Topic 02: Root Finding

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Rooting Finding

Iterative techniques for solving $f(x) = 0$ for x.

Bisection: start with an interval $[a, b]$ bracketing the root. Evaluate the midpoint. Replace one end, maintaining a root bracket. Linear convergence. Slow but **robust**.

Newton's Method: $x_{k+1} = x_k - f(x_k)/f'(x_k)$. Faster, quadratic convergence (number of correct decimals places doubles each iteration).

Downsides of Newton's Method: need derivative info, and additional smoothness. Convergence usually not guaranteed unless "sufficiently close": not **robust**.

Systems

$f(x) = 0$, but now $f : \mathbb{R}^n \to \mathbb{R}^n$.

This is a system of nonlinear equations. Denote a solution as $\alpha \in \mathbb{R}^n$.

Derivation: Taylor expansion about x

$$0 = f(\alpha) = f(x) + J(x)(\alpha - x) + \text{h.o.t.}$$

where $J(x)$ is the Jacobian matrix…

Pretend h.o.t. are 0, so instead of α we find x_{k+1}:

$$0 = f(x_k) + J(x_k)(x_{k+1} - x_k)$$

In principle, can rearrange to solve for x_k but better to solve

$$J_k \delta = -f(x_k)$$

That is, solve "$Ax = b$". Then update:

$$x_{k+1} := x_k + \delta$$

Optimization

A *huge* area, but to get started, consider calculus: finding min/max points by setting the derivative equal to zero.