Last day: Eqn of a plane

$\vec{n}$: fixed, normal vector $\langle a,b,c\rangle$
$\vec{r}_0$: fixed, a point on the plane (x_0, y_0, z_0)

$\vec{r}$: variables, any point (x,y,z) which is on the plane.

~~Thm~~ Thm: (x,y,z) is on the plane if and only if $(\vec{r}-\vec{r}_0)\cdot\vec{n}=0$

Equiv: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

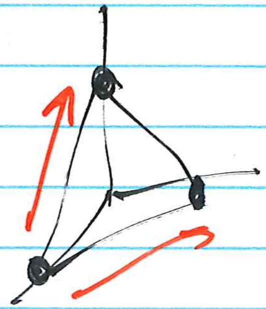
Equiv: $\boxed{ax+by+cz+d=0}$

d: a, x_0, b, etc

note linear eqn

coefficients encode the normal.

Ex Find eqn of a plane thru $(1,0,0)$, $(0,2,0)$, $(0,0,3)$



$\vec{n} = \langle -1,2,0\rangle \times \langle -1,0,3\rangle = \ldots$
$= \langle 6,3,2\rangle$

$\Rightarrow 6x+3y+2z+d=0$
sub in a point like $(1,0,0)$
$6\cdot 1+0+0+d=0$

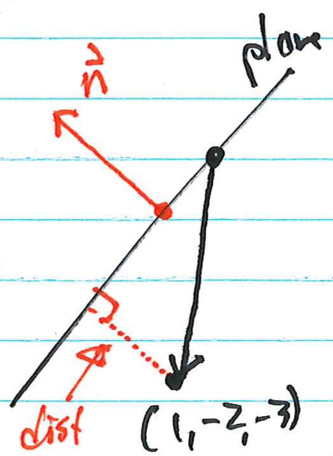
$\Rightarrow \boxed{6x+3y+2z-6=0}$

Ex find the (min) distance from the previous plane to $(1,-2,-3)$

① Find vector from $(1,-2,-3)$ to any point in plane.

$$\vec{u} := \langle 1,0,0\rangle - \langle 1,-2,-3\rangle = \langle 0,2,3\rangle$$

② Now $\underbrace{\text{scalar projection}}_{\text{comp}_{\vec{n}}\vec{u}}$ of $\vec{u}$ onto $\vec{n}$



$$\text{dist} = \left|\cancel{\text{proj}_{\vec{n}}\vec{u}}\right| = \left|\frac{\vec{u}\cdot\vec{n}}{|\vec{n}|}\right| = \frac{0\cdot 6 + 2\cdot 3 + 3\cdot 2}{7} = \frac{12}{7}$$

$7 = \sqrt{49}$

answer could be negative

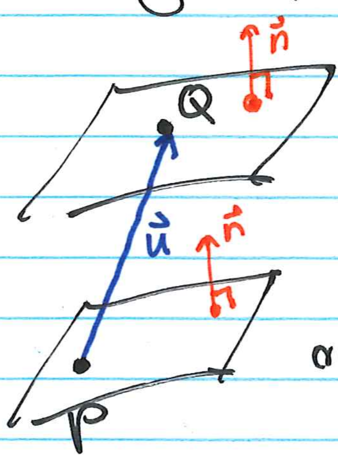
exercise: use pt $(0,0,3)$

<u>Ex</u> Find dist b/w two parallel planes $x+y-2z=2$

$x+y-2z=4$

① Use the normal (to both) $\vec{n}=\langle 1,1,-2\rangle$

② Find any point P in plane 1



$\hookrightarrow$ $y=z=0 \Rightarrow$ solve for x

$x+0-2\cdot 0=2$

$\Rightarrow P=(2,0,0)$

any point on the other plane

$\Rightarrow Q=(4,0,0)$

③ vector $\vec{u} := \overrightarrow{PQ} = \langle 2,0,0\rangle$

Scalar projection of $\vec{u}$ onto $\vec{n}$

$$\mathrm{Comp}_{\vec{n}}\vec{u} = \frac{\vec{u}\cdot\vec{n}}{|\vec{n}|} = \frac{2}{\sqrt{6}} = \sqrt{2/3}$$

<u>Note</u>: might need abs value.

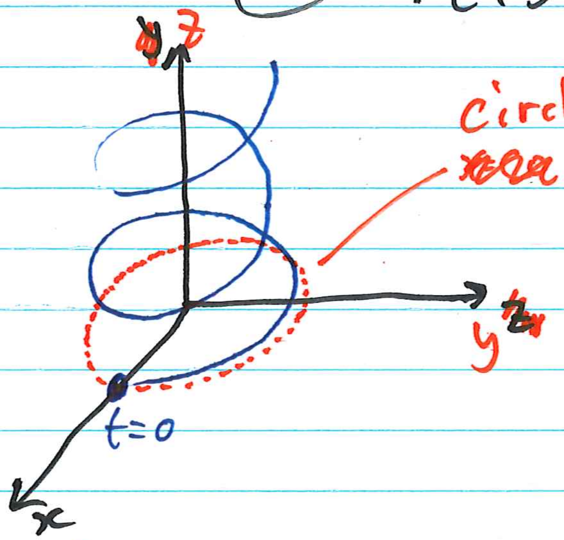
Curves §11.1 (and some Calculus §11.2)

We've seen parametric eqn for line $\vec{r}(t) = \vec{r}_0 + t\vec{v}$. More generally, $\vec{r}(t)$ can describe a curve in $\mathbb{R}^3$:

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

Ex ① eq'n of a line

② $\vec{r}(t) = \langle \cos t, \sin t, \frac{1}{4\pi} t \rangle$



Derivative of a parametrized curve is easy to calculate:

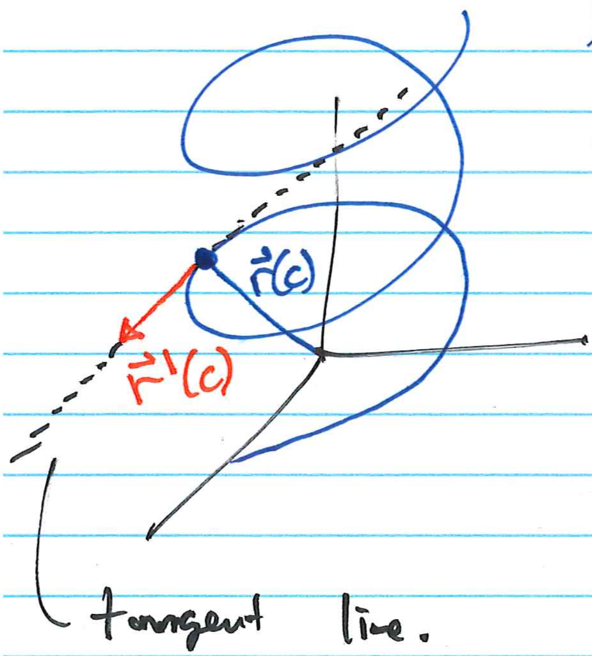
$$\frac{d}{dt}\vec{r}(t) = \vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$$

One meaning is the velocity vector of a particle following the curve, $\vec{r}(t)$.

From this, we can get the tangent line to $\vec{r}(t)$ at a point $\vec{r}(c)$

$$\vec{l}(s) = \vec{r}(c) + s\,\vec{r}'(c)$$

parametric eqn for line



Caution: book reuses "t" here in def'n 71.

How to be careful?

$s = t - c$

esp if t is physical, eg. time.