Math 253

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web: www.math.ubc.ca/~cbm/math253/2017

Multivariable Calculus

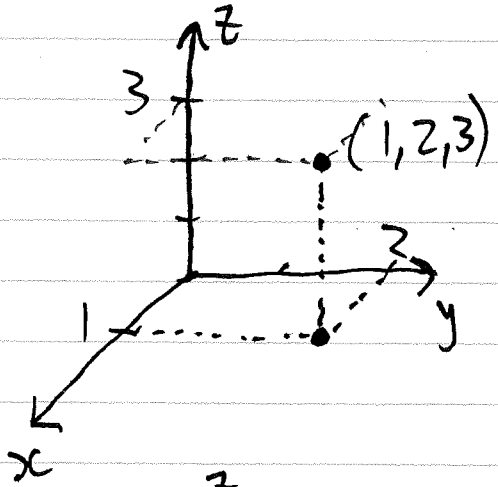
$f(x, y, z)$ change

language/tool for discussing/modelling rates of change in complex multidimensional problems

↳ Sci/engineering/finance data science/etc

3D viz and geometry is important.

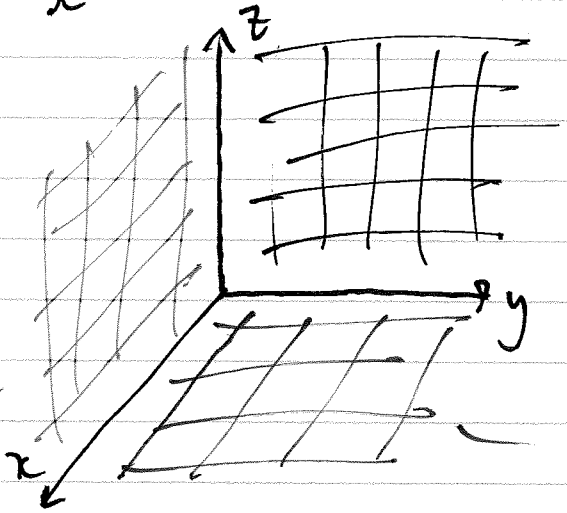
§10.1 3D coordinates ($\mathbb{R}^3$)



Right-hand system

point "pt" $(1, 2, 3)$

Ex: planes



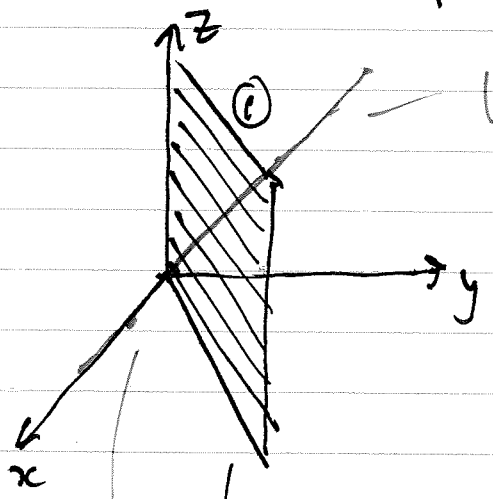
y-z plane, eqn $\boxed{x=0}$

x-y plane, $\boxed{z=0}$

x-z plane, $\boxed{y=0}$

(?)

Ex two planes: ① $x=y$ and ② $y=z$



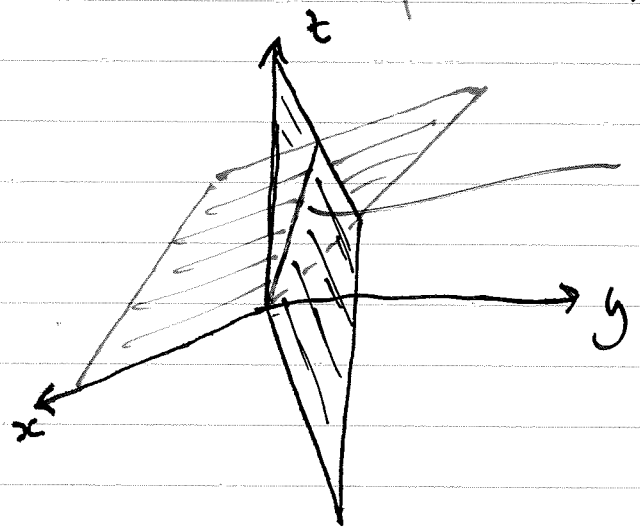
① — line $y=z$ when $x=0$

② — draw in y-z plane

"edge on" plane

line $x=y$ when $z=0$

↳ oops! Try again:



intersection: line $x=y$ and $y=z$

$\Rightarrow \boxed{x=y=z}$

Example Eqn of a sphere
Centre $C=(a,b,c)$
radius r

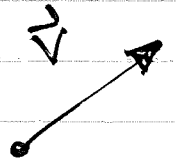
all $P=(x,y,z)$ such that $|PC|^2 = r^2$

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

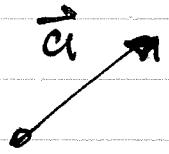
[back to §10.1 later]

§10.2 Vectors

- Def'n: a vector is a directed line segment



$\vec{v}$ has length/magnitude and direction



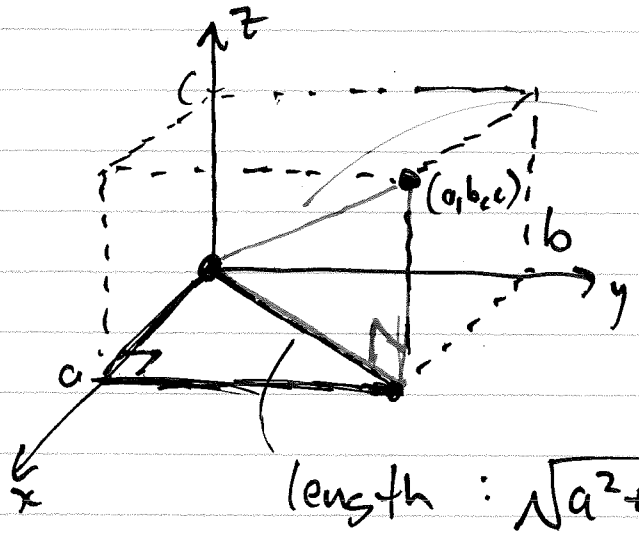
eg. force, velocity

Here $\vec{a} = \vec{v}$ b/c they have same mag. and dir.

- Notation: $\overrightarrow{PQ}$ vector connecting pt P to pt Q

- zero vector $\vec{0}$ has zero length (and no direction)

Distance from (0, 0, 0) to (a, b, c)



length : $\sqrt{a^2+b^2}$

distance : $\sqrt{\left(\sqrt{a^2+b^2}\right)^2+c^2}$

$= \sqrt{a^2+b^2+c^2}$

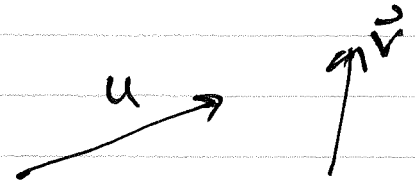
Distance from $P_1=(x_1, y_1, z_1)$ to $P_2=(x_2, y_2, z_2)$

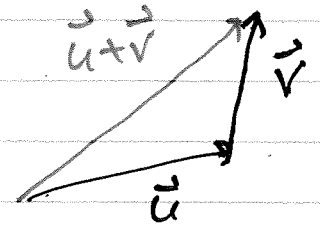
is $\|\overline{P_1P_2}\| = |P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

(book) alt. notation

- draw diagram
- shift P_1 to the origin

Def'n $\vec{u}+\vec{v}$:





note: $\vec{u}+\vec{v}=\vec{v}+\vec{u}$