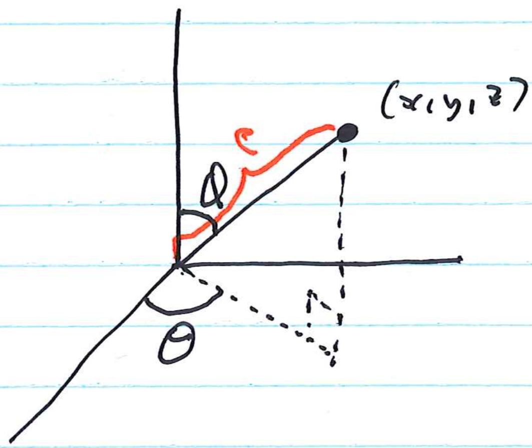
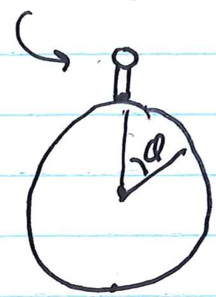
Spherical Coors

$$(x, y, z) \longrightarrow (\rho \sin\phi \cos\theta,\ \rho \sin\phi \sin\theta,\ \rho\cos\phi)$$



North Pole $\phi = 0$

$x = 0$
$y = 0$
$z = \rho$



SP

$\phi = ?\ \pi$

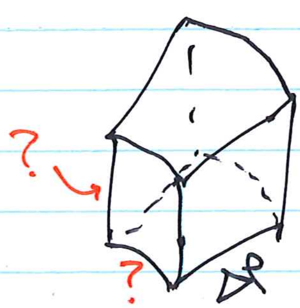
$x = 0$
$y = 0$
$z = -\rho$

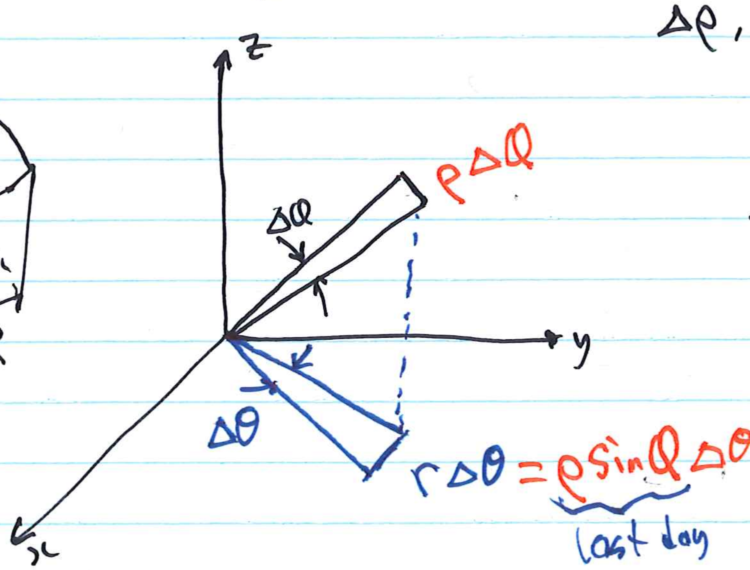
$$\left.\begin{array}{l} 0 \le \phi \le \pi \\ 0 \le \theta \le 2\pi \\ 0 \le \rho \le \infty \end{array}\right\} \text{all of } \mathbb{R}^3$$

Careful:
"Hollow Earth"
Rudy Rucker
don't count twice

$$\boxed{\text{We want } \iiint_E f\,dV}$$

Riemann : small intervals
$\Delta\rho,\ \Delta\phi,\ \Delta\theta$





$$\Delta V = (\rho\,\Delta\phi)\cdot(\Delta\rho)\cdot(\rho\sin\phi\,\Delta\theta)$$
$$= \rho^2 \sin\phi\,\Delta\rho\,\Delta\phi\,\Delta\theta$$

$$\iiint_E f\,dV = \int_?^? \int_?^? \int_?^? f\,\underbrace{\rho^2 \sin\phi}_{\text{like "r" in } r\,dr\,d\theta}\,d\rho\,d\phi\,d\theta$$

Ex

$$I = \iiint_B e^{(\underbrace{x^2+y^2+z^2}_{\rho^2})^{3/2}}\,dV$$ where B is a ball radius 1

$$= \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} \int_{\rho=0}^{1} e^{\rho^3}\,\underbrace{\rho^2 \sin\phi\,d\rho\,d\phi\,d\theta}_{dV}$$

$$= \int_0^{2\pi} d\theta \int_{\phi=0}^{\pi} \sin\phi\,d\phi \int_0^1 e^{\rho^3}\rho^2\,d\rho$$

$u = \rho^3$

$$= 2\pi \cdot 2 \cdot \frac{1}{3}(e-1)$$

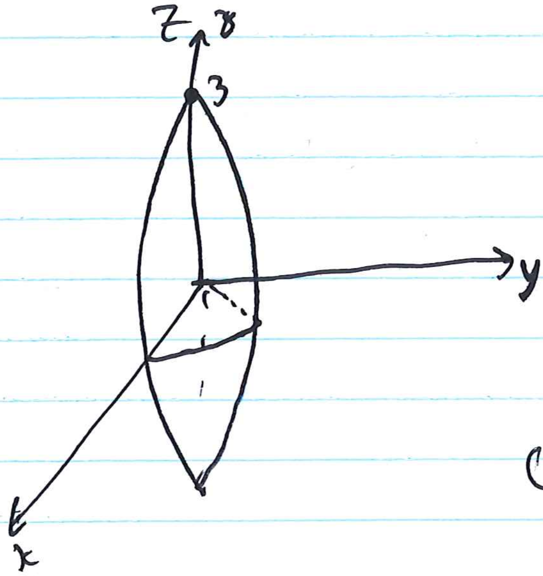
$$= \frac{4\pi}{3}(e-1)$$

Constrast:

$$I = \int_{x=-1}^{1} \int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{z=-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} e^{(\quad)^{3/2}}\,dz\,dy\,dx$$

Ex Find the volume of region E where

$$E = \{ (\rho, \phi, \theta) : 0 \le \rho \le 3,\ 0 \le \phi \le \pi,\ 0 \le \theta \le \tfrac{2\pi}{20} \}$$



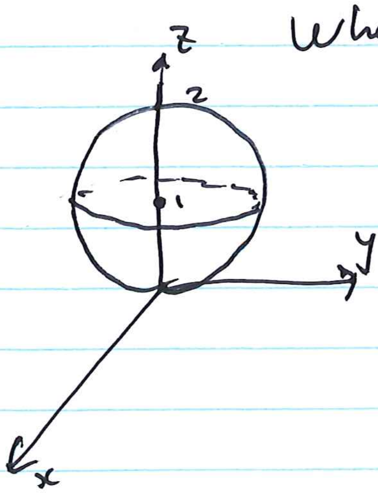
$$V = \int_0^{\frac{2\pi}{20}} \int_0^{\pi} \int_0^{3} 1\ \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

↳ part of a Terry's Chocolate Orange [tm]

↑ send free samples!

Ex Find the limits for $I = \iiint_E f \, dV$

where $E = \{ x^2 + y^2 + (z-1)^2 \le 1 \}$

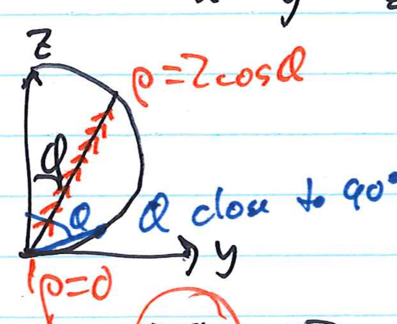


$$x^2 + y^2 + z^2 - 2z + \cancel{1} \le \cancel{1}$$

$$x^2 + y^2 + z^2 \le 2z$$

$$\rho^2 \le 2\rho\cos\phi$$

$$\rho \le 2\cos\phi$$

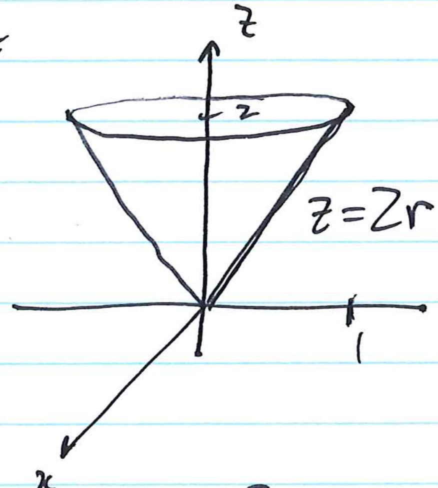


$$I = \int_0^{2\pi} \int_0^{\pi/2} \int_0^{2\cos\phi} f \, \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

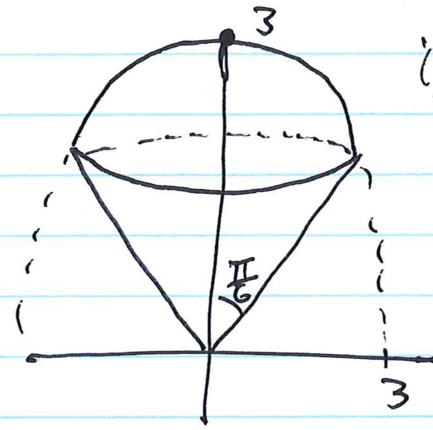
not π!

Spherical or Cylindrical? if I see x^2+y^2 and rotational symmetry, its probably either spherical or cylindrical but which?

Ex **cone**



icecream cone



Cylindrical?

$$\cancel{0\le} \; 2r \le z \le 2$$
$$0 \le r \le 1$$
$$0 \le \theta \le 2\pi$$

Spherical?

$$0 \le \phi \le \pi/6$$
$$0 \le \rho \le 3$$
$$0 \le \theta \le 2\pi$$

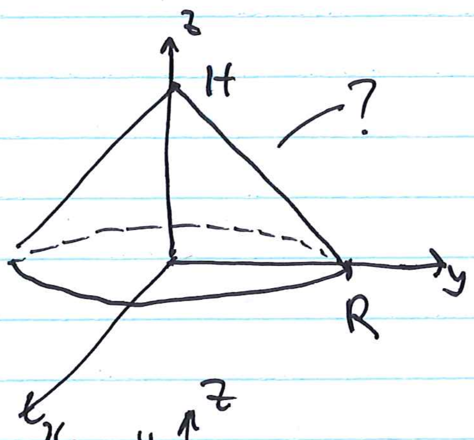
But also depends on the integrand!

Ex Stewart § 15.8 #31

How much work to raise Mt Fuji from sea level?
↳ Cone!

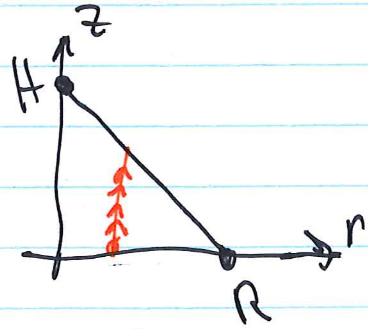
radius $R = 19000\,m$
height $H = 4000\,m$
density $\rho = 3000\,kg/m^3$

Work to lift volume ΔV to height z is $\underbrace{F}_{\text{force}}\underbrace{d}_{\text{distance}} = \underbrace{\rho \Delta V}_{\text{mass}}\ \underbrace{g}_{9.8\,m/s^2}\ z$ $\quad \underbrace{[J]}_{kg\cdot\frac{m}{s^2}\cdot m}$



$$W = \iiint_{Mt\ Fuji} \rho g z \, dV$$

$$= \rho g \int_0^{2\pi}\int_0^R \underbrace{\int_0^{\frac{H}{R}(R-r)} z\,dz}_{\frac{1}{2}\left(\frac{H}{R}(R-r)\right)^2}\ \underbrace{r\,dr\,d\theta}_{dV}$$



$$z = \frac{H}{R}(R-r)$$

$$= \ldots = \frac{\pi}{12}\rho g H^2 R^2$$

$$= \frac{\pi}{12} \cdot 3\times10^3 \cdot 16\times10^6 \cdot 19^2\times10^6 \cdot 9.8$$

$$= 4.45\times10^{19}\ [J]$$