

ASSIGNMENT 1.11 (Section 2) Due: Friday, November 15

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions (in full sentences) with complete arguments and justifications in a linear, coherent manner. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. Your work must be your own and must be self-contained. Assignments must be stapled, with your name and student number at the top of each page. The assignment is due at the beginning of class on the due date.

1. Let $f(x) = x^n e^x$. Find all the values of n for which $f(x)$ satisfies the differential equation

$$f'''(x) - 3f''(x) + 3f'(x) - f(x) = 0.$$

2. In the lectures this past week we have seen that

$$(c^x)' = c^x \cdot \lim_{h \rightarrow 0} \frac{c^h - 1}{h}.$$

In this problem we will explore the mysterious quantity

$$\lim_{h \rightarrow 0} \frac{c^h - 1}{h}.$$

For $c > 0$ let $f(c)$ be defined as

$$f(c) = \lim_{h \rightarrow 0} \frac{c^h - 1}{h}.$$

- (a) Find $f(1)$.
- (b) Find the value of $f(2)$ accurate to three decimal places. To do so, use your calculator to find the values of $\frac{2^h - 1}{h}$ for increasingly small values of h .
- (c) Find the value of $f(4)$ accurate to three decimal places using a similar computation.
- (d) Show that for $c > 0$ and $d > 0$, the equality $f(c \cdot d) = f(c) + f(d)$ holds.

Hint: you might find the following identity useful for this purpose: $(cd)^h - 1 = (c^h - 1)d^h + (d^h - 1)$.

- (e) We have recently talked about another function $g(c)$ defined for $c > 0$ and satisfying $g(cd) = g(c) + g(d)$ for any $c > 0$ and $d > 0$. What is this function?
- (f) Can you guess an algebraic expression for $f(c)$ (i.e. an expression that does not involve limits)? Check that this expression agrees with the values of $f(1)$, $f(2)$ and $f(4)$ which you found before.
3. This question will be based on data in the Wikipedia article “List of cities in the European Union with more than 100,000 inhabitants” located at

http://en.wikipedia.org/wiki/List_of_cities_in_the_European_Union_with_more_than_100,000_inhabitants

- (a) Let $N(P)$ be the number of cities in the list with population at least P , where P is measured in hundred thousands of people. For instance $N(1)$ is the number of cities with population at least 100,000.

For $P = 1$, $P = 2$, $P = 4$, $P = 8$, $P = 16$, $P = 32$ and $P = 64$ find N , $\ln P$ and $\ln N$. Organize your answers in the following table.

P	N	$\ln P$	$\ln N$
1			
2			
4			
8			
16			
32			
64			

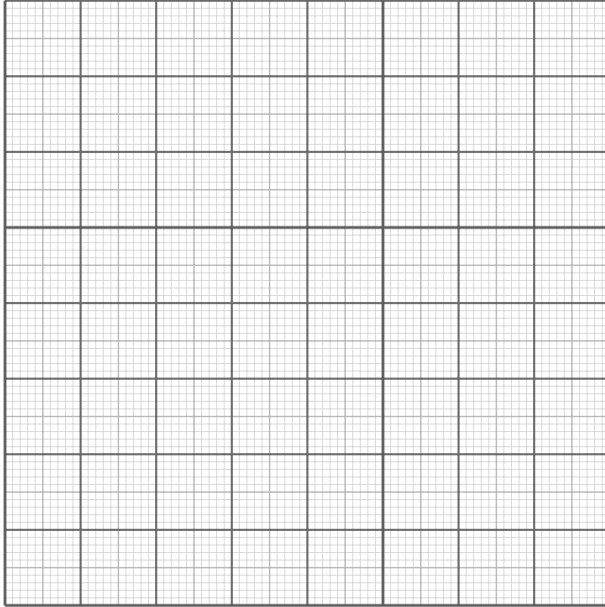
(b) A hypothesis has been proposed that N and P might be related by a formula of the form

$$N = C \cdot P^m$$

for some C and m .

Let $y = \ln N$ and $x = \ln P$. Express y in terms of x assuming that $N = C \cdot P^m$.

(c) Using data from part (a), plot 7 points on the graph of y as a function of x .



(d) Using the graph from previous part, estimate the values of m and C .