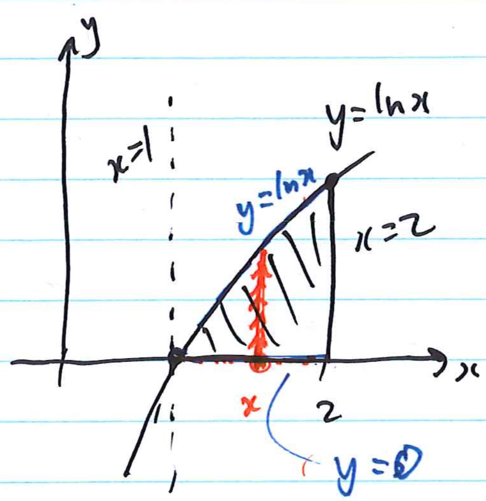
Last day: reversing order of integration $dx\,dy \to dy\,dx$

Ex $$I = \int_1^2 \int_0^{\ln x} f(x,y)\,dy\,dx$$

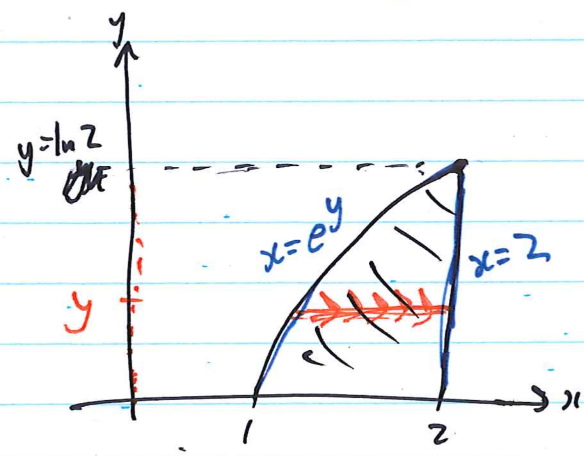
Reverse the order of integration. $\iint_D f\,dA$



$D = \{(x,y) \quad 1 \le x \le 2$ and $0 \le y \le \ln x\}$

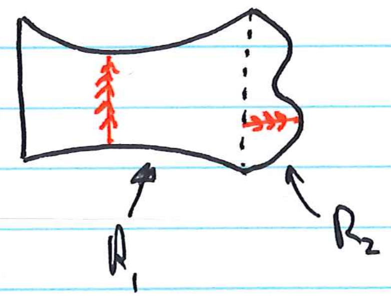
Curves: $x=1$, $x=2$, $y=0$, $y=\ln x$

$x = e^y$



$$I = \int_0^{\ln 2} \int_{e^y}^{2} f(x,y)\,dx\,dy$$

Note not every problem is type I or type II.

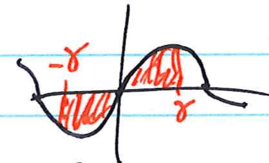
R: 

Cut domain R into type I piece and type II piece.

$$\iint_R f(x,y)\,dA = \iint_{R_1} f(x,y)\,dy\,dx + \iint_{R_2} f(x,y)\,dx\,dy$$

Symmetry Tricks

review in 1-variable: $\displaystyle\int_{-r}^{r} \underbrace{\sin(x)}_{\text{odd-in-}x}\,dx = 0$



(signed) areas cancel.

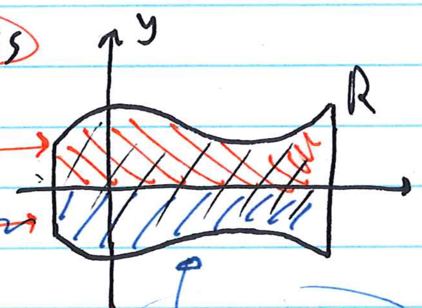
2-vars: 2 cases we consider:

(1) Region R symmetric about x-axis ($y=0$)
and $f(x,y)$ is odd-in-y
$f(x,-y) = -f(x,y)$

$$\Rightarrow \iint_R f(x,y)\,dA = 0$$

signed volume above

signed volume below



= − signed volume below

Eg: y^3: odd-in-y b/c $(-y)^3 = -y^3$

(review) y^2: even-in-y b/c $(-y)^2 = y^2$

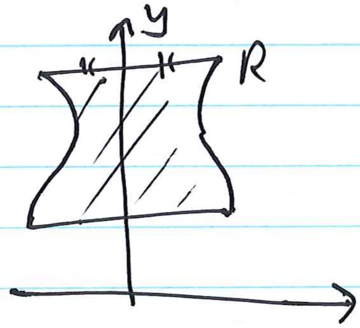
e^y: neither even nor odd $e^{-y} \neq -e^y$, $e^{-y} \neq e^y$

even·odd = odd, even·even = even, odd·odd = even

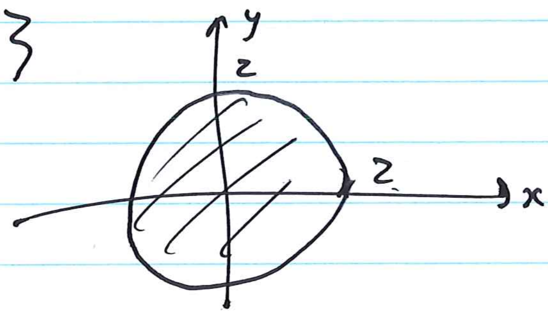
neither·odd = neither, neither·even = neither.

(2) R is symmetric about $x=0$ (the y-axis) and $f(x,y)$ is odd-in-x then

$$\iint_R f(x,y)\, dA = 0$$



Ex $R = \{(x,y) : x^2+y^2 \le 4\}$



Evaluate: $I =$

$$\iint_R \underbrace{e^y \underset{\text{even}}{x^2} \underset{\text{odd}}{\tan x}}_{\text{odd in } x} + \underbrace{\sin(e^{x}y^3)}_{\text{odd in } y} + 5 \, dx\,dy$$

$$= \iint_R \text{''} \, dx\,dy \to 0 \; + \iint \text{''} \, dx\,dy \to 0 \; + \iint 5 \, dx\,dy$$

integrand: odd-in-x
R symmetric about $x=0$

integrand: odd in y
R symmetric about $y=0$

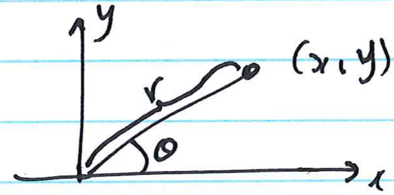
$$= 5 \underbrace{\iint_R dx\,dy}_{\text{area}(R)}$$

$$= 5 \cdot 4 \cdot \pi$$

Midterm 2.

Double integrals over polar regions §13.3

review $(x, y) \to (r, \theta)$



$x = r\cos\theta$
$y = r\sin\theta$

$r^2 = x^2 + y^2$
$\tan\theta = y/x$ ← atan2 on the computer
atan2(y, x) = arctan(y/x) except that it works for $\theta > \pi/2$

curves: $r = \cancel{f(\theta)}$
$= r(\theta)$

Ex What shape ~~is~~ in x, y is given by $r = 2\cos\theta$, $\theta \in [-\pi/2, \pi/2]$

$r \cdot r = 2r\cos\theta$ (mult LHS, RHS by r)
$r^2 = 2x$
$x^2 + y^2 = 2x$
$x^2 - 2x + 1 + y^2 = 1$
$(x-1)^2 + y^2 = 1$ circle centered at $(x, y) = (1, 0)$