

Mathematics 446 — partial solutions to the fourth assignment

1. The following line from the table mentioned in last week's assignment is said by Knuth somewhere not to be correct. Is it or not? Explain your answer.

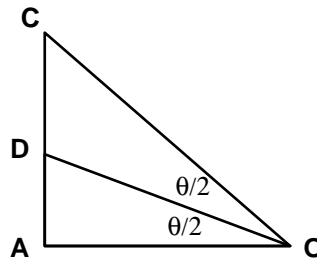


The correct inverse of the left hand side ends with 38 : 24 not 33 : 45 which is an easy and understandable error in copying.

2. Memorize the Greek alphabetic numeral system.

Well?

3. Read the selection in English from Archimedes that I handed out in class, as well as the account by Arndt and Haenel. Write up Archimedes' procedure in your own words, taking no more than one page, explaining the reference by Arndt and Haenel to recursion, and tying it in explicitly to the original by Archimedes. You should think of this as a guide to reading Archimedes.



Proposition VI.3 of Euclid says that

$$\begin{aligned} \frac{OC}{OA} &= \frac{CD}{DA} \\ \frac{OC}{OA} + \frac{OA}{OA} &= \frac{CD}{DA} + \frac{DA}{DA} \\ &= \frac{CA}{DA} \\ \frac{OC}{CA} + \frac{OA}{CA} &= \frac{OA}{AD} \end{aligned}$$

and

$$OA^2 + AD^2 = OD^2, \quad \frac{OA^2}{AD^2} + 1 = \frac{OD^2}{AD^2}$$

If we let

$$a_\theta = \frac{1}{\tan \theta}, \quad b_\theta = \frac{1}{\sin \theta}$$

then these equations become

$$\begin{aligned} a_{\theta/2} &= a_\theta + b_\theta \\ b_{\theta/2} &= \sqrt{a_{\theta/2}^2 + 1} \end{aligned}$$

If $\theta = \pi/n$ and $\theta/2 = \pi/2n$ then the circumference of the circumscribed n -polygon is $2n \tan(\pi/n) = 2n/a_{\pi/n}$, and the inscribed one has circumference $2n/b_{\pi/n}$.

Archimedes starts with $n = 6$, and in this case $\tan \pi/6 = 1/\sqrt{3}$ and $a_{\pi/6} = \sqrt{3}$, $\sin \pi/6 = 1/2$.

We get this table:

n	a_n	b_n	$C_n = 2n/a_n$	$I_n = 2n/b_n$	$C_n - I_n$
6	1.732051	2.000000	3.464102	3.000000	0.464102
12	3.732051	3.863703	3.215390	3.105829	0.109562
24	7.595754	7.661298	3.159660	3.132629	0.027031
48	15.257052	15.289788	3.146086	3.139350	0.006736
96	30.546204	30.563204	3.142715	3.141032	0.001683

Note that the difference, essentially half the error, gets cut in 4 in each step as predicted later on.

4. In the selection in Greek, find all the numbers in it, and tell what they are in decimal. (Refer to them by page and line, writing them first as the Greek text does, then in decimal.)

Look at the combined Greek and Latin text on the course web page.

5. Use Archimedes' technique to find the inner and outer perimeters for polygons of 768 sides.

$$C_{768} = 3.141610, \quad I_{768} = 3.141584.$$

6. How many sides of the hexagon would you need to get the difference between inner and outer perimeters to be less than 10^{-8} ? 10^{-16} ?

If the error for $n = 96$ is 0.0008 and it gets cut in 4 each doubling, that for $96 \cdot 2^k$ steps is . . . ?

7. The text of the English Archimedes refers to Euclid VI.3. Locate the statement on the internet and rewrite Euclid's statement and proof in your own words with lots of pictures. Your aim is to convince me you understand why the result is true. Include whatever you need to do that. Only do the part that Archimedes needs—that if we bisect the angle then the ratios have the property they are supposed to.

Almost everybody got this on the quiz, so I won't do it here. The trick is to reduce the question to slices across parallel lines by drawing isosceles triangles.