

## Mathematics 446 — third assignment — due Monday, October 6

**Exercise 1.** Locate through the UBC library and JSTOR the article ‘Of the theory of circulating decimal fractions’ by John Robertson in the 1768 volume of *Philosophical Transactions*. Summarize what it says in 300 words or less, including some calculation examples.

**Exercise 2.** Write down and prove **by mathematical induction** the formula for the finite geometric sum

$$1 + q + q^2 + \cdots + q^n .$$

**Exercise 3.** Prove by mathematical induction the binomial theorem

$$(x + 1)^n = x^n + nx^{n-1} + \frac{n(n-1)}{2}x^{n-2} + \cdots + nx + 1$$

where  $n$  is an arbitrary positive integer and the coefficient of  $x^k$  is

$$\frac{n(n-1)\cdots(n-(r-1))}{1 \cdot 2 \cdot 3 \cdots r}$$

**Exercise 4.** Find  $a$  such that

$$a \equiv 14 \text{ modulo } 71$$

$$a \equiv 17 \text{ modulo } 91$$

**Exercise 5.** Write down a **complete** proof that  $\sqrt{3}$  is not a fraction.

**Exercise 6.** Write an essay of 100 words explaining why the Chinese Remainder Theorem is called what it is.

**Exercise 7.** Find all the powers of 60 modulo 37. Use this to find the repeating fraction for  $1/37$  in base 60.

**Exercise 8.** What is the length of the repeating fraction  $1/91$  in base 60?

**Exercise 9.** Read Book VII. Proposition 1 of Euclid’s *Elements* (on line at Joyce’s site). Restate the Proposition and rewrite the proof in your own words, using modern algebraic notation.

**Exercise 10.** Find the continued fraction expansion of (a)  $\sqrt{5}$ ; (b)  $\sqrt{3}$ ; (c)  $\sqrt{19}$ .