

ASSIGNMENT 1· 12 (Section 002) Due: Friday, November 22

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions (in full sentences) with complete arguments and justifications in a linear, coherent manner. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. Your work must be your own and must be self-contained. Assignments must be stapled, with your name and student number at the top of each page. The assignment is due at the beginning of class on the due date.

1. Let $g(x) = (f(-\cos(x)))^3$. Assume the following values for $f(x)$ and $f'(x)$ given in the following table.

x	$f(x)$	$f'(x)$
$-\frac{\sqrt{2}}{2}$	0	2
$-\frac{1}{2}$	-7	-9
$-\frac{\sqrt{3}}{2}$	2	$-\sqrt{5}$

Find the equation of the tangent line to $g(x)$ at $x = \frac{\pi}{6}$.

2. It's my birthday and I want an inflatable bouncy castle at my party. The volume of the bouncy castle after inflating it for t minutes is given by the function:

$$V(t) = \frac{e^{\frac{t}{10}}}{t^2 + 1}$$

The height of the bouncy castle in inches is given by the function:

$$h(V) = \sqrt[3]{V + 2}$$

The party is about to start and I'm worried about how long the structure is going to take to fully inflate. Calculate the instantaneous rate of change of the height with respect to time at $t = 30$ minutes.

3. The following functions play an important role in the study of differential equations. In particular, they are used to study heat transfer over metal surfaces.

(a) If n is a positive integer, prove that $\frac{d}{dx}(\sin^n(x) \cos(nx)) = n \sin^{n-1}(x) \cos((n+1)x)$.

(b) Find a similar formula for $\frac{d}{dx}(\cos^n(x) \cos(nx))$.

(c) BONUS: Find $\frac{d}{dn}(\sin^n(x) \cos(nx))$. Yes, *differentiate the function with respect to n* .

Hint: For the above problems you may find it useful to recall the following trigonometric identities:

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\sin(x + y) = \sin x \cos y + \sin y \cos x$$

4. **Not for marks.** We will be doing review during the last week of class. It would be great if you could submit (on a separate sheet of paper; disjoint from your assignment) problems from the course that you would like me to cover in review. These problems can come from (or be similar to) past webworks, written assignments, practice problems in the text, examples done in class, the midterm or any other source. You could also ask for me to cover a particular kind of problem or review a certain concept.