Math 152, Lecture 11

Last day: <u>matrices</u>

<u>Ex</u> scalar x: $1x = x$

matrices? $\mathbb{1}A = A$? ✗

matrix of 1's

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11} + a_{21} & \cdots \\ & \end{bmatrix}$$

2×2 case: want $IA = A$

$$\begin{bmatrix} i_{11} & i_{12} \\ i_{21} & i_{22} \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} i_{11}a_{11} + i_{12}a_{21} & \\ & \cdots \end{bmatrix}$$

a_{11}?

$i_{11} = 1$
$i_{12} = 0$

$$\ldots \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Generally, we have <u>the</u> <u>identity matrix</u>

$$I_{n \times n} = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & \ddots \; 1 \end{bmatrix}$$

<u>Note</u>: $IA = AI = A$

Ex notes.pdf, Prob 4.4, functions of matrices

b) $A^2 = AA$, $A^3 = AAA$, etc

What if $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$? What is $A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

$I^k = I$

c) e^{tA}, where t scalar, A square matrix.

not the usual exp(x)

Idea: Taylor (from calculus)

scalar

$$\exp(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \ldots$$

generalize: $\operatorname{expm}(B) := I + B + \frac{1}{2}B^2 + \frac{1}{6}B^3 + \frac{1}{24}B^4 + \ldots$

↳ $B = tA$

matrix exponential

Such abstract functions of matrices are useful (we'll see a bit more later).

↳ Computing these in practice is an area of research in linear algebra.

Linear Transforms §4.2?

Scalar fcns: $f(x)$, scalar output ex
scalar input $\cos(x)$
or $\boxed{42x}$

generalize: $\vec{f}(\vec{x})$, vector output, m components
vector input
n components

$$\vec{x} \in \mathbb{R}^n, \quad \vec{f}(\vec{x}) \in \mathbb{R}^m$$

$\vec{f}$ is called a vector function, or transformation or map

Multiplication by $m \times n$ matrix defines a linear transform from $\mathbb{R}^n$ to $\mathbb{R}^m$

$$\vec{f}(\vec{x}) := A\vec{x}$$

(i) $\vec{f}(\vec{x}+\vec{y}) = \vec{f}(\vec{x}) + \vec{f}(\vec{y})$ for all $\vec{x}, \vec{y} \in \mathbb{R}^n$

(ii) $\vec{f}(s\vec{x}) = s\vec{f}(\vec{x})$ for all $\vec{x} \in \mathbb{R}^n$, $s \in \mathbb{R}$

$\vec{f}(\vec{x}) = A\vec{x}$ satisfies these requirements?

follow from 12 properties last day. $\left\{ \begin{array}{l} A(\vec{x}+\vec{y}) = A\vec{x} + A\vec{y} \quad \checkmark \\ A(s\vec{x}) = sA\vec{x} \quad \checkmark \end{array} \right.$

Notes

I $\vec{f}(\vec{x}) = A\vec{x}$ defines the output of a linear system with input $\vec{x}$

So $A\vec{x} = \vec{b}$ is an Inverse problem: find the input $\vec{x}$ for a given output $\vec{b}$

II If $A\vec{x}_1 = \vec{b}_1$ and $A\vec{x}_2 = \vec{b}_2$

then $\vec{x} = \vec{x}_1 + \vec{x}_2$ is a solution of $A\vec{x} = \vec{b}$ with $\vec{b} = \vec{b}_1 + \vec{b}_2$

III Say $\vec{x}_1$ and $\vec{x}_2$ both solve $A\vec{x} = \vec{b}$

then $A(\vec{x}_1 - \vec{x}_2) = A\vec{x}_1 - A\vec{x}_2 = \vec{b} - \vec{b} = \vec{0}$,

that is difference of sol'ns of a linear system solves the associated homog. lin. sys.

E.O.L.