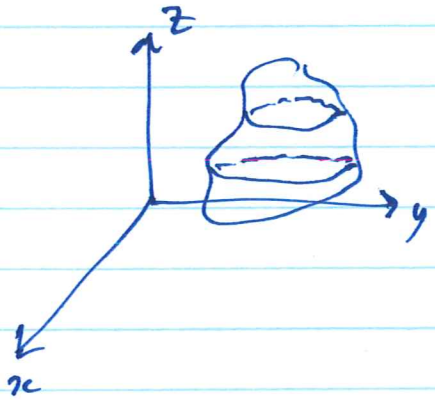
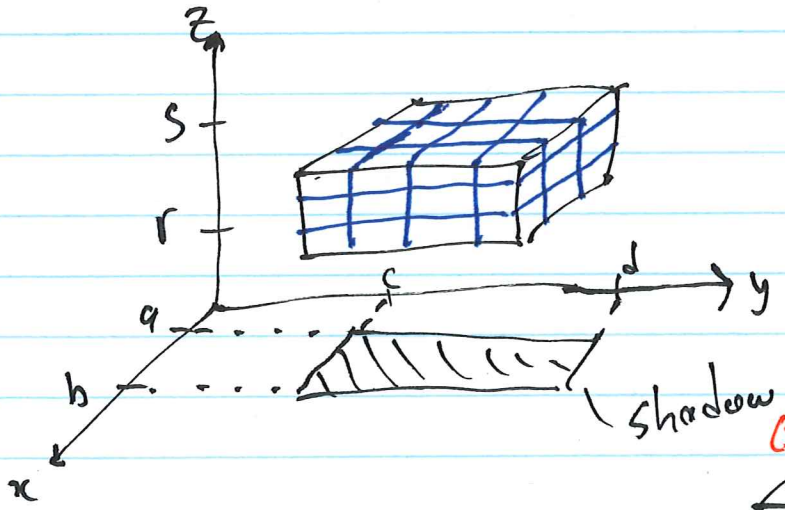
Errata last day: $SA = \iint_D \sqrt{1+f_x^2+f_y^2}\, dA$ is always positive.

Today: Triple Integrals $\iiint_E f(x,y,z)\, dV$ (dV: $dz\,dy\,dx$ (eg.))

f: fcn of 3 vars



Simplest region E = box $= [a,b] \times [c,d] \times [r,s]$

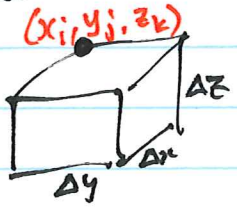


- Example for $f(x,y,z)$ is density

$\iiint_E f\, dV$ = mass

- Example for $f(x,y,z) \equiv 1$

$\iiint_E 1\, dV$ = volume



Typical subbox: B_{ijk}

Divide $[a,b]$ into ℓ subintervals length $\Delta x = \frac{b-a}{\ell}$

" $[c,d]$ " m " " $\Delta y = \frac{d-c}{m}$

" $[r,s]$ " n " " $\Delta z = \frac{s-r}{n}$

Define $\iiint_E f(x,y,z)\,dV = \lim_{l,m,n\to\infty} \sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_i, y_j, z_k)\underbrace{\Delta x \Delta y \Delta z}_{\Delta V}$

E = Box: $\iiint_E f\,dV = \int_r^s \int_c^d \int_a^b f(x,y,z)\,dx\,dy\,dz$

= 5 other iterated integrals

$dx\,dz\,dy$
$dy\,dx\,dz$
$dy\,dz\,dx$
$dz\,dx\,dy$
$dz\,dy\,dx$

Ex $\int_{-1}^{1}\int_{-1}^{1}\int_{0}^{2} \rho^{(x,y,z)}\,dz\,dy\,dx$

$\rho = x + 2y + z^3 + 3$

Android : mine test

IP: 128. 189. 248. 224.

diamond $\rho = 1$ $\#= 13$
obsidian $\rho = 3$ $\#= ?$
gold $\rho = 7$ $\#= ?$
$\rho = 9$ $\#= ?$
$\rho = 11$ $\#= ?$
$\rho = 13$ $\#= ?$

Mass = ?

$\Delta z = 1/3$

$\Delta x = \Delta y = 2/7$

Ex

$$\int_{-1}^{1}\int_{-1}^{1}\int_{0}^{2} \overbrace{x + 2y + z^2 + 3}^{P}\, dz\, dy\, dx$$

$$\int_{-1}^{1}\int_{-1}^{1}\left[xz + 2yz + \frac{1}{3}z^3 + 3z \right]_0^2 dy\, dx$$

$$\int_{-1}^{1}\int_{-1}^{1} 2x + 4y + \frac{8}{3} + 6 \; dy\, dx$$

$$\int_{-1}^{1} 2xy + 2y^2 + \frac{8}{3}y + 6y \Big|_{-1}^{1} dx$$

$$\int_{-1}^{1} \quad - \quad - \quad - \quad dx$$

$$= \cdots = 104/3 \approx 34.667$$

ρ Δx Δy Δz

$$\text{Mass} = 13 \cdot 1 \cdot 2/7 \cdot 2/7 \cdot 1/3$$

$$+ \boxed{?} \cdot 3 \cdot 2/7 \cdot 2/7 \cdot 1/3$$

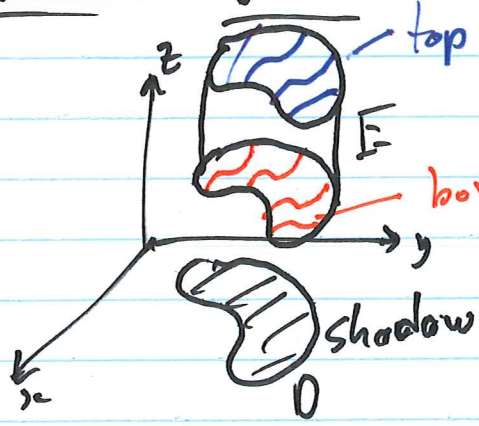
$$+ \boxed{?} \cdot 7 \cdot 2/7 \cdot 2/7 \cdot 1/3$$

$$+ \ldots$$

$$=? \quad \left(\text{hopefully close to } 104/3\right)$$

More general regions

Type 1



E is the 3D solid region between the surfaces $z = u_1(x,y)$ and $z = u_2(x,y)$ and above D, a region in x-y plane

$$\iiint_E f(x,y,z)\,dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\,dz \right] dA$$

- for each $x,y \in D$ ...
- compute this integral (for fixed x,y).
- eg. $dx\,dy$, or $dy\,dx$, $r\,dr\,d\theta$

Type 2: $E = \{(x,y,z) : (x,z) \in D,\ u_1(x,z) \le y \le u_2(x,z)\}$