ASSIGNMENT 1·10 (Section 002) Due: Friday, November 8

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions (in full sentences) with complete arguments and justifications in a linear, coherent manner. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. Your work must be your own and must be self-contained. Assignments must be stapled, with your name and student number at the top of each page. The assignment is due at the beginning of class on the due date.

1. Let

$$f(x) = \frac{\sin x \cos x}{\sin x + \cos x}.$$

   Find the equation of the line tangent to f through the point $(\frac{\pi}{2}, 0)$.

2. Recall that given a function, say f, we can think about its derivative, which we denote $f'(x)$ (providing the function is differentiable). The derivative $f'(x)$ is itself a function and so conceivably we could take its derivative (providing it is also differentiable). We call this new function the second derivative of f and denote it by $f''(x)$. In other words $f''(x) = (f'(x))'$. For each part in this problem make sure you justify your answer.

   (a) Find a function $f(x)$ such that

$$f''(x) = -f(x). \tag{1}$$

   (b) Find another function satisfying (1).

   (c) Find infinitely many functions satisfying (1).

   (d) Find a function satisfying (1) with the property that $f(0) = 0$ and $f(\pi/2) = 2$.

3. Let

$$f(x) = \begin{cases} (x^2+1)\sin x & x < 0 \\ g(x) & 0 \le x \le \pi \\ \cos^2 x & x > \pi \end{cases}$$

   Find $g(x)$ so that f is differentiable everywhere. Justify your answer.