Mathematics 446 — second homework

This is due next Monday, January 17.

1. The Egyptians always expressed fractions as sums of unit fractions $1/N$. This raises some mildly interesting mathematical questions.

(a) Is it true that every fraction f between 0 and 1 can be expressed as a sum of distinct unit fractions? If so, prove it. If not, give an example, and explain which can be expressed in this way (with poroofs).

(b) Can f have an infinite number of such expressions? An infinite number witha given number of terms?

(c) Find all such expressions for $2/45$ involving two terms; three terms.

(d) Find all such expressions for $2/47$ involving two or three terms. For $2/53$.

2. Find the base 60 expressions for (a) 180, (b) 456, (c) $5,000$, and (d) $314,678$.

3. Write in detail a proof that if B is an integer larger than 1, every positive integer n can be expressed uniquely as a sum

$$n = n_0 + n_1 B + n_2 B^2 + \cdots + n_k B^k$$

with $0 \leq n_i, n_k > 0$. Write down an explicit algorithm for find the n_i.

4. Find the infinite sexagesimal expansion for $1/3$, $1/5$, $1/11$, $1/13$.

5. Find the first 8 'digits' of $\sqrt{2}$ in base 60.

6. Read the selection by Newman. Tell me what the problem being solved on the two-page spread is, and what and where the solution is on those pages.