Name (print):

Student number:

Section *(Please circle one)*: 001 002 003 004

University of British Columbia

MATH 110 (Section 001): FEBRUARY EXAM

Date: *February 12, 2013*

Number of pages: *9 (including cover page)*

Exam type: *Closed book*

Aids: *No calculators or other electronic aids*

Question	Points	Score
1	4	
2	4	
3	4	
4	5	
5	6	
6	5	
7	7	
Total:	35	

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

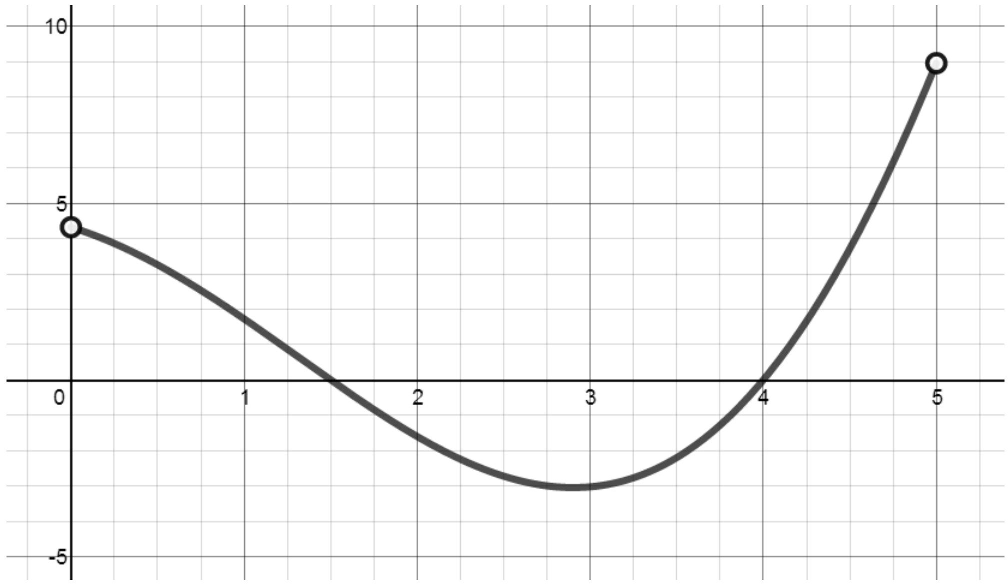
- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

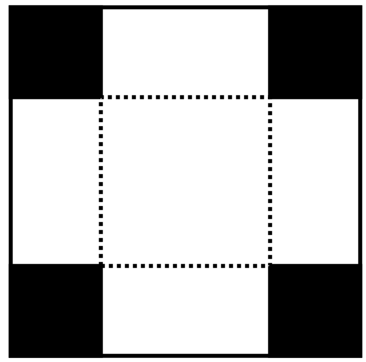
Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

4 marks 1. Find all local maximum and minimum values for the function $f(x) = \dfrac{4x}{x^2+1}$.

$\boxed{\text{4 marks}}$ 2. Below is a sketch of the graph of $f'(x)$ (note that it is the graph of the *derivative* of $f(x)$). Clearly label every x value at which $f(x)$ has a local maximum, local minimum or inflection point. Below the picture, justify each of your labels in one or two sentences.



4 marks 3. Suppose you are asked to construct a box out of a sheet of paper measuring 8.5 inches by 8.5 inches by cutting off four corners of the sheet and folding along the dotted lines, as in the diagram below.



If the four corner squares have a side length of x inches, the resulting box has a volume of

$$V(x) = x(8.5 - 2x)^2$$

cubic inches. The derivative may be taken to be

$$V'(x) = (8.5 - 2x)(8.5 - 6x).$$

Explain carefully how you may conclude that in order to construct a box of maximal volume, you should cut off four corner squares of side length $x = \frac{8.5}{6}$ inches.

4. Let $f(x) = \sin(2x) + x$.

$\boxed{\text{4 marks}}$ (a) Determine where $f(x)$ is concave up on the interval $[0, \pi]$.

$\boxed{\text{1 mark}}$ (b) Describe where $f(x)$ is concave up on the interval $(-\infty, \infty)$.

5. Let $f(x) = e^x + 2x^3$.

2 marks (a) Use the Intermediate Value Theorem to show that $f(x)$ has at least one root.

2 marks (b) Explain why $f(x)$ is increasing on the interval $(-\infty, \infty)$.

2 marks (c) Use the Mean Value Theorem or Rolle's Theorem, along with your answer from part (b), to show that $f(x)$ cannot have two roots. (Hint: If there are two roots, what might go wrong?)

6. Complete the answers to the following true or false questions by providing a counterexample or explanation as requested. (In the case of providing a counterexample, a picture suffices.)

1 mark (a) The following statement is false. Provide a counterexample demonstrating that it is false.

If a function is defined everywhere, increasing on $(-\infty, 0)$ *and decreasing on* $(0, \infty)$*, then the function has a global maximum at* 0.

1 mark (b) The following statement is false. Provide a counterexample demonstrating that it is false.

If $f(0) = 0$ *and* $f(1) = 1$*, then there is a number* c *in the interval* $(0, 1)$ *such that* $f'(c) = 1$.

1 mark (c) The following statement is false. Provide a counterexample demonstrating that it is false.

A critical point can never be an inflection point.

2 marks (d) The following statement is true. Explain why.

The sum of two decreasing functions is a decreasing function.

7. For each of the following parts, sketch a continuous function with domain $(0, 5)$ satisfying the conditions:

$\boxed{\text{2 marks}}$ (a) A function $f(x)$ with a local maximum, a local minimum but no global maximum and no global minimum.

$\boxed{\text{2 marks}}$ (b) A function $g(x)$ with $g'(x) > 0$ for all $x \neq 2$ and $g'(2)$ undefined.

$\boxed{\text{3 marks}}$ (c) A function $h(x)$ with $h''(x) < 0$ for all $x \neq 3$. Further, $h'(3)$ does not exist and $h'(x) < 0$ for $x < 3$.

This page may be used for rough work. It will not be marked.