Mathematics 446 — Fall 2005 — sixth assignment

This is due next Monday, November 7.

Read Dedekind's essay and my notes on it. Note that Dedekind includes negative numbers, whereas I don't.

Every proof must depend on results already proven, or on clearly stated properties of rational numbers.

1. Prove that if A satisfies (a1)–(a2) and B is its complement, then the conditions (a1)–(a2) are equivalent to (c1)–(c3).

2. If x and y are positive real numbers, define $x \leq y$ to mean $A_x \subseteq A_y$, and $x < y$ to mean $x \leq y$ but $x \neq y$. Prove that if x and y are any positive real numbers, the either $x < y$, $x = y$, or $y < x$.

3. If $x < y$, define $y - x$. Verify that $x + (y - x) = y$.

4. Define what it means for a sequence x_i to converge to 0; for it to converge to a (positive) real number x.

5. Write out in your own words the proof that if x_i is a sequence bounded from above then it converges to a real number.

6. Prove that if

$$x_1 - x_2 + x_3 - \cdots$$

is a series with $0 < x_{i+1} < x_i$ and x_i converges to 0, then the series converges to a limit.