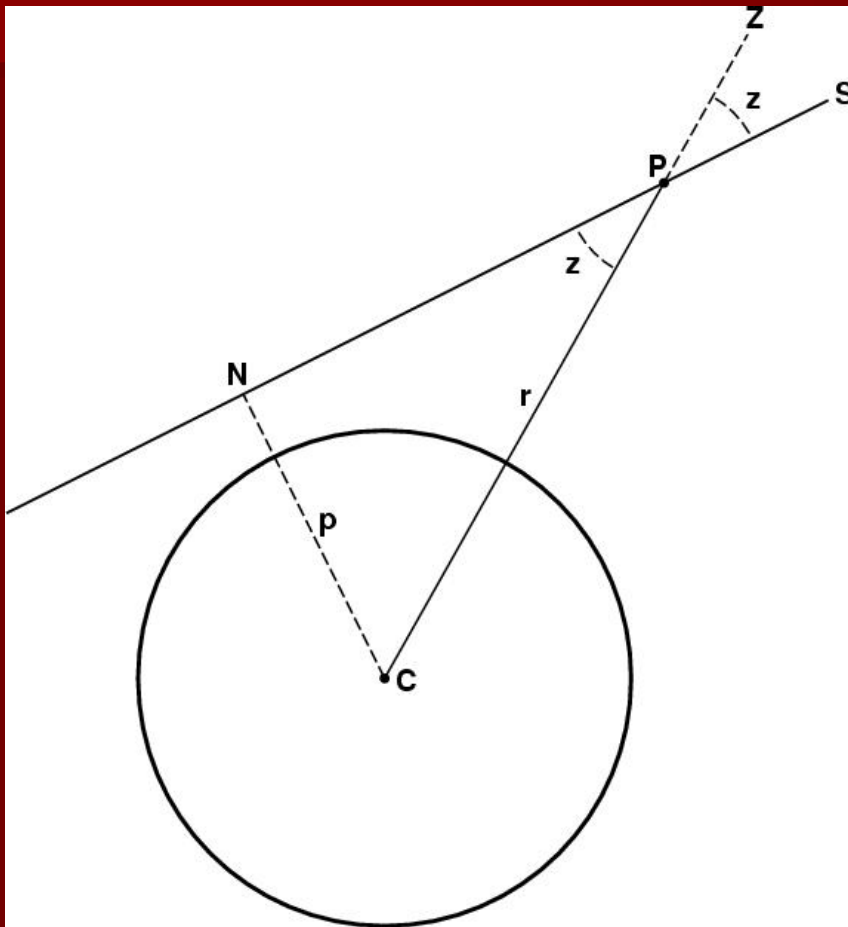
1. Refractive invariant
2. The atmosphere and its index of refraction
3. Ray tracing through atmosphere
4. Sunset distortions
5. Green flash

# Spherical Earth Approximation

For the purposes of this discussion, we will assume that the earth is a sphere

- n Earth is not really a sphere, but it is close enough for our purposes
- n Ray tracing using this model is very easy because there is a conserved quantity along the entire ray's trajectory

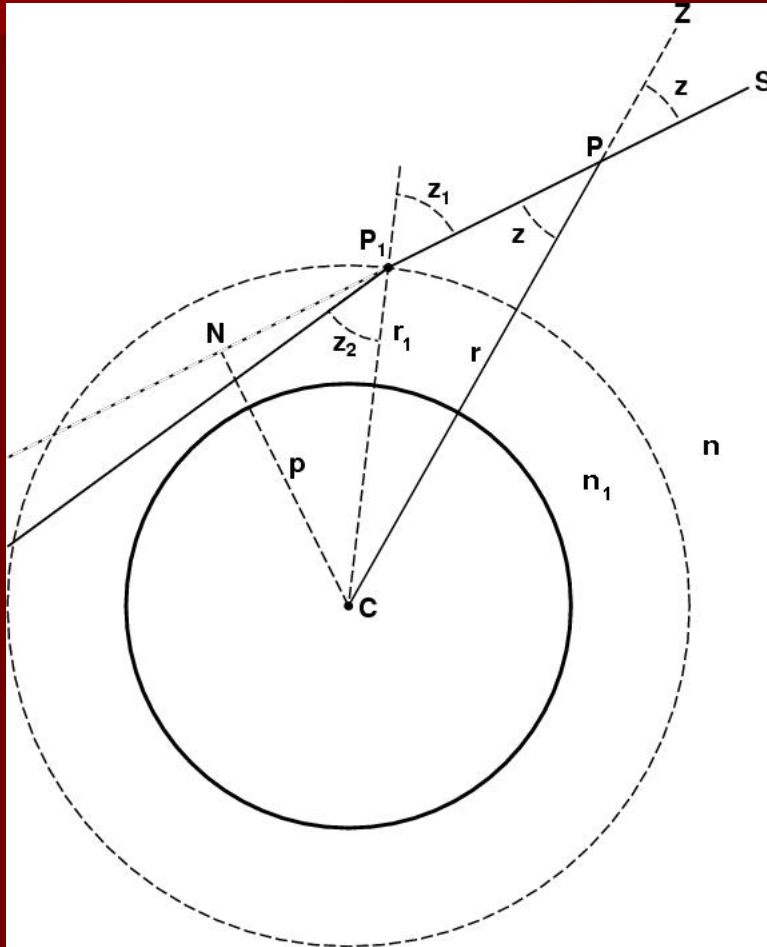
# Refractive invariant



If there were no atmosphere,  
 $\sin(z) = p/r$   
 $r \sin(z) = p$   
where  $p$  is the distance of the  
ray's closest approach.

This quantity,  $p$ , is invariant for  
any point  $P$  along the ray.

# Refractive invariant, cont



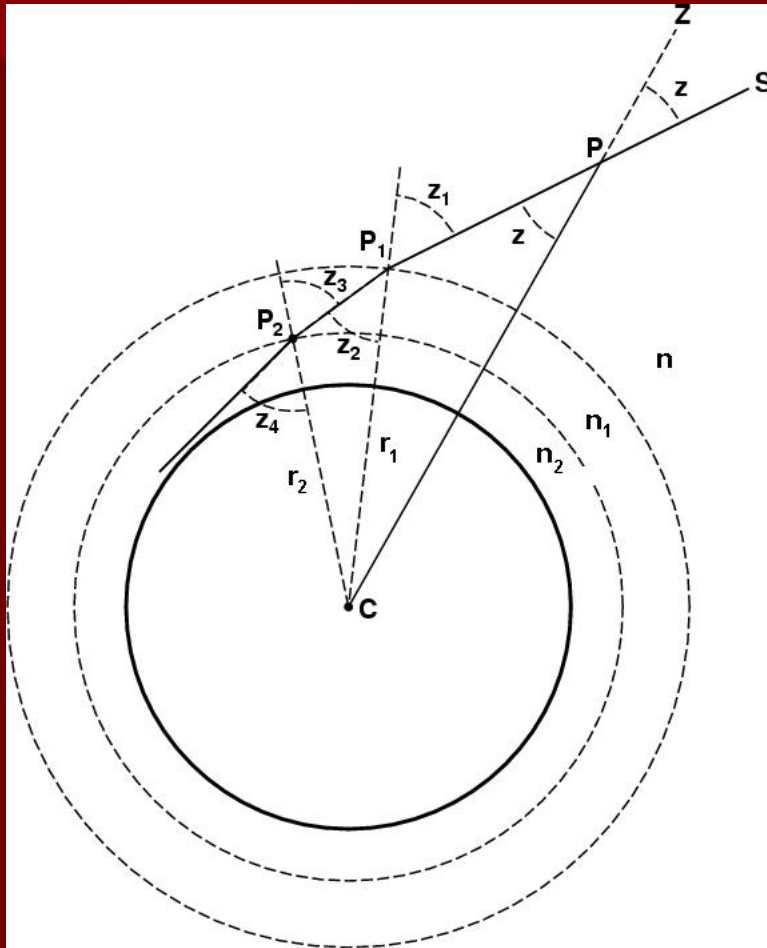
For a ray entering a homogeneous atmosphere,  
 $n_1 \sin(z_2) = \sin(z_1)$

$$\begin{aligned} n_1 r_1 \sin(z_2) &= r_1 \sin(z_1) \\ &= p \end{aligned}$$

But  $r_1 \sin(z_2) = p_1$ , which is invariant along refracted ray.

$$\begin{aligned} p &= n r \sin(z) \\ &\text{(before and after refraction)} \end{aligned}$$

# Refractive Invariant, cont



For an atmosphere with 2 layers

$$n_2 \sin (z_4) = n_1 \sin (z_3)$$

$$\begin{aligned} n_2 r_2 \sin (z_4) &= n_1 r_2 \sin (z_3) \\ &= n_1 r_1 \sin (z_2) \\ &= n_1 p_1 \\ &= p \end{aligned}$$

But  $r_2 \sin (z_4) = p_2$ , which is invariant along refracted path.

$$p = n r \sin (z)$$

(at any point along ray)

# Refractive Invariant, cont

n This process can be repeated for any number of layers with different index of refraction.

n For any given ray, calculate  $p$  then solve for  $z$ :

$$z(r) = \arcsin(p/n(r)r)$$

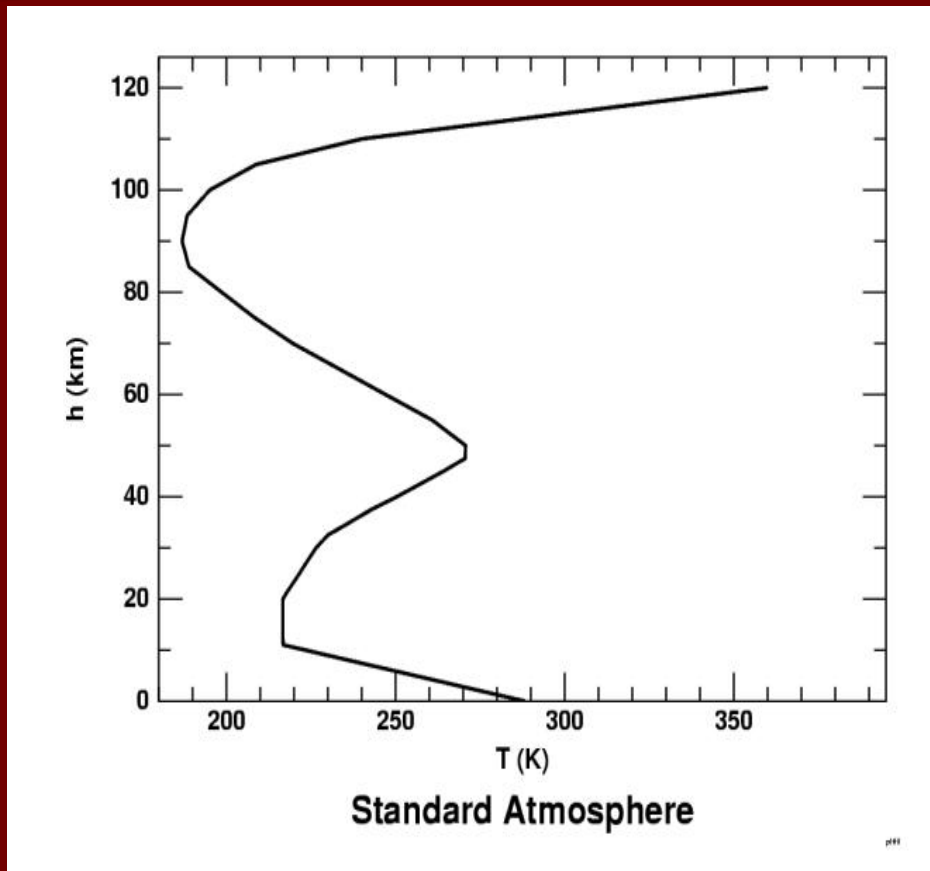
– if  $n(r)r$  is decreasing monotonically along ray's path,  $z$  must be increasing monotonically

    \ 2 rays that converge on a point can never again cross

– since  $\sin(z)$  is symmetric about  $90^\circ$ , if ray's path has a minimum then it is symmetric about this minimum

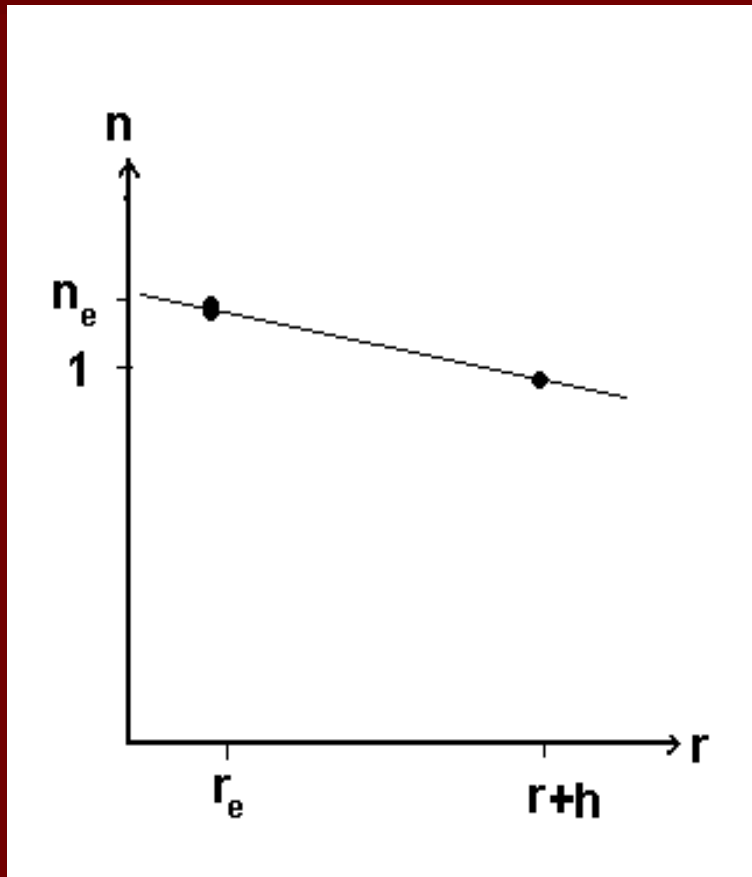
# Standard Atmosphere Model

- n based on meteorological data
- n temperature related to index, but only for lower 8-10 km
- n small contribution from upper layers of atmosphere





# Trevor's Model of Atmosphere



- n** like the lower portion of standard atmosphere
- n** specify:
  - $h_a$  (height of atmosphere)
  - $r_a$  (radius of earth)
  - $n_e$  (index at surface)
- n** Gives index as a function of height

$$n(r) = 1 + (r - r_e)(n_e - 1)/h$$

# Ray Tracing

1. Calculate  $p$  based on  $(x_0, y_0, \theta_0)$
2. Calculate intersection of ray with next layer of atmosphere  $(x_1, y_1)$
3. Calculate index,  $n_1$ , at next layer based on model of atmosphere
4. Calculate new zenith angle,  $z_1$ , using refractive invariant,  $p$
5. Calculate  $\theta_1$
6. repeat steps 2-6

# Ray Tracing, cont

1. Calculate  $p$  given  $(x_0, y_0, \theta_0)$  :

$$\begin{aligned}\underline{P}(t) &= \underline{P}_0 + t \underline{v} \\ &= [x_0 + t \cos(\theta_0), y_0 + t \sin(\theta_0)]\end{aligned}$$

$$\begin{aligned}x(t)^2 + y(t)^2 &= r_0^2 \\ &= x_0^2 + y_0^2 \\ t_{a,b} &= 0, -2(x_0 \cos(\theta_0) + y_0 \sin(\theta_0))\end{aligned}$$

The ray is closest when  $t = t_p = (t_a + t_b)/2$

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$$p = \sqrt{x(t_p)^2 + y(t_p)^2}$$

# Ray Tracing , cont

2. Calculate where ray intersects next layer of atmosphere at  $r_1$ :

$$\begin{aligned}\underline{P}_1(t) &= \underline{P}_0 + t \underline{v} \\ &= (x_0 + t \cos(\theta_0), y_0 + t \sin(\theta_0))\end{aligned}$$

$$x(t)^2 + y(t)^2 = r_1^2$$

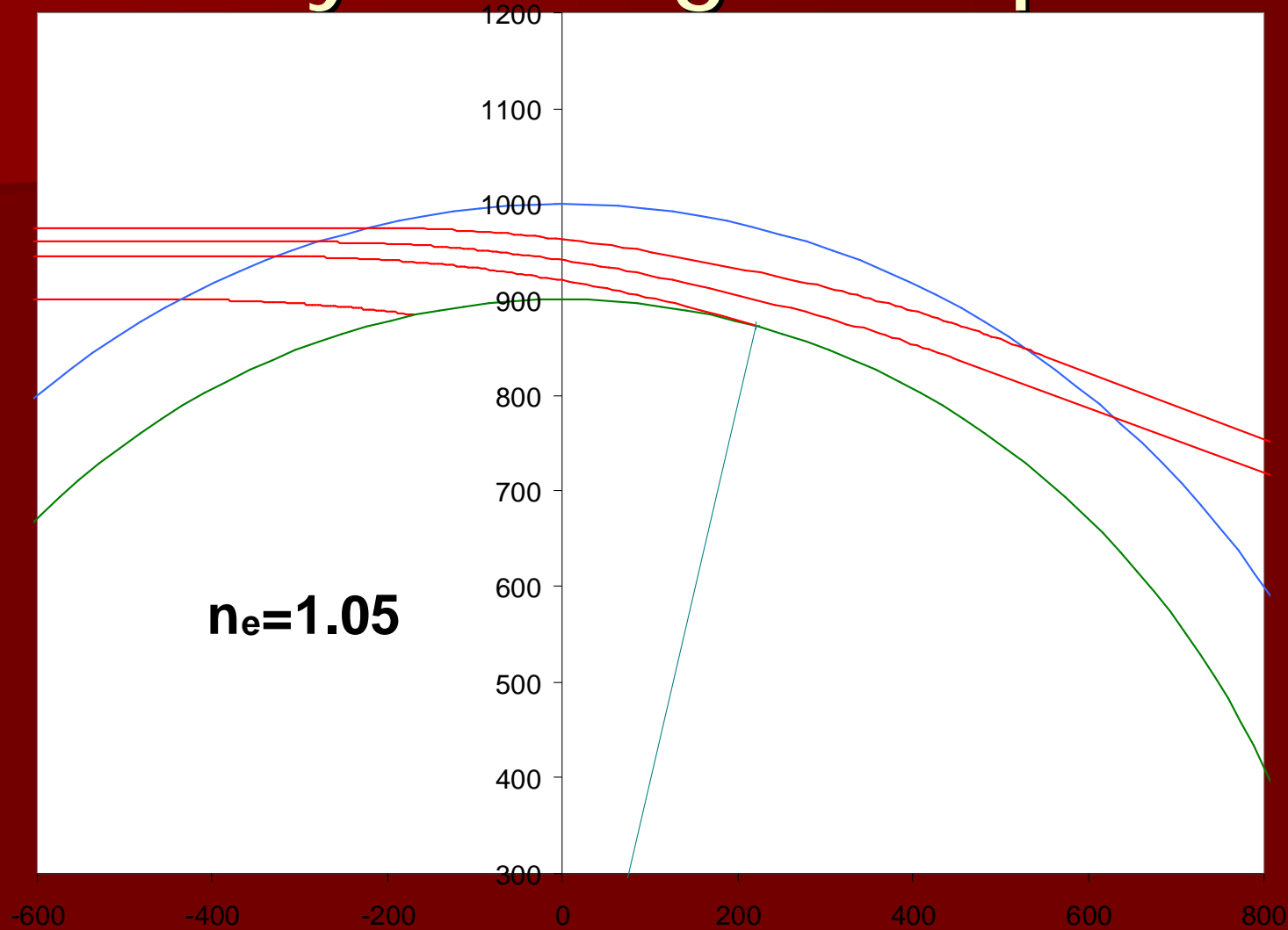
$$t_{a,b} = \frac{-((x_0 \cos(\theta_0) + y_0 \sin(\theta_0)) \pm \sqrt{(x_0 \cos(\theta_0) + y_0 \sin(\theta_0))^2 - (x_0^2 + y_0^2 - R_1^2)}}{p}$$

$$\underline{P}_1 = (x_1, y_1)$$

# Ray Tracing, cont

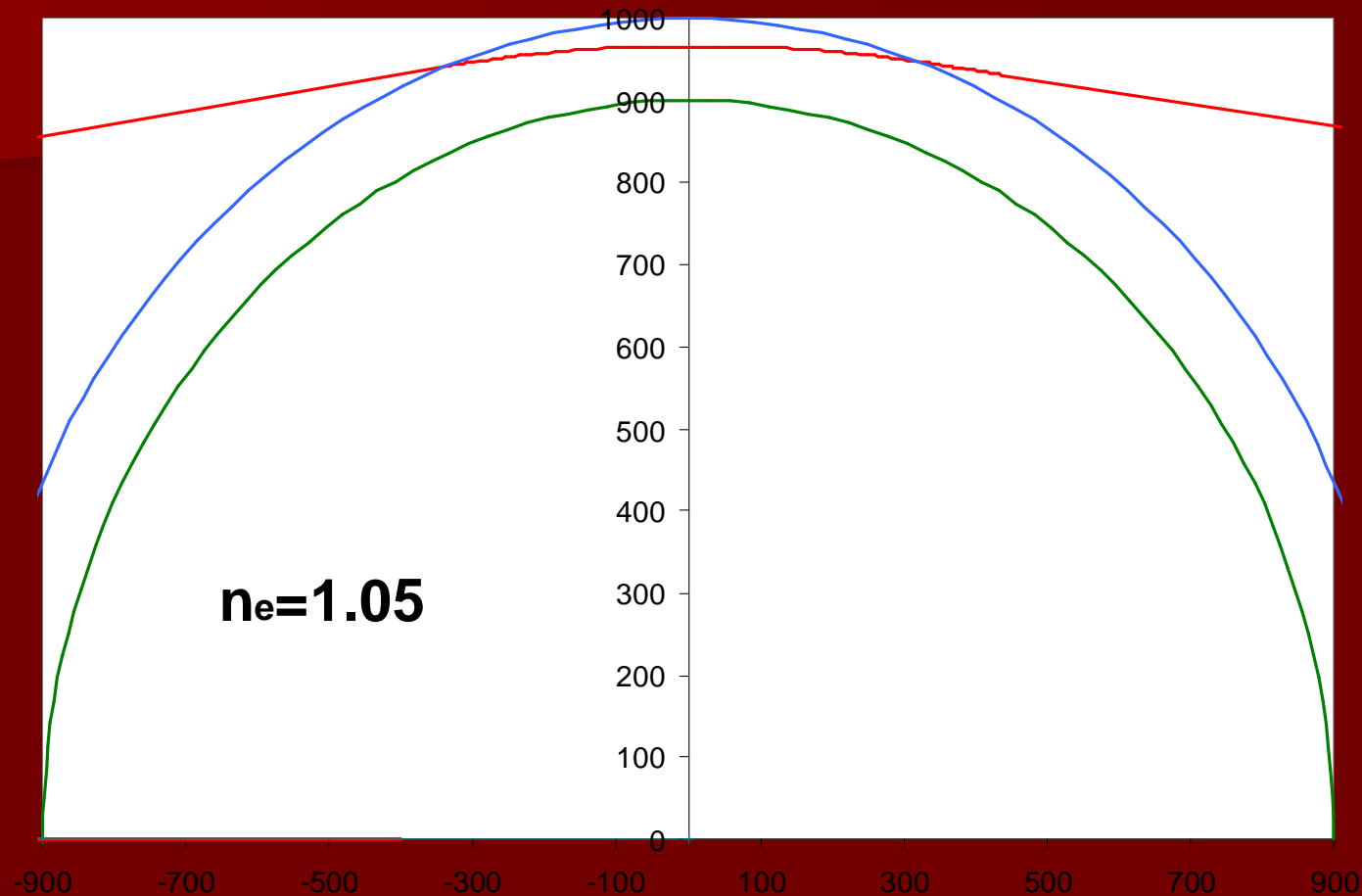
3. Calculate index of refraction for next strata,  $n_1$ , using  $n=n(r)$
4. Calculate  $z_1$  using refractive invariant  $p$   
$$p = n r \sin(z)$$
$$z_1 = \arcsin (p/n_1 r_1)$$
5. Calculate  $\theta_1$   
$$\theta_1 = z_1 + \arctan(y_1/x_1) - \mathbf{B}$$
6. Repeat steps 2-6

# Ray Tracing Example



- n Ray tracing based on my model with  $n=1.05$  at surface
- n notice that rays don't cross

# Ray Tracing Example 2



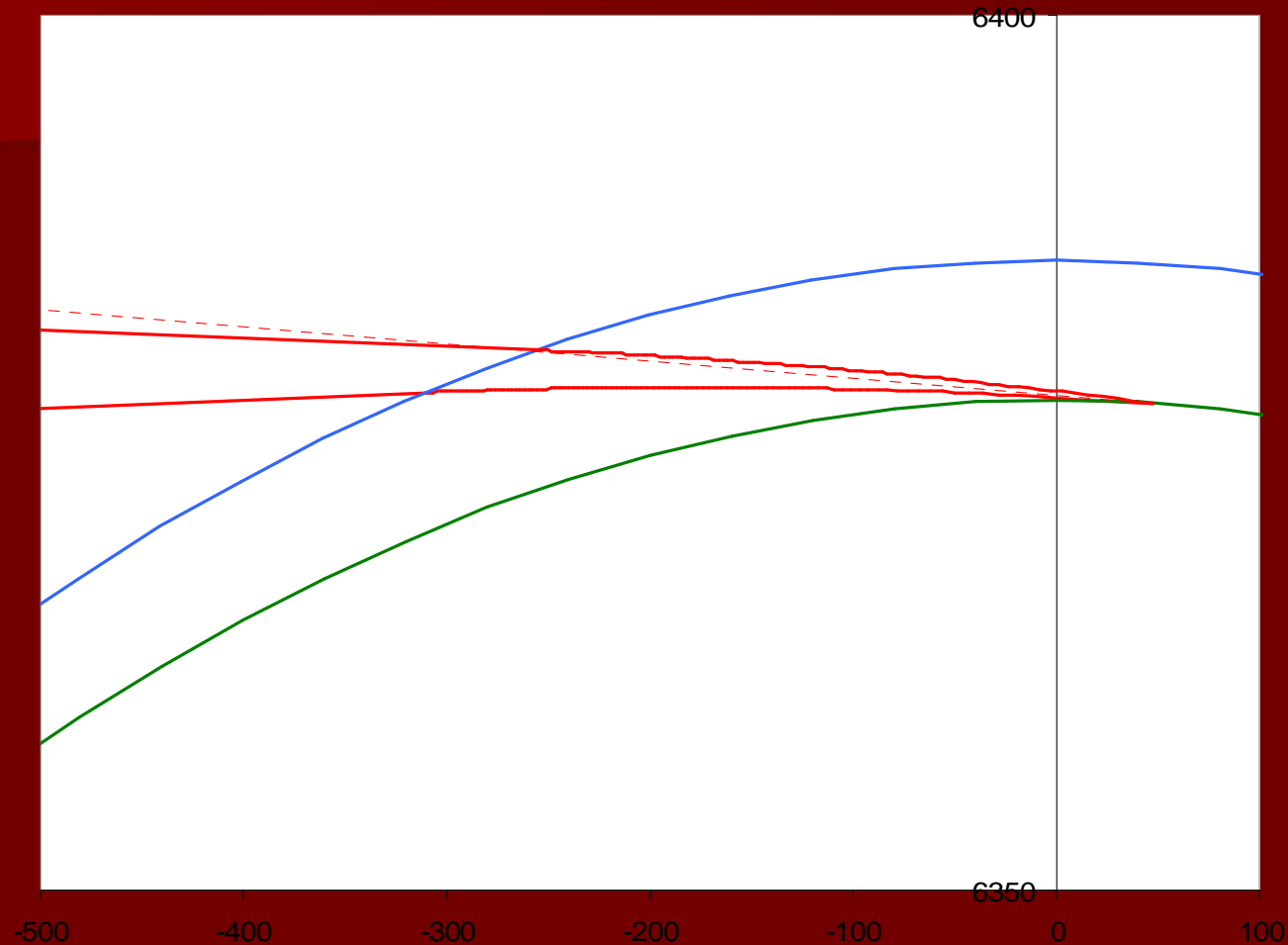
- n Ray tracing based on my model with  $n=1.05$  at surface
- n Notice that ray is symmetric about its minimum

# Distortions of Sunsets

1. Position of sun
2. Angular size of sun
3. Rate of sun setting



# Position and Angular Size of Sun



- n Rays traced using my model with  $n=1.00029$  at surface,  $h_{\text{atm}}=8$  km, and actual sizes of earth and sun
- n Sun appears higher and smaller than it really is

# Angular Size of Sun

## n Vertical size of sun

- actual angular size of sun =  $0.54E$
- at sunset, measured to be =  $0.45E$
- at sunset, my model predicts size =  $0.34E$

## n Horizontal size

- reduced by refraction because its extremities are both moved up towards the zenith on converging great circles, bringing them closer together
- very small effect (approx 0.03%)

# Rate of Sun Setting

The speed at which the sun goes down is determined by:

- the rate of rotation of the earth,

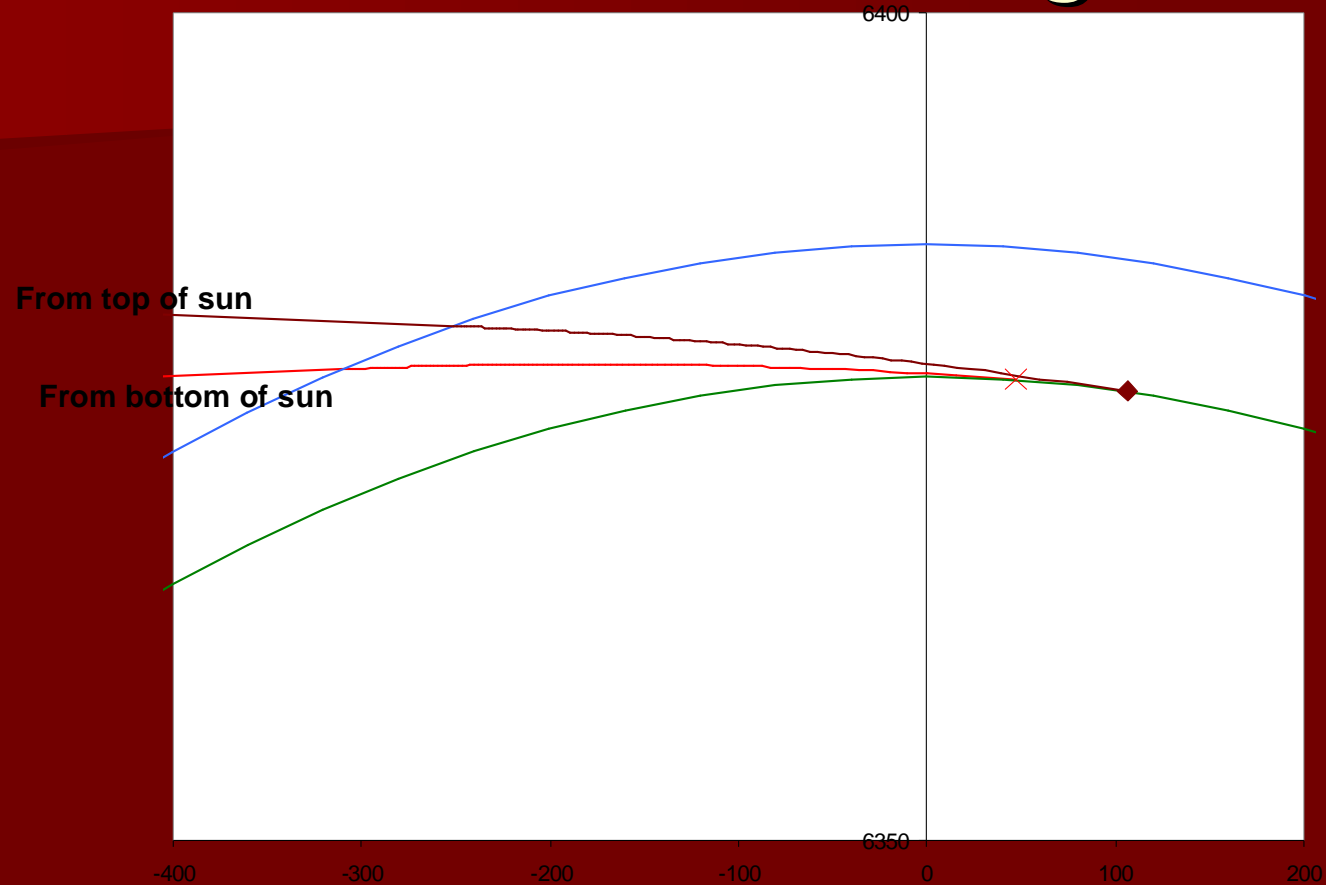
$$dN/dt = 360E/(24*60*60 \text{ s})$$

- the angular size of the sun

$$F = 0.54E$$

- the latitude of the observer
- the season (inclination of earth toward sun)
- the effects of atmospheric refraction

# Rate of Sun Setting, cont



- n plot shows the points where the top and bottom rays from sun set
- n angle between them ) N

# Rate of sun setting, cont

On March 21 if there were no refraction the time for sun to set would be (at 50E lat):

$$T = F / (\cos(\theta) dN/dt) = 197.5 \text{ s}$$

On March 14, I timed sunset to take

$$T = 150 \text{ s}$$

My model predicts  $\theta = 0.533$

$$T = \theta N / dN/dt = 127.9 \text{ s}$$

# The Green Flash

- n sun appears green just as it crosses horizon at sunrise and sunset
- n only seen if conditions are just right
- n there is a long history of wrong or incomplete explanations
  - refraction through waves
  - after image in eye after seeing red sunset
- n due mainly to:
  - differential refraction of different wavelengths
  - Rayleigh scattering
  - temperature inversion causing mirage

# Differential Refraction

**n** index of refraction varies slightly for different wavelengths of light

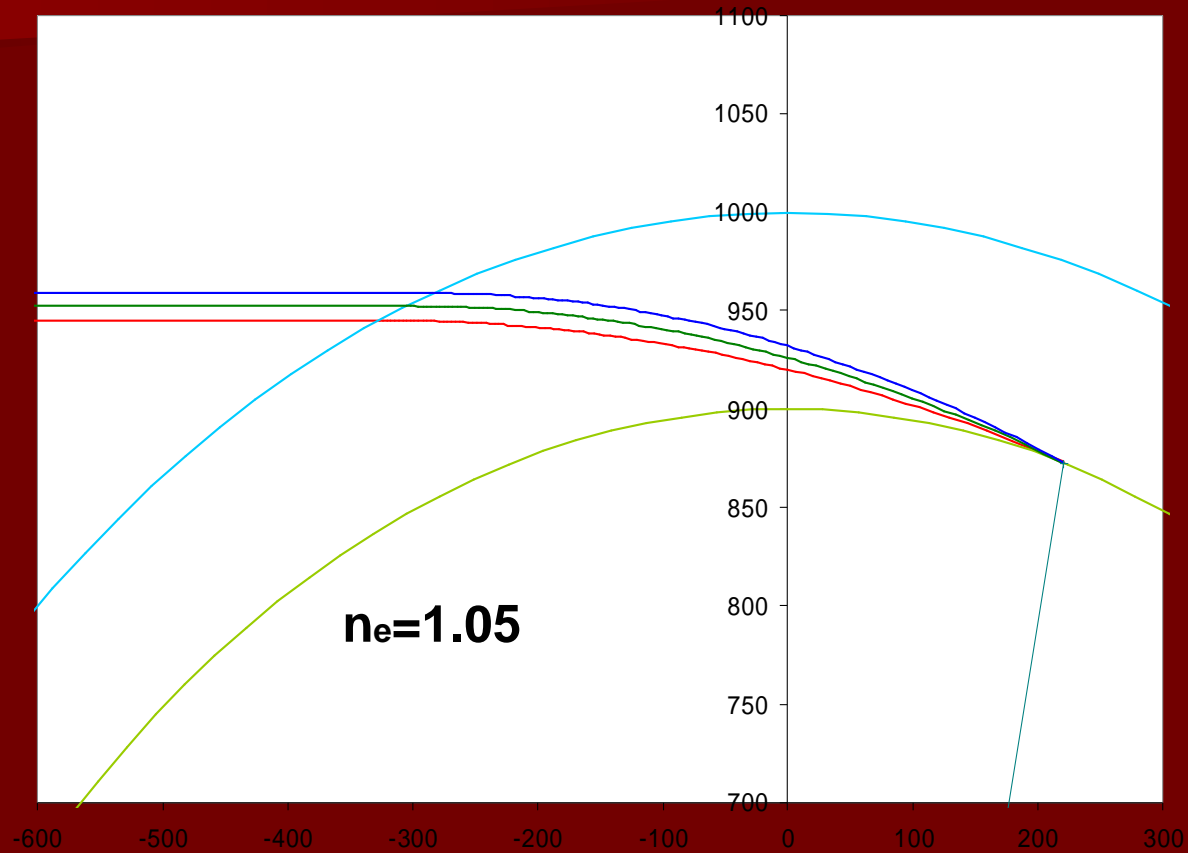
$$n_{\text{red}} = 1.000291$$

$$n_{\text{green}} = 1.000295$$

$$n_{\text{blue}} = 1.000299$$

**n** shorter wavelengths experience more refraction, thus set later than longer wavelengths

# Differential Refraction



Red light has set, but blue and green light is still above horizon



# Rayleigh Scattering

- n light interacts with particles that are of similar dimension as wavelength of light
- n light absorbed then re-emitted in a different direction
- n air particles happen to be right size to scatter blue light
- n reason that sky is blue
- n light coming directly from sun that we see at sunset almost devoid of blue light
  - safe to look at sunset, as harmful UV rays are removed by scattering through thick slice of atmosphere

# Basic Explanation of Green Flash

1. as red light (long  $\lambda$ ) from sun is just about to disappear, the blue and green light (shorter  $\lambda$ ) is still above horizon
2. the blue light has been removed by Rayleigh scattering, leaving green light

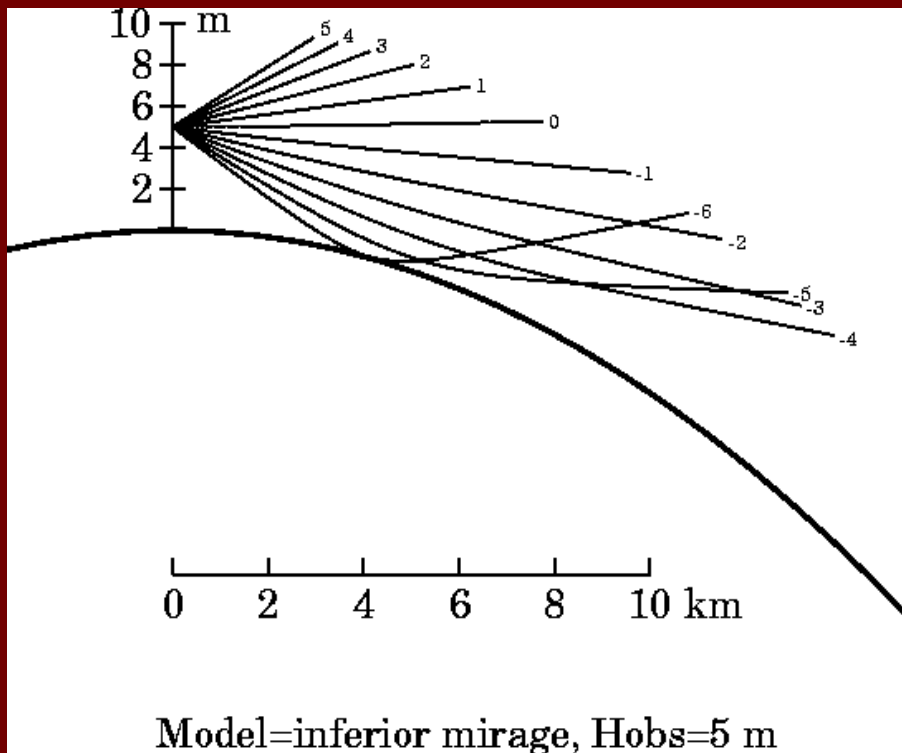
This explanation is correct, but not complete. The effect it produces is 10X too weak to be seen with the naked eye.

3. Something else must be magnifying the green light to make it visible...

# Mirages

- n Mirages are what is seen when light from a single point takes more than one path to the observer. The observer sees this as multiple images of the source object.
- n 2 basic types:
  - inferior mirage
  - superior mirage
- n Inferior mirage is what typically makes the green flash visible

# Inferior Mirage



- n Occurs when temperature at surface is very high (low  $n$ ) and observer looking down at heated region
  - n rays bend upward toward the region of higher  $n$  upward
  - n  $nr$  no longer monotonic, meaning rays below observer can cross, creating mirage
- $$\sin(z) = p/nr$$
- n Green rays that have not yet set are bent toward observer
  - n This is what makes green flash visible to naked eye

# Green Flash, no Mirage

## TEXTBOOK SUNSET

for the  
U. S. Standard Atmosphere  
seen from 4 m



# Green Flash, with Mirage

**INFERIOR-MIRAGE  
SUNSET**

for

$L = -10$  m

seen from 4 m



**good night**