Last day

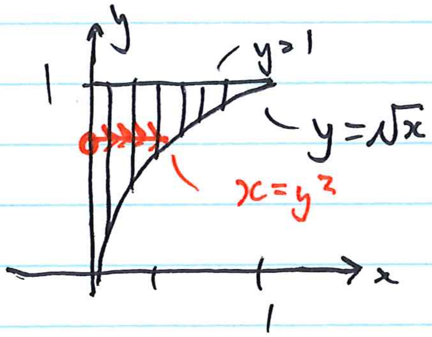
Ex $$I = \int_{x=0}^{1} \int_{y=\sqrt{x}}^{1} \boxed{\int_{z=0}^{1-y} f \, dz} \underbrace{dy\,dx}_{dA} \qquad (1)$$

Q: Write down all <u>five</u> other iterated integrals.

(We will do without visualizing E)

Strategies: ~~visualize~~ ① visualize in 3D
② Fubini
③ Swap the outer two
④ Swap the inner two

② $dA = dy\,dx \rightarrow dA = dx\,dy$ (ignoring the inner integral)



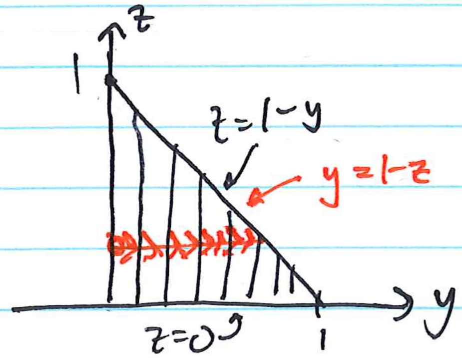
$$\iint_D (\quad)\, dA$$

$$\int_{y=0}^{1} \underbrace{\int_{x=0}^{y^2} \boxed{\int_{z=0}^{1-y} f \, dz} \, dx}_{\text{inner two integrals over a rectangular region (different rectangles for different } y)} dy \qquad (2)$$

③ Fubini Thm

$$\int_{y=0}^{1} \int_{z=0}^{1-y} \int_{x=0}^{y^2} f \, dx\, dz\, dy$$

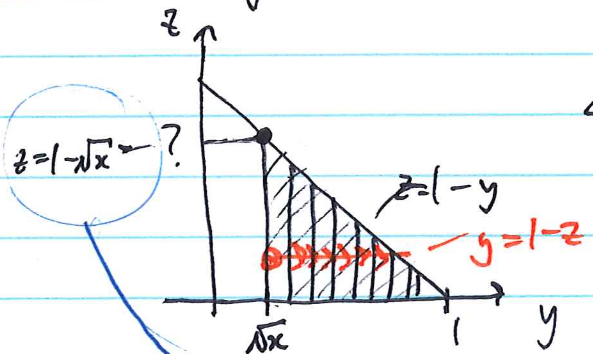
(4) $\iint_D (\quad)\, dA$ with $dA = dz\, dy$



swap outer two.

$$\int_{z=0}^{1} \int_{y=0}^{1-z} \left(\int_{x=0}^{y^2} f \, dx \right) dy \, dz$$

(5) want dy in inner position, go back to (1), swap inner two



← we have some fixed x.

$$\iint_D f \, dz\, dy$$

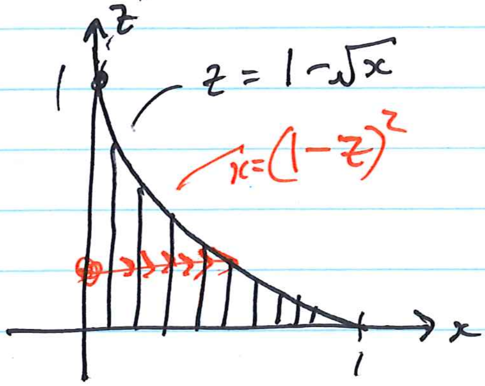
D depends on x

$$\int_{y=\sqrt{x}}^{1} \int_{z=0}^{1-y} f \, dz\, dy$$

$$\int_{z=0}^{1-\sqrt{x}} \int_{y=\sqrt{x}}^{1-z} f \, dy\, dz$$

$$\Rightarrow I = \int_{x=0}^{1} \int_{z=0}^{1-\sqrt{x}} \int_{y=\sqrt{x}}^{1-z} f \, dy\, dz\, dx \qquad (5)$$

6 ~~...~~ Swap $dz\,dx$ in 5 $\iint_D (\quad)\,dz\,dx$



$$\int_{z=0}^{1} \int_{x=0}^{(1-z)^2} \left(\int_{y=\sqrt{x}}^{1-z} f \, dy \right) dx\,dz$$

See another example APEX pg 801
Ex 4.7.5.

$\iiint$ in Cylindrical ~~[illegible]~~ Coors :

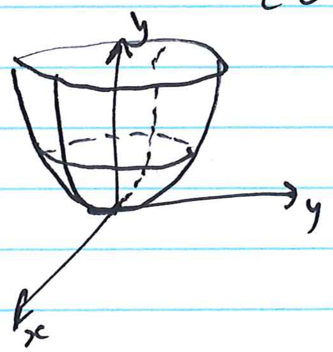
not in APEX
website has text
by Strang §14.4

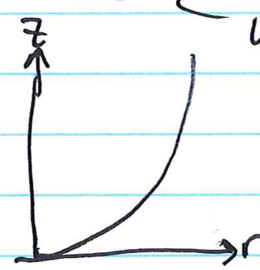
Defn: (x,y,z) with (x,y) in polar (r,θ)
(and z unchanged)

$$(x,y,z) \Downarrow (r,\theta,z)$$

Ex paraboloid $z = x^2+y^2$ in cylindrical coors: $z = r^2$

rotational symmetry,
no θ dependence.

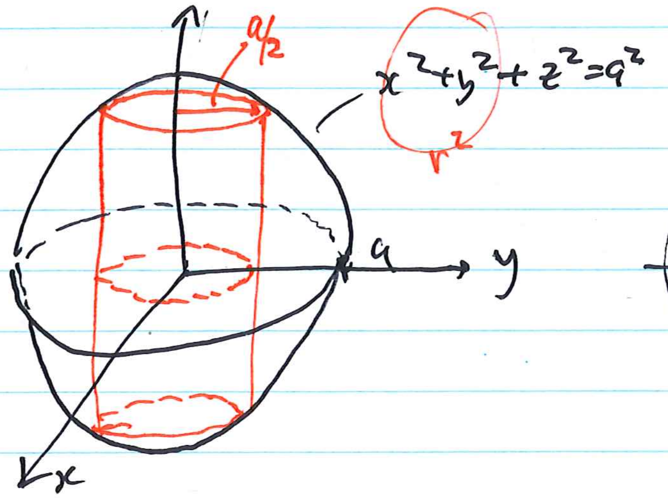


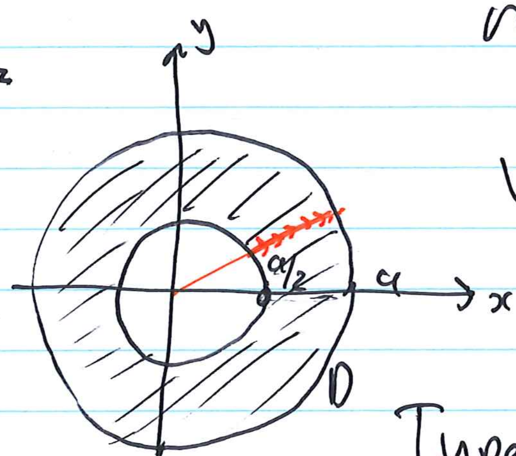


$\iiint_E$ in (r,θ,z) useful if f fcn of r z
x^2+y^2

and E is type 1 above a polar rectangle in x-y plane,

Ex sphere radius a, drill hole with diameter a. Find the ~~remaind~~ remaining volume.



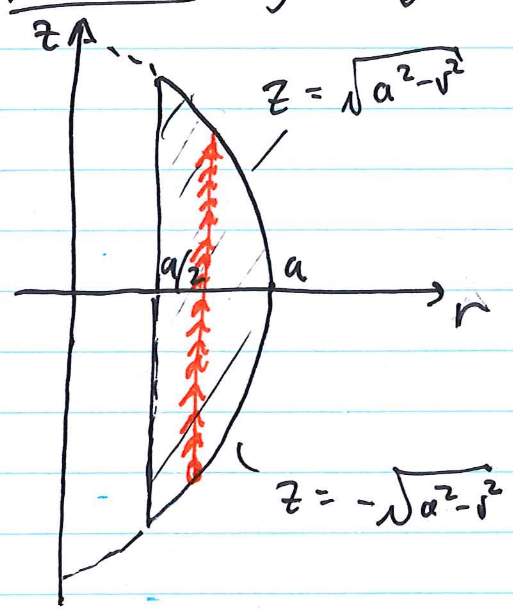


$$V=\iiint_E 1\,dV$$

Type 1?

$$\iint_D \int_{z=?\ (u_1(r,\theta))}^{?\ (z=u_2(r,\theta))} 1\,dz\,\underbrace{dA}_{r\,dr\,d\theta}$$

Rotationally symmetric



$$V=\int_{\theta=0}^{2\pi}\int_{r=a/2}^{a}\int_{z=-\sqrt{a^2-r^2}}^{\sqrt{a^2-r^2}} 1\,dz\,r\,dr\,d\theta$$

$$=2\pi\int_{r=a/2}^{a} 2\sqrt{a^2-r^2}\,r\,dr$$

sub $u=a^2-r^2$

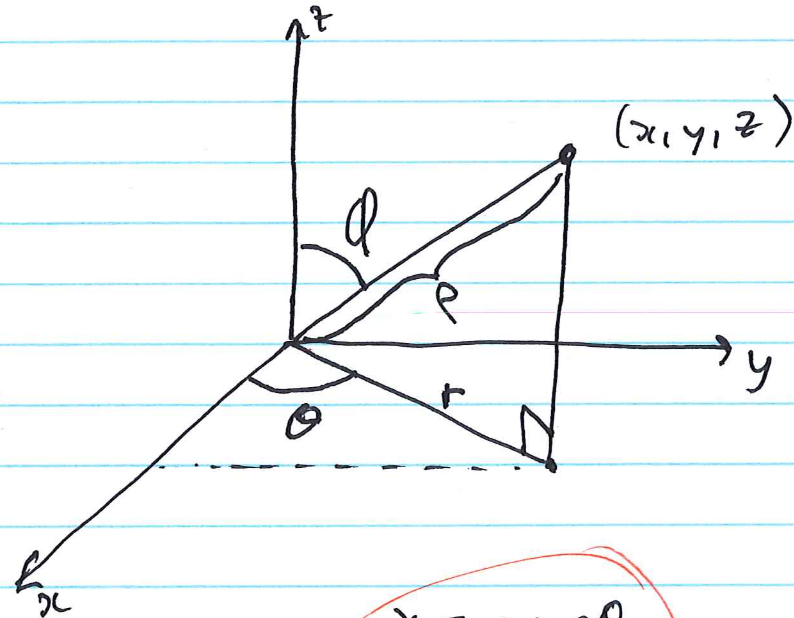
$du=-2r\,dr$

$= \ldots$

$$=\frac{\sqrt{3}}{2}\pi a^3$$

Spherical Coordinates (SSS)

↳ see § 14.4 Strang text.



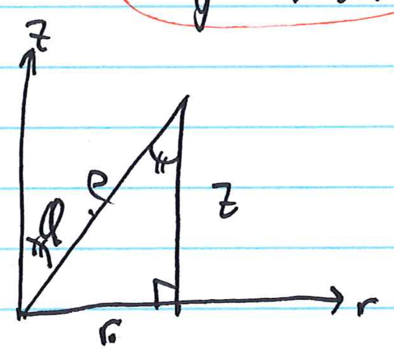
ρ : "rho" distance from origin

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

φ : "phi" angle from z-axis

θ : "theta" angle of rotation from x-axis

$$x = r\cos\theta$$
$$y = r\sin\theta$$



$$\boxed{r = \rho\sin\phi}$$
$$z = \rho\cos\phi$$

$$\boxed{(x, y, z) \longrightarrow (\rho\sin\phi\cos\theta,\ \rho\sin\phi\sin\theta,\ \rho\cos\phi)}$$

Caution: some sources swap φ, θ