

Mathematics 446 — Spring 2005 — third assignment

This is due next Monday, January 24.

1. Redo the questions from this week about unit fractions, to the extent discussed below. First explain in detail why any fraction $2/N$ only has a at most finite number of expressions as the sum of two unit fractions. Explicitly, if

$$\frac{2}{N} = \frac{1}{a} + \frac{1}{b}$$

show lower and upper bounds on a and b . What we did in class wasn't quite accurate, but the same basic idea works: try $a = n = (N + 1)/2, n + 1, \dots, N$, and then show that you need not look further. Do this by first swapping a and b if necessary to get $b > a$ and then show that $a < N$.

Next do the same for three terms by showing bounds for $a < b < c$ with

$$\frac{2}{N} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

In class I was also little sloppy on this last part, but you can still find some simple bounds by being careful. Again $(N + 1)/2 < a$ is easy to see. And you can also find an easy upper bound on a . This means that only a finite number of a are possible. But then for each a only a finite number of b and c are possible.

2. Read the paper 'Ancient Babylonian algorithms' by Donald Knuth, in *Communications of the Association for Computing Machinery* volume **15**, issue 7 (1972), pages 671–677. Also the correction to it in the letter on page 108 of volume **19**, issue 2 (1976). These are available electronically through UBC elinks. The correction is in the letter section that starts on page 105 of the issue, and you might have to download the whole section.

Translate line by line into algebra and English the procedure exhibited on page 672 and also the first, second, and fourth on page 673. Explain Knuth's remark at bottom right on page 673 that "the stated parameters 1 and 5 cannot possibly correspond . . ." Your primary goal is to make it clear to a modern reader what the Babylonian text is talking about. Do not deviate more than necessary from the Babylonian text.

3. Transliterate the numbers in rows 5–10 (not counting the head) of the tablet **thureau-dangin.pdf** available on the course web site—first into sexagesimal form, then into decimal. The article by Knuth lists some of the first few rows from that tablet, to help you see what's going on.

4. Do the problem from the last assignment about deciphering the two-page spread from the paper by James R. Newman, if you have not already done it. Keep in mind that although it is not easy to figure out what the problem is, the solution is exhibited somewhere on the page, so you have a good check on things. It ought to be an interesting exercise to figure out what almost every part of the spread means. Whatever your solution, justify it in your own words.