Name (print):

Student number:

Section *(Please circle one)*:   001   002   003   004

University of British Columbia
MATH 110: MIDTERM TEST 1

Date: *October 16, 2013*
Time: *6:00 p.m. to 7:30 p.m.*
Number of pages: *9 (including cover page)*
Exam type: *Closed book*
Aids: *No calculators or other electronic aids*

For examiners' use only		
Question	Mark	Possible marks
1		7
2		4
3		5
4		6
5		4
6		8
7		6
Total		40

1. Let P be the point $(1, 2)$ and C be the circle described by the equation

$$(x - 5)^2 + (y - 5)^2 = 25.$$

All three parts of this question refer to the point P and the circle C.

2 marks   (a) Does the point P lie on the circle C? Justify your answer.

2 marks   (b) Find the equation of the line that passes through the point P and the centre of the circle C.

3 marks   (c) The line whose equation you found in part (b) intersects the circle C at exactly two points. Find the coordinates of those points. *Hint: Draw a picture.*

2. Both parts of this question refer to the functions

$$f(x) = \sqrt{2x+1} \ \text{ and } \ g(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases} .$$

2 marks   (a) On its entire domain, the function $(f \circ g)(x)$ has a *range*, or output, of exactly two numbers. Find those numbers, and explain your answers.

2 marks   (b) On its entire domain, the function $(g \circ f)(x)$ also has a range of exactly two numbers. Find those numbers, and explain your answers.

3. Suppose the position of a particle with respect to time is described by the function

$$p(t) = 2t^2 - 3t + 4,$$

where position is measured in metres and time is measured in seconds. All three parts of this question refer to this particle.

2 marks   (a) Calculate the average velocity of the particle between $t = 2$ seconds and $t = 3$ seconds.

2 marks   (b) Calculate the average velocity of the particle between $t = 2$ seconds and $t = 2 + h$ seconds, where h is a small positive number.

1 mark   (c) Estimate the instantaneous velocity of the particle at $t = 2$ seconds, and explain your answer.

4. Evaluate each of the following limits or show that the limit does not exist.

2 marks   (a) $\lim\limits_{x \to 4} 2x^2 - 5x - 12$

2 marks   (b) $\lim\limits_{x \to 4} \dfrac{2x^2 - 5x - 12}{x - 4}$

2 marks   (c) $\lim\limits_{x \to 4} \dfrac{2x^2 - 5x - 12}{|x - 4|}$

4 marks 5. Show that the function $f(x) = x^3 - 3x^2 + 1$ has at least three roots in the interval $[-1, 3]$.

[**1 bonus mark**] Explain why $f(x)$ cannot have more than three roots in the interval $[-1, 3]$.

6. Determine whether each of the following statements is true or false. If it is true, provide justification. If it is false, provide a counterexample.

| 2 marks | (a) The graph of $f(x) = \frac{1}{2}\left((x+3)^2 - 2\right)$ crosses the x-axis.

| 2 marks | (b) If $\lim\limits_{x \to 1} f(x) = 4$, then $f(1) = 4$.

| 2 marks | (c) If $f\left(\frac{1}{10}\right) = 2$, $f\left(\frac{1}{100}\right) = 2$, $f\left(\frac{1}{1000}\right) = 2$, and in fact $f\left(\frac{1}{10^n}\right) = 2$ for every integer n, then $f(0) = 2$.

| 2 marks | (d) $f(x) = \begin{cases} \frac{3}{x+2} & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}$ is continuous at all real numbers.

6 marks 7. On the grid below, sketch a function $f(x)$ satisfying all of the following 6 properties:

- The domain is $[-10, -6) \cup (-6, 10]$.
- For every value x in the domain, $-4 \leq f(x) \leq 6$.
- $\lim\limits_{x \to 6} f(x) = 4$
- $f(6) = 0$
- $\lim\limits_{x \to -2^-} f(x) = 2$
- $\lim\limits_{x \to -2^+} f(x) = -2$

You are not required to come up with an algebraic equation for the function.

This page may be used for rough work. It will not be marked.