The University of British Columbia

# MATH 253

# Midterm 2

14 November 2012

Time: 50 minutes

FIRST NAME: _____ LAST NAME : _____

STUDENT #: _____

This Examination paper consists of 7 pages (including this one). Make sure you have all 7.

INSTRUCTIONS:

No memory aids allowed. No calculators allowed. No communication devices allowed.
**PLEASE CIRCLE YOUR INSTRUCTOR'S NAME BELOW**

MARKING:

| | |
|---|---|
| **Q1** | /10 |
| **Q2** | /12 |
| **Q3** | /12 |
| **Q4** | /16 |
| TOTAL | /50 |

Names of Instructors: Jim Bryan, Dale Peterson, Ian Hewitt, Yariv Dror-Mizrahi, Ed Richmond

**Q1** [10 marks]

Find the volume of the region in 3-space which is below the surface $z = 1 + 3x^2y^2$ and lies above the region in the xy-plane enclosed by the curves $x = y^2$ and $x = 1$.

**Q2** [12 marks]

Suppose $T(x, y, z) = xy^2 - x + x^2z + yz^2$ gives the temperature at the point $(x, y, z)$ in space.

($a$) Find an equation of the plane tangent at $(1, 2, 1)$ to the level surface of $T$ passing through that point.[4pts]

($b$) At time $t = 0$, a fly passes through $(1, 2, 1)$ moving toward the point $(4, 2, 5)$ at speed of 1 unit/sec. Calculate $\dfrac{dT}{dt}$ at $t = 0$ for the fly.[4pts]

(*c*) A worm crawling on the plane $2x - y + 2z = 2$ passes through the point $(1, 2, 1)$. The worm wishes to keep his temperature constant while increasing $z$. In which direction should the worm move? Express your answer as a unit vector. [4pts]

**Q3** [12 marks]

Consider the following iterated integral

$$\int_0^2 \int_{2-\sqrt{4-x^2}}^{x} f(x,y)\,dydx.$$

(a) Sketch the region of integration. Be sure to label your axes and clearly mark x and y values on the axes. Give the coordinates of any intersection points. [4pt]

(b) Change the order of integration to $dx\,dy$. [4pt]

(c) Convert the integral to polar coordinates.[4pt]

**Q4** [16 marks]

Consider the function $f(x, y) = (4y + 7)e^{-x^2-y^2}$ on the domain $x^2 + y^2 \leq 1$.

($a$) Find all critical points of $f$ which are inside the domain [4 pts]

($b$) Classify each of the critical points on the inside of the domain as a "local maximum", "local minimum", "saddle points", or "discriminant is zero". [4 pts]

(*c*) Use the method of Lagrange multipliers to find the maximum and minimum values of $f$ on the boundary of the domain. [6pt]

(*d*) Find the absolute maximum and minimum values of $f$ on its whole domain. You may use the fact that $e^{15/16} > 11/8$. [2 pts]