Math 152 - lecture 10

Applications : domain knowledge

$\Downarrow$

math model

$\Downarrow$

linear system

$\Downarrow$

solve & interprete

Ex Chemistry: Methane Combustion

$$a\,CH_4 + b\,O_2 \rightarrow c\,H_2O + d\,CO_2$$

integer

\# of molecules

homog

Balance

$$C: a = d \qquad H: 4a = 2c \qquad O: 2b = c + 2d \quad \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 4 & 0 & -2 & 0 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} \boxed{1} & 0 & 0 & -1 & 0 \\ 0 & \boxed{1} & 0 & -2 & 0 \\ 0 & 0 & \boxed{1} & -2 & 0 \end{array}\right]$$

free variable

$d = t,\ c = 2t,\ b = 2t,\ a = t$

choose $t = 1$ to get smallest integer sol'n.

could be hard in general

$$CH_4 + 2O_2 \rightarrow 2\,H_2O + CO_2$$

Matrices : §4

$$A = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$m \times n$ matrix

rows — columns

entries/components : a_{ij}

ith row — jth column.

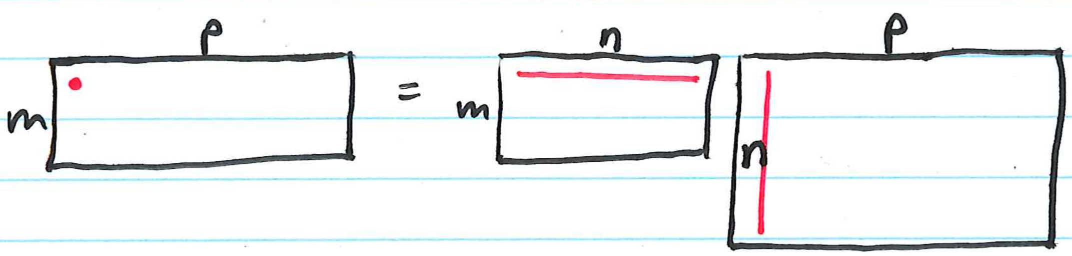
$1 \times n$ matrix is a "row vector"
$m \times 1$ " " " column vector

- Can add matrices $A+B$: add the entries $a_{ij} + b_{ij}$
- Can scalar multiply matrices sA : scalar mult. each entry. sa_{ij}

↳ set $m \times n$ matrices is a vector space.

But can also multiply (some) matrices:

$$\underbrace{C}_{m\times p} = \underbrace{A}_{m\times n}\,\underbrace{B}_{n\times p}$$



C is a $m \times p$ matrix with $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$

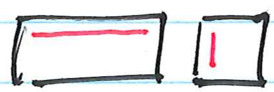
= dot prod of ith row of A and jth column of B

Ex $\quad A = \begin{array}{cc} & 2 \\ 2 & \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \end{array} \qquad \text{ok} \qquad B = \begin{array}{cc} & 3 \\ 2 & \begin{bmatrix} 1 & 0 & 5 \\ 2 & 3 & 1 \end{bmatrix} \end{array}$

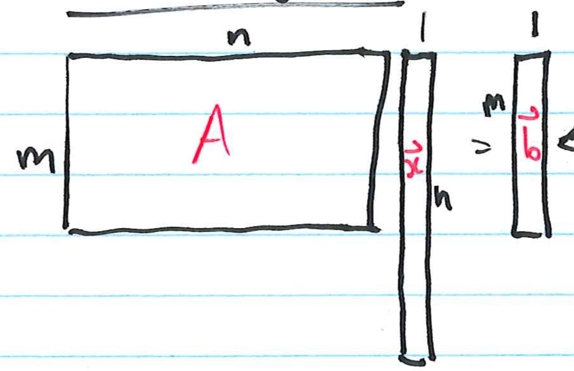
$C = AB = 2 \times 3$ matrix

$$= \begin{array}{ccc} (2,1)\cdot(1,2) & (2,1)\cdot(0,3) & (2,1)\cdot(5,1) \\ \multicolumn{3}{c}{\begin{bmatrix} 4 & \quad 3 \quad & 11 \\ 5 & \quad 6 \quad & 7 \end{bmatrix}} \\ (1,2)\cdot(1,2) & (1,2)\cdot(0,3) & (1,2)\cdot(5,1) \end{array}$$

<u>Note</u>: BA undefined (!)



Linear Systems



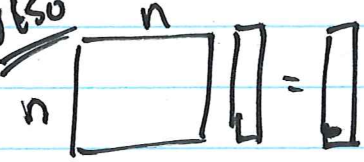
$$A\vec{x} = \vec{b} \iff \overbrace{\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{array}}^{\text{lin sys.}}$$

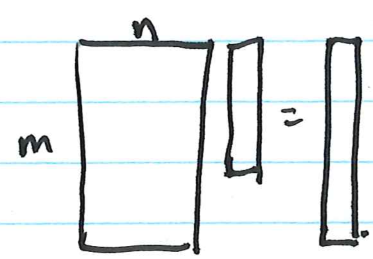
A: $m \times n$ matrix

$\vec{x}$: column vector of n unknowns

$\vec{b}$: column vector of m RHS values

also





Multiplication of Matrices is weird

E.g. $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $BA = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

- $AB \neq BA$ (usually) even when both defined.
- $AB = \emptyset$ even though $A \neq \emptyset$ and $B \neq \emptyset$

zero matrix, all entries 0

i.e. $AB = \emptyset \not\Rightarrow A = \emptyset$ or $B = \emptyset$

12 properties of matrix operations:

(1) - (8) inherited b/c we have a vector space: $A + B = B + A$

(9) $A(B+C) = AB + AC$ (sizes must be compatible)

(10) $(A+B)C = AC + BC$

(11) $A(BC) = (AB)C = ABC$

(12) $s(AB) = (sA)B = A(sB)$

~~sc~~ scalar

Most important: remember that $AB \neq BA$