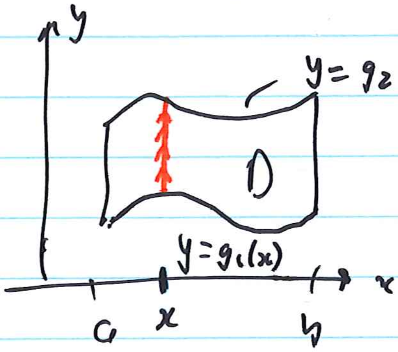
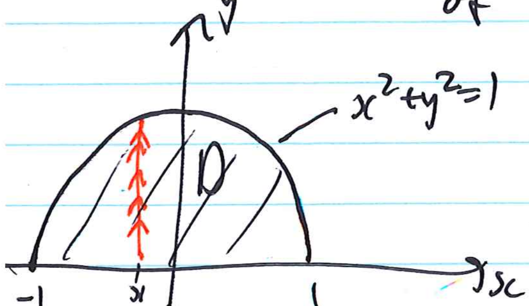
Last day: $\iint$, type I vesion



$$D = \{(x,y) : a \le x \le b \quad g_1(x) \le y \le g_2(x)\}$$

$$\iint_D f(x,y)\,dA = \int_a^b \left(\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy \right) dx$$

Ex ~~Suppose~~ Colin has a loaf of bread occupies a unit semidisc in the xy-plane and the loaf has height $f(x,y) = (1-x^2)^{3/2}$. What is the volume of the loaf



$$V = \int_{-1}^{1} \underbrace{\int_0^{\sqrt{1-x^2}} (1-x^2)^{3/2}\,dy}_{} \,dx$$

$B(x)$ area of one slice of bread

$$V = \lim_{m \to \infty} \sum_{i=1}^{m} B(x_i)\,\Delta x$$

thickness of the slice

$$B(x) = \int_0^{\sqrt{1-x^2}} (1-x^2)^{3/2}\, dy = (1-x^2)^{3/2} \int_0^{\sqrt{1-x^2}} dy$$

$$= (1-x^2)^{3/2}\, y \Big|_0^{\sqrt{1-x^2}}$$

$$= (1-x^2)^{3/2} (1-x^2)^{1/2}$$

$$= (1-x^2)^2$$

Now the outer integral:

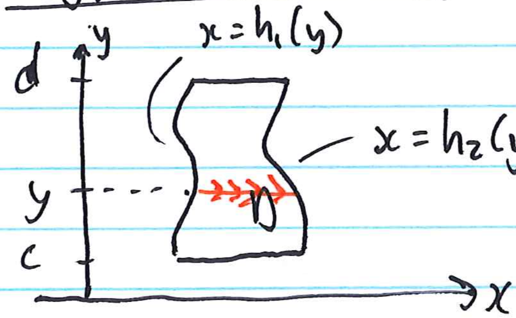
$$\int_{-1}^{1} B(x)\, dx = \int_{-1}^{1} (1-x^2)^2\, dx = 2\int_0^1 (1-x^2)^2\, dx$$

Symmetric interval — Even fcn of x

$$= 2\int_0^1 1 - 2x^2 + x^4\, dx$$

$$= \ldots = 32/15 \;(?)$$

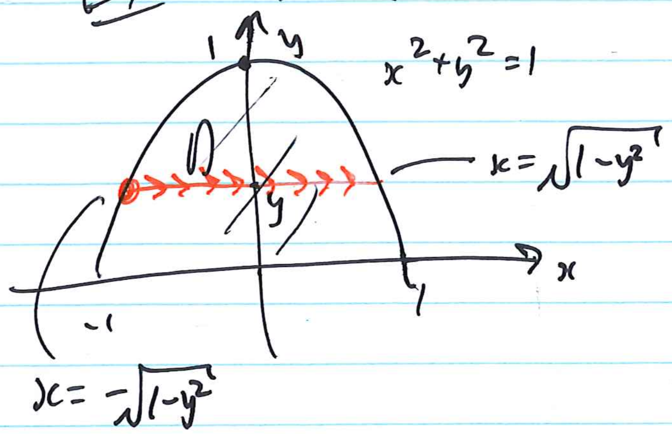
Type II region



$$D = \{(x,y) : c \le y \le d,\; h_1(y) \le x \le h_2(y)\}$$

$$\iint_D f(x,y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dx\, dy$$

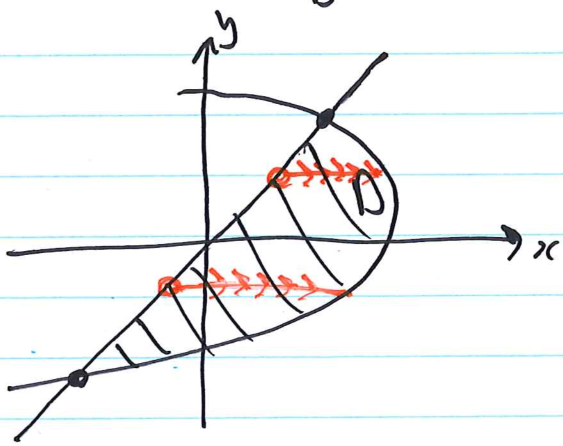
Ex Same bread



$$V=\iint_D (1-x^2)^{3/2}\,dA$$

$$\int_0^1 \underbrace{\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (1-x^2)^{3/2}\,dx}_{\text{harder integral!}}\,dy = \frac{32}{15}\ (?)$$

Ex $\iint_D y\,dA$ over the region D bounded by $x=y$ and $x=2-y^2$.



$$\int_?^? \int_{x=y}^{x=2-y^2} y\,dx\,dy$$

where the curves intersect. (in y)

$y=2-y^2$

$y^2+y-2=0 \qquad y=1,-2$

$$\int_{-2}^{1}\int_y^{2-y^2} y\,dx\,dy$$

$$=\int_{-2}^{1} y\left(x\Big|_y^{2-y^2}\right)dy=\int_{-2}^{1} y(2-y^2-y)\,dy$$

$$=\dots$$

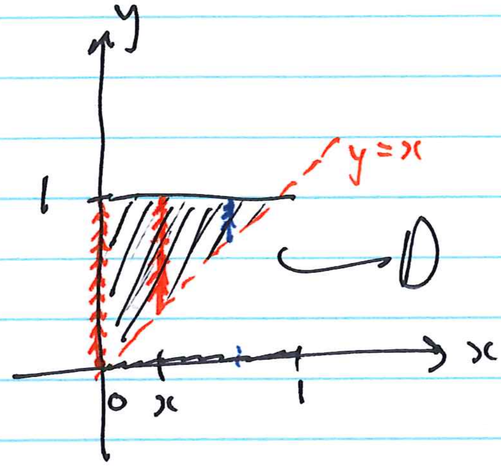
Ex

$$I = \int_{x=0}^{x=1}\int_{y=x}^{y=1} \sin(y^2)\,dy\,dx$$

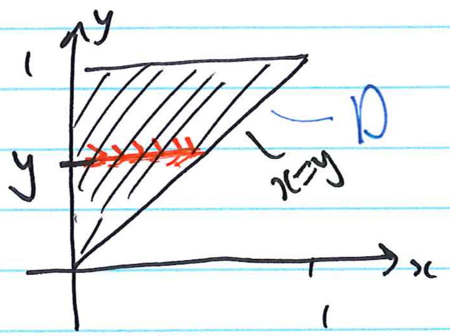
no elementary anti-derivative.

bad idea $\cancel{\overset{?}{=}} \int_x^1\int_0^1 \sin y^2\,dx\,dy$

Fubini only works for rectangles.



$$\int_{y=0}^{y=1}\int_{x=0}^{x=y} \sin y^2 \,\cancel{dy\,dx}\,dx\,dy$$



$$\iint_D f(x,y)\,dA$$

$$\int_0^1 \underbrace{\int_0^y \sin y^2\,dx}\,dy$$

$$= \int_0^1 \left[\sin y^2 \int_0^y dx\right] dy = \int_0^1 \sin y^2 \cdot x \Big|_0^y dy$$

$$= \int_0^1 \sin y^2 \; y\,dy$$

let $u = y^2$ $du = 2y\,dy$

$$= \frac{1}{2}\int_{u=0}^{u=1} \sin u\,du = \frac{-\cos(y^2)}{2}\Bigg|_{y=0}^{y=1} \approx 0.2298$$