Math 152 <u>Linear Systems</u>
Section 208

Colin Macdonald

Common webpage: www.math.ubc.ca/~koru/m152
section page: /~cbm/math152

Linear systems, a.k.a. linear algebra "linalg"
matrix algebra, etc

- <u>Unified</u> treatment of linear problems
  $x+y=6$
  $2x+3y=10$
- Foundational in Sci / Eng / OR / Finance
  operations research
  data science, etc
- History of solving linear eqns
  uniquely connected to history of computers
  Eg. 1950's: "big" system: 17 eqns.
  ⋮
  modern big system: billions of eqns.
  Eg. "top 500 list" → comparison based on "linpack" 1970's to solve linear systems

- Abstract + basically no ~~prere~~ prerequiests.
  - combine simple ideas ~~&~~ in abstract ways
    ↳ you need to keep on top of the material.

WebWork : weekly. ~~&~~ see webpage.

Computer Labs – fortnightly starting in ~~wk~~ week 2 / 3.
- see registration for where/when.
- MATLAB Software
- GNU Octave is a free / open source alternative.

Midterms: evenings Thu: Feb 8 6pm–7pm
Mar 15 "

Notes: on common webpage.

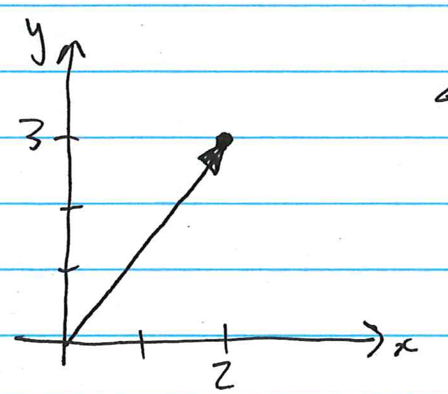
Vectors

Def'n a vector is an ordered set of n real numbers.

For now, $n=2$

Ex. $[2, 3]$ is a vector in $\mathbb{R}^2$ — "vector space" of all possible vectors for $n=2$

(x, y) coordinates of the vector



geometric representation

- Sometimes we think of the vector $[2, 3]$ as the point $(2, 3)$

usually $\longrightarrow$
- more often, the vector is the arrow.

Notation: (a_1, a_2), $[a_1, a_2]$, $\langle a_1, a_2 \rangle$,

$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ ← column vector – useful later.

$\mathbf{a}$, $\underline{a}$, $\vec{a}$, $\vec{a}$

bold a

$a_1 \hat{i} + a_2 \hat{j}$

$a_1 \hat{x} + a_2 \hat{y}$

$a_1 \vec{e}_1 + a_2 \vec{e}_2$

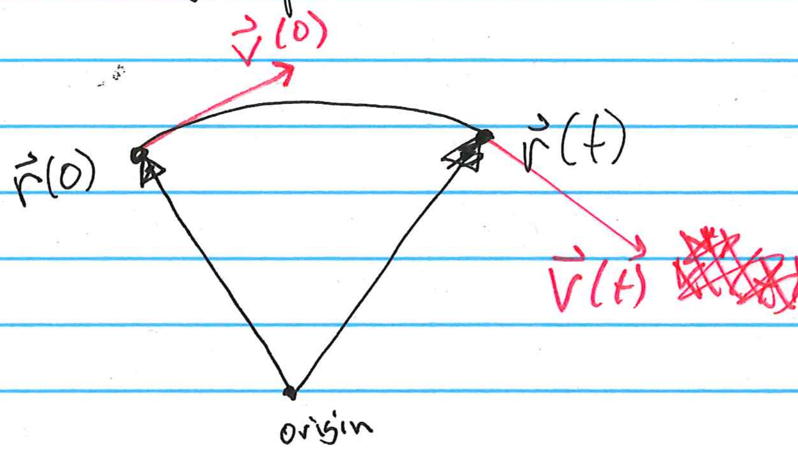
Q: what are vectors used for?

- physics, anything with <u>magnitude</u> and <u>direction</u>
  - force, position, velocity, accel.

vectors can be long ~~arbitrarily~~

- computer graphics
- data science
- approx. theory / numerical analysis

<u>Ex</u> moving particle.



$\vec{r}(t)$ position of particle.

$\vec{v}(t)$ velocity

tangent to the path

<u>note</u>: we drew $\vec{v}(t)$ "where it has meaning".

the vector is the arrow!

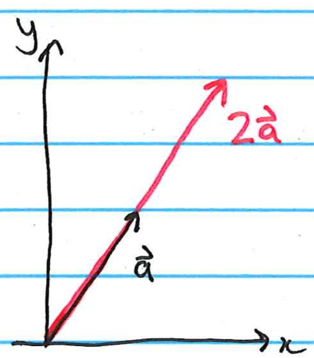
Q: if particle moves in circular motion with uniform speed, where does the acceleration vector point?

see XKCD comic :)

Math on vectors

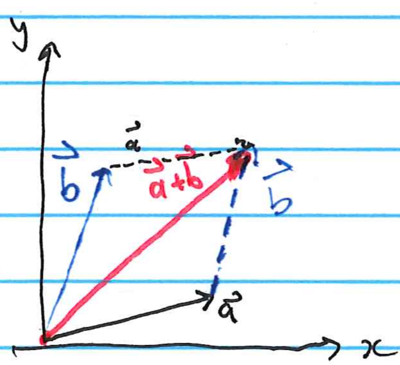
1. multiply vectors by "scalars".
2. add vectors.

Ex $\underbrace{2}_{\text{scalar}}\underbrace{[2,3]}_{\text{vec}} = [2\cdot 2,\ 2\cdot 3] = [4,6]$



Geometry: mult. by scalar preserves direction but changes length.
~~scalr~~ scales
"scalar multiplication"

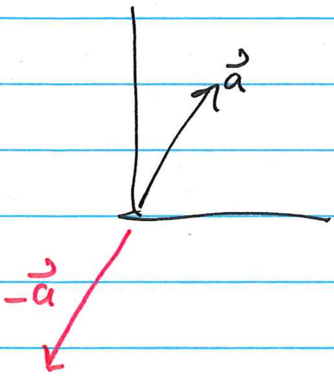
Ex $\langle 3,1\rangle + \langle 1,4\rangle = \langle 3+1,\ 1+4\rangle$
$= \langle 4,\ 5\rangle$



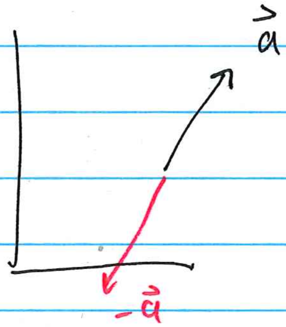
Geometry: put tail to head, add.

Note: two forces on object $\Rightarrow$ two vectors
$\Rightarrow$ total force is the vector addition.

Ex $\quad -[2,3] = -1\,[2,3] = [-2,-3]$



Geometry: we said scalar multiplication preserves direction... "same dir" means lie on the same line ~~through the origin.~~



8 Algebraic Facts.

$$\vec{a}+\vec{b} = \vec{b}+\vec{a} \qquad \text{for all } \vec{a},\vec{b} \qquad (1)$$

start w/ LHS ... RHS

$$\left[\text{proof in } \mathbb{R}^2: \ \vec{a}+\vec{b} = (a_1+b_1,\ a_2+b_2) = (b_1+a_1,\ b_2+a_2) = \vec{b}+\vec{a} \ \blacksquare \right]$$

proved in notes

$$\vec{a}+(\vec{b}+\vec{c}) = (\vec{a}+\vec{b})+\vec{c} \qquad \text{for all } \vec{a},\vec{b},\vec{c} \qquad (2)$$

$$\vec{a}+\vec{0} = \vec{a} \qquad \text{for all } \vec{a} \qquad (3)$$

$$t(\vec{a}+\vec{b}) = t\vec{a}+t\vec{b} \qquad \text{for all } \vec{a},\vec{b},t \qquad (4)$$

$$(t+s)\vec{a} = t\vec{a}+s\vec{a} \qquad \text{for all } \vec{a},t,s \qquad (5)$$

$$t(s\vec{a}) = (ts)\vec{a} \qquad \text{''} \qquad (6)$$

$$1\vec{a} = \vec{a} \qquad \text{for all } \vec{a} \qquad (7)$$

$$\vec{a}-\vec{a} = \vec{0} \qquad \text{''} \qquad (8)$$

Q: what operation is missing?
(Compared to arithmetic)

↳ vector mult. → later!

<u>Abstraction</u>: Any collection of objects (vectors) for which scalar mult and vector add can be defined to satisfy (1)–(8) is called a <u>vector space</u>

<u>Ex</u>
- "our" $\mathbb{R}^2$: 2D Euclidean space.
- $\mathbb{R}^3$ ordered sets of 3 real numbers.
- $\mathbb{R}^{50}$ " " " 50 " " .

  $\vec{a} \in \mathbb{R}^{50}$ salaries of 50 employees.

  Q: $(1.05)\vec{a}$? } meaning in this example.
  $\vec{a} + \vec{b}$?
- $\mathbb{R}^n$
- <u>more</u> <u>abstract</u>: set of a polynomials.

  $\vec{a} = 1 + x^2$, $\vec{b} = 3 + x^3 + 2x^5$

  ~~vector~~ scalar mult: $2\vec{a} = 2 + 2x^2$

  vector add: $\vec{a} + \vec{b} = 4 + x^2 + x^3 + 2x^5$

(can show (1) – (8) are satisfied)