3 marks 6. (a) The average of a function $f(x, y)$ on a planar region D is given by the formula $f_{\text{ave}} = \frac{1}{\text{Area}(D)} \int\int_D f dA$. Find the average distance between a point lying inside a circle of radius 1 and the centre of the circle.

Answer:

3 marks

(b) Consider the planar region $D = \{(x, y) \mid x \geq 0, (x-1)^2 + y^2 \geq 1, x^2 + y^2 \leq 4\}$. Sketch the region D and describe it in polar coordinates.

2 marks

(c) Using any method, compute the area of D.

Answer:

5 marks 7. (a) Find the area of the portion of the cone $z^2 = x^2 + y^2$ lying between the planes $z = 2$ and $z = 3$.

Answer:

5 marks

(b) Find the centre of mass of a triangular lamina with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$ and density $\rho(x, y) = x + y$.

Answer: