# Mathematics 446 — fourth assignment solutions

**Exercise 1.** *Prove by mathematical induction this formula for the $n$-th Fibonacci number:*

$$F_n = \frac{\Phi^n - (-1)^n/\Phi^n}{\sqrt{5}}$$

*where $\Phi = 1.618\ldots$ is the golden ratio. Then use it to prove that the ratio of $F_{n+1}/F_n$ converges to $\Phi$ by answering this question explicitly: given a small number $\varepsilon$, how large does $n$ have to be in order that*

$$|F_{n+1}/F_n - \Phi| \le \varepsilon \text{ ?}$$

The formula can be checked explicitly for $n = 0$ and 1. Then look at #3 in Assignment 6 to see what to do next.

As for the last part:

$$\begin{aligned}
\frac{F_{n+1}}{F_n} &= \frac{\Phi^{n+1} - (-1)^{n+1}\Phi^{-n-1}}{\Phi^n - (-1)^n\Phi^{-n}} \\
\frac{F_{n+1}}{F_n} - \Phi &= \frac{\Phi^{n+1} - (-1)^{n+1}\Phi^{-n-1}}{\Phi^n - (-1)^n\Phi^{-n}} - \Phi \\
&= \frac{(-1)^n\Phi^{-n+1} - (-1)^{n+1}\Phi^{-n-1}}{\Phi^n - (-1)^n\Phi^{-n}} \\
&= (-1)^n \, \frac{\Phi + \Phi^{-1}}{\Phi^{2n} - (-1)^n} \\
&= (-1)^n \, \frac{1}{\Phi^{2n} - (-1)^n}
\end{aligned}$$

Its size is about $\Phi^{-2n}$. This will be about $\varepsilon$ when $2n \ln \Phi = -\ln \varepsilon$.

**Exercise 2.**