Mathematics 446 — third homework

This is due next Friday, September 30.

1. The Egyptians always expressed fractions as sums of unit fractions $1/N$. This raises some mildly interesting mathematical questions.

(a) Is it true that every fraction f between 0 and 1 can be expressed as a sum of distinct unit fractions? If so, prove it. If not, give an example, and explain which can be expressed in this way (with poroofs).

(b) Can f have an infinite number of such expressions? An infinite number witha given number of terms?

(c) Find all such expressions for $2/45$ involving two terms; three terms.

(d) Find all such expressions for $2/47$ involving two or three terms. For $2/53$.

First explain in detail why any fraction $2/N$ only has a at most finite number of expressions as the sum of two unit fractions. Explicitly, if

$$\frac{2}{N} = \frac{1}{a} + \frac{1}{b}$$

show lower and upper bounds on a and b. The basic idea is this: try $a = n = (N+1)/2, n+1, \ldots, N$, and then show that you need not look further. Do this by first swapping a and b if necessary to get $b > a$ and then show that $a < N$.

Next do the same for three terms by showing bounds for $a < b < c$ with

$$\frac{2}{N} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ .$$

You can find some simple bounds by being careful. Again $(N+1)/2 < a$ is easy to see. And you can also find an easy upper bound on a. This means that only a finite number of a are possible. But then for each a only a finite number of b and c are possible.

2. Read the selection by Newman. Tell me what the problem being solved on the two-page spread is, and what and where the solution is on those pages. Keeo in mind that although it is not easy to figure out what the problem is, the solution is exhibited somewhere on the page, so you have a good check on things. It ought to be an interesting exercise to figure out what almost every part of the spread means. Whatever your solution, justify it in your own words.

3. Carry out the next step in the calculation of the square root of 720 suggested by Heron in the excerpt quoted by Fowler & Robson.