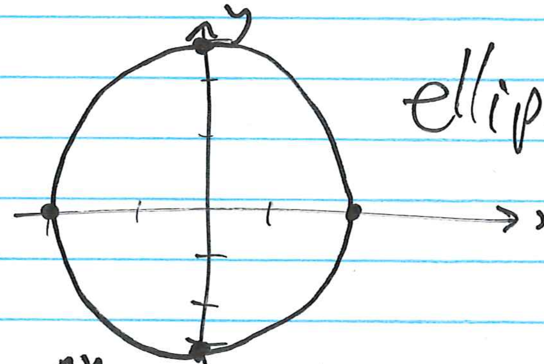
§10.1 Cylinders and Quadric Surfaces

Review of 2-d: (in x-y plane) eqns written in terms of $\{x^2, y^2, xy, x, y, \text{constants}\}$ are called conic sections (circles, ellipses, parabolas, hyperbolae)

Ex a) $\frac{x^2}{4} + \frac{y^2}{9} = 1$



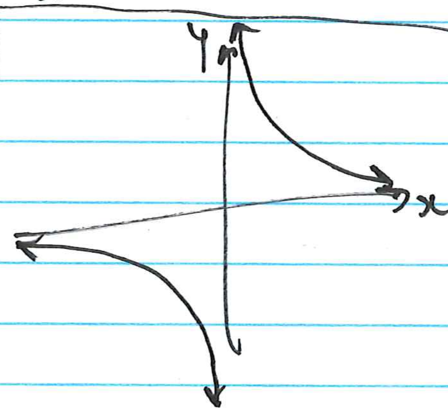
ellipse

Ex b) $x^2 - y^2 = 1$

note $\frac{y^2}{x^2} = 1 - \frac{1}{x^2}$

as $x^2 \to \infty$, $\frac{y^2}{x^2} \to 1 \Rightarrow y \sim \pm x$ as $x \to \pm\infty$

c) $xy = 1$



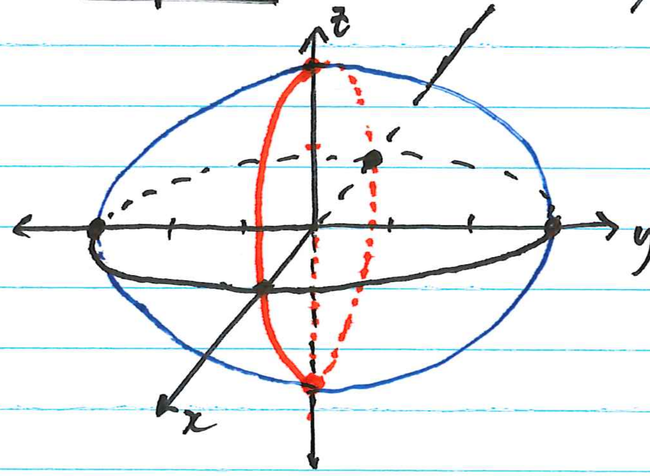
asymptotes of $x = 0$ and $y = 0$

In 3-d: surface given as eqn in $\{x^2, y^2, z^2, xy, xz, yz, x, y, z, \text{const}\}$ is a <u>quadric surface</u>

<u>Ex</u> <u>Sphere</u> $x^2+y^2+z^2=r^2$

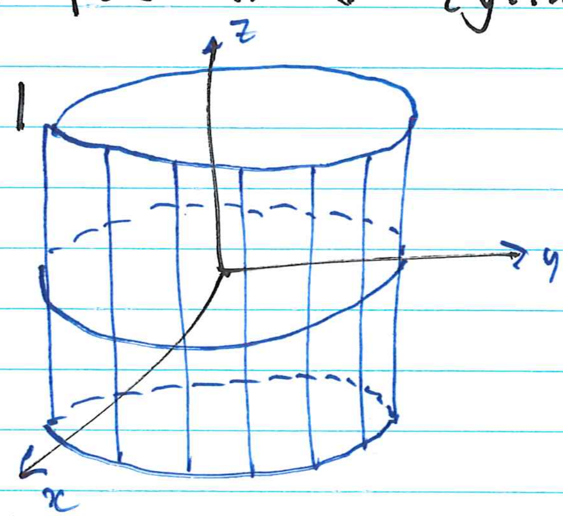
<u>Plane</u> $ax+by+cz+d=0$

<u>Ellipsoid</u> $x^2+y^2/9+z^2/4=1$

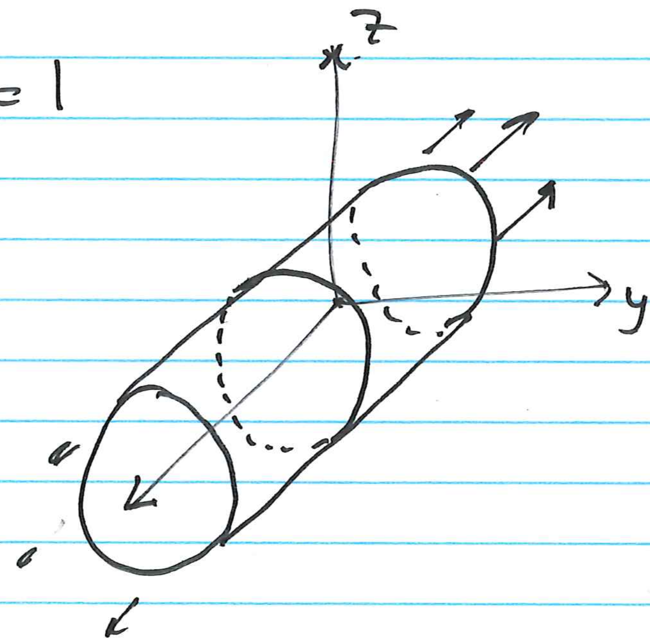


<u>Special case</u> : <u>Cylinder</u> : if the eqn ~~involve~~ involves only 2 of the 3 variables the ~~eq~~ surface is a cylinder.

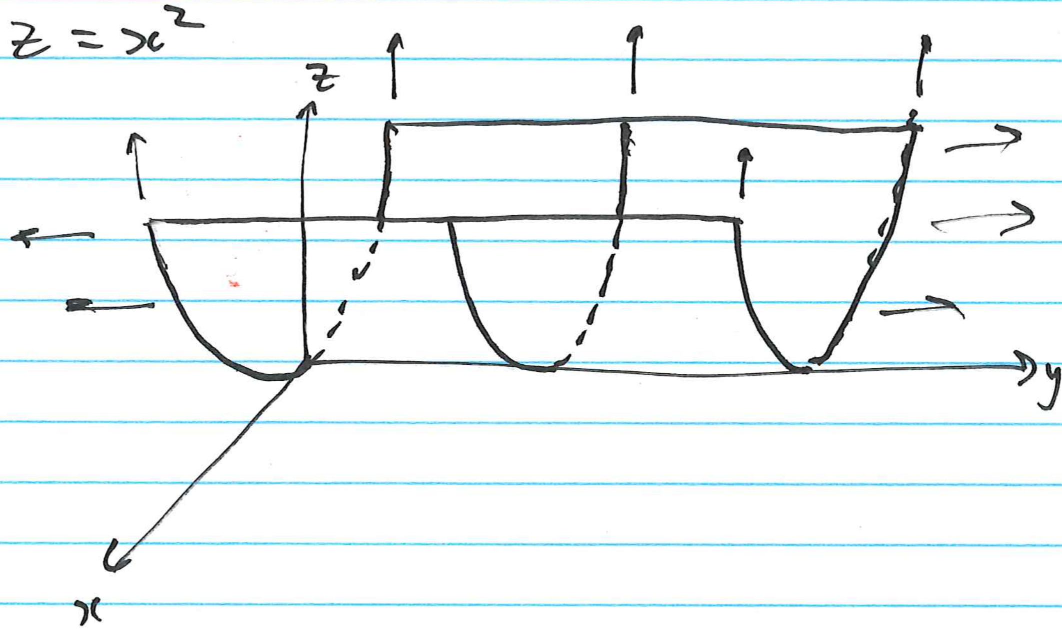
<u>Ex</u> (a) $x^2+\frac{y^2}{9}=1$



b) $y^2 + z^2 = 1$



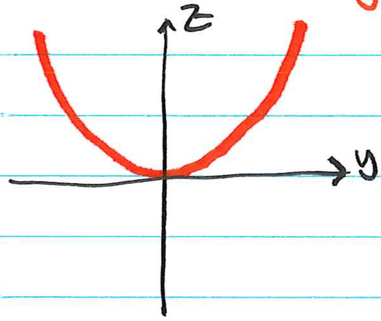
c) $z = x^2$



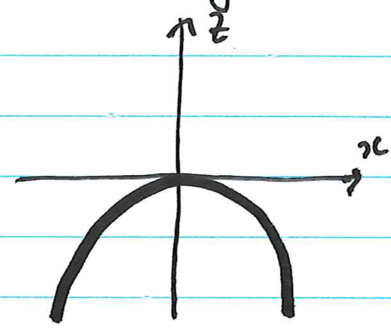
Trace Curves: "cut" the surface by a plane parallel to a coordinate axis.

Ex Sketch $z = y^2 - x^2$

trace $x=0$: $z=y^2$

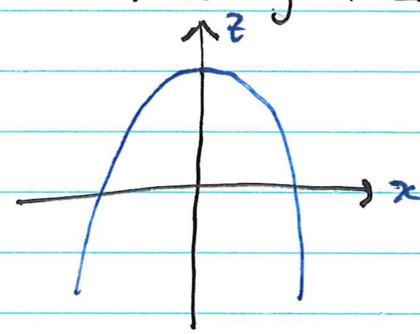


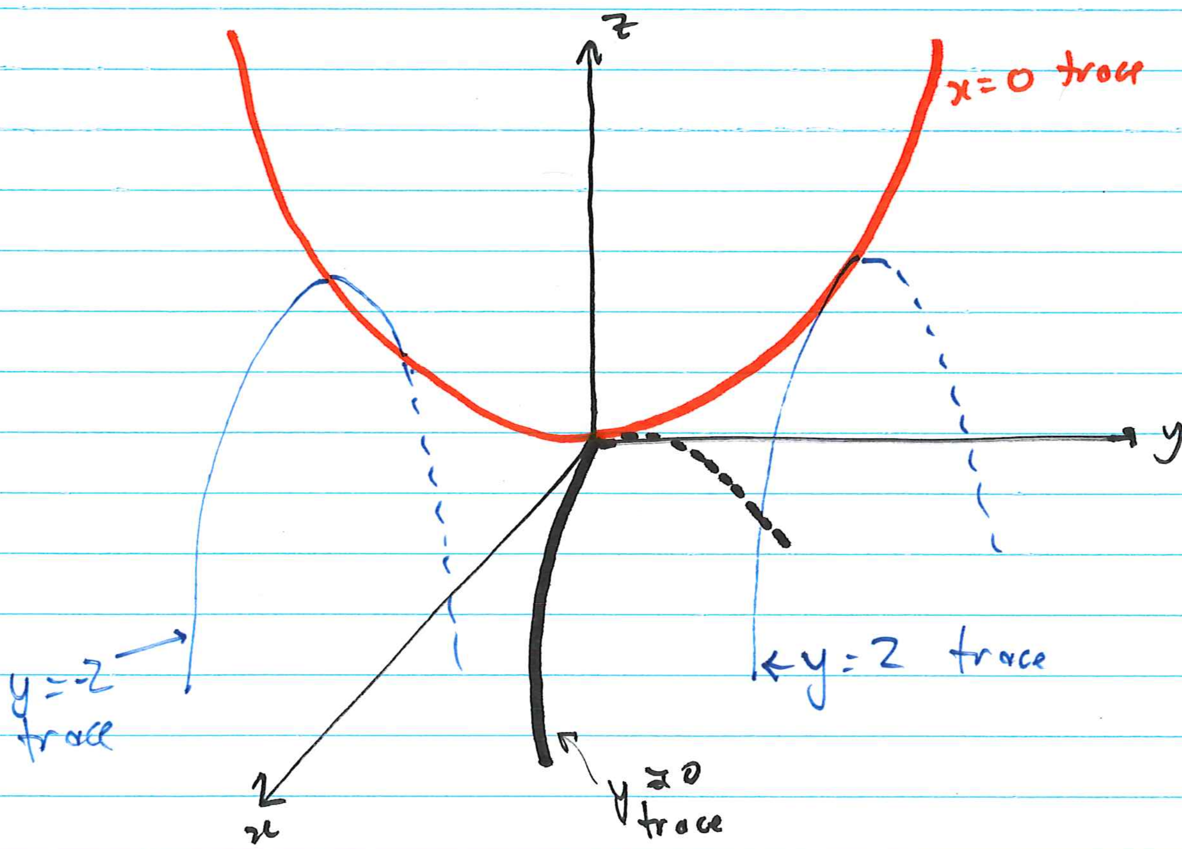
trace $y=0$: $z=-x^2$



$z=4-x^2$

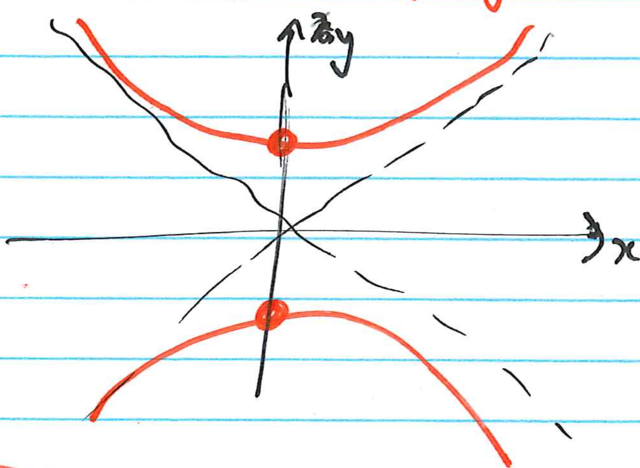
trace: $y=\pm 2$



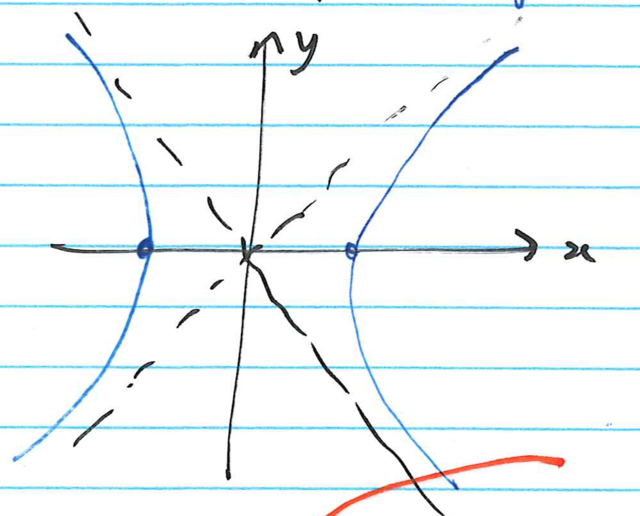


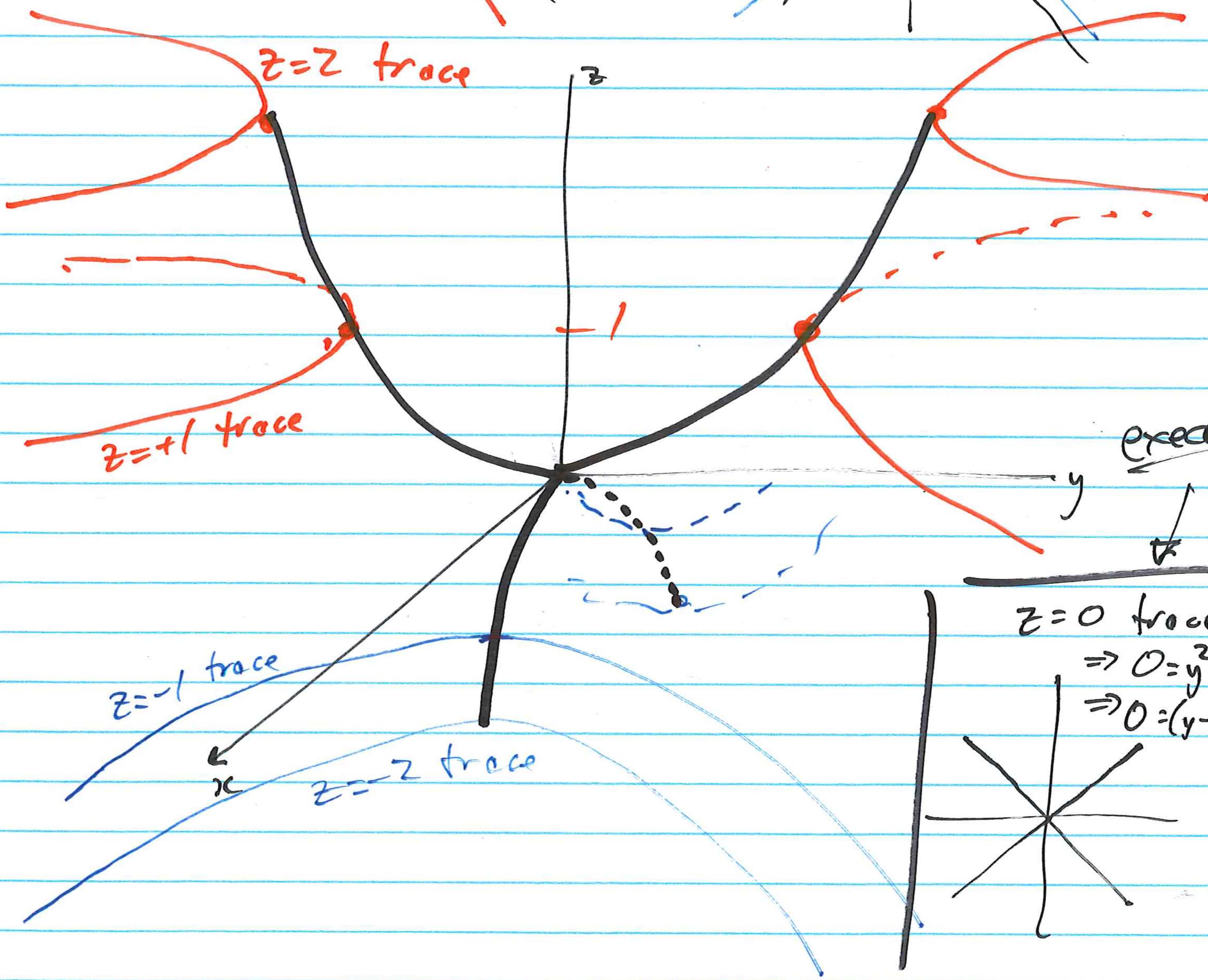
Ex <u>same</u> <u>again</u>: keep $x=0$, $y=0$ traces

trace $z=1$: $1 = y^2 - x^2$



trace $z=-1$: $-1 = y^2 - x^2$





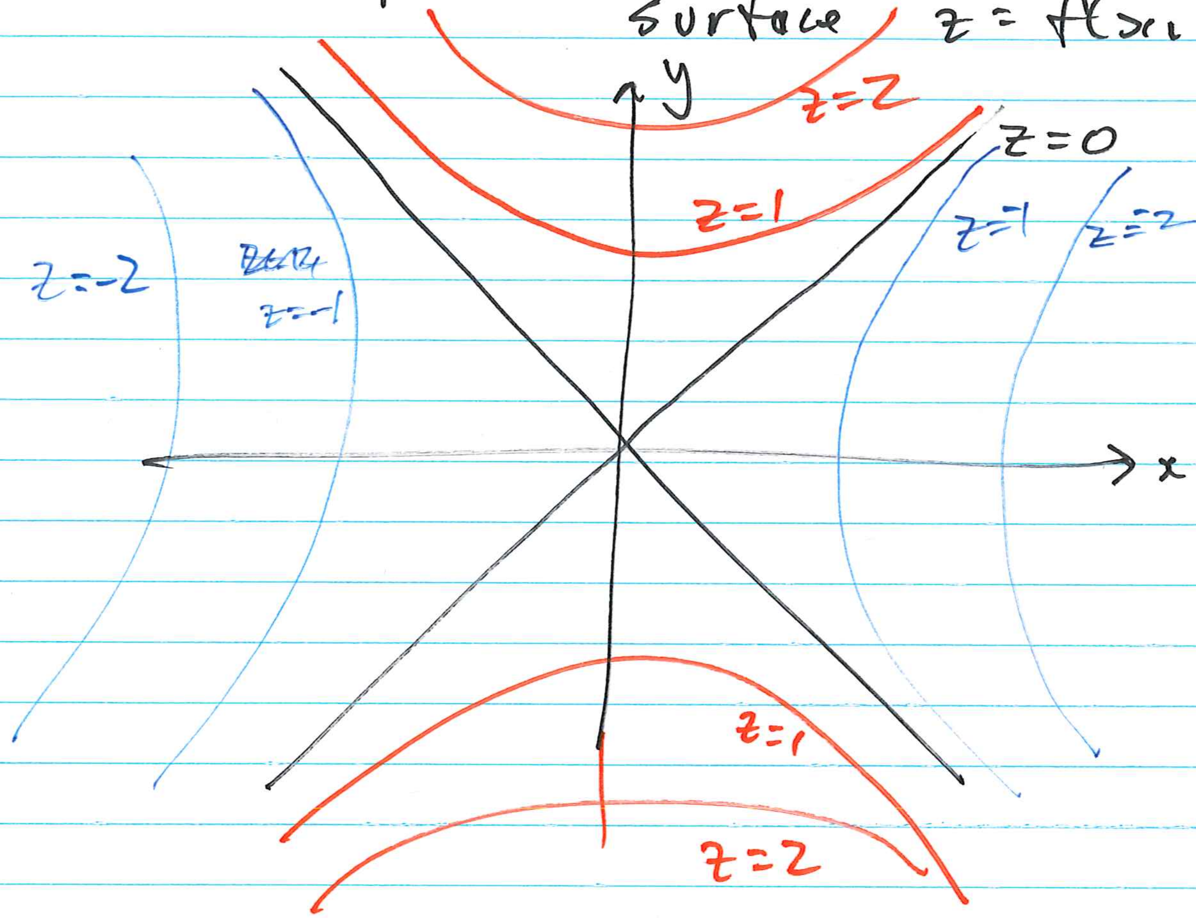
exercise

$z=0$ trace

$\Rightarrow 0 = y^2 - x^2$

$\Rightarrow 0 = (y-x)(y+x)$

Contour plot : level curves of the surface $z = f(x,y) = y^2 - x^2$



This particular example is a hyperbolic paraboloid

↳ Example of a saddle