Instructions:

- There are two parts to this assignment. The first part is on WeBWorK.
- The second part consists of questions on this page.
- This assignment **does not** have to be submitted.
- Solutions will be posted at the end of the week.
- You are encouraged to attempt these problems as this content will be covered in the exam without a formal assignment for it.

Questions:

1. **The Lemniscate of Bernoulli:** Consider the curve given by

$$(x^2 + y^2)^2 = 2a^2(x^2 - y^2).$$

   (a) Find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ using any technique covered in class.

   (b) Fix $a = 2.5$. Find the equation of the tangent line passing through the point $(3, 1)$.

   (c) Plot a graph of the curve and the tangent line from the previous part (possibly with the aid of a computer program).

   (d) Using the graph, describe what happens if we try to find the tangent line a the origin. How is that reflected in the derivative we've found in the previous part?

2. **The pitfalls of implicit differentiation:** Consider the following equation:

$$\frac{4(x-y)}{x+y} = \frac{x}{y} + 1$$

   (a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ using implicit differentiation. *Hint: Fractions are ugly since they imply quotient rule. Simplify first.*

   (b) Show that there are no points on that curve. That is, there are no possible pair of points $P = (x, y)$ that satisfy the equation.

   Note: This highlights an important aspect of implicit differentiation. Without a point on a curve, the derivative we have found is meaningless.

3. **The rise of Android:** In 2010, A popular internet search company revealed 100 million activations for Android superphones. In 2012, the same study revealed 300 million activations for Android superphones. Denote $N(t)$ be the number of activations of Android superphones (in millions) since 2008.

   (a) Justify why it makes sense to use an exponential model to measure $N(t)$?

   (b) Use an exponential model to find the number of activations in 2008 (when the first android superphone was released).

   (c) What will be the projected number of activations for 2015?

   (d) According to this model, at what point will the number of activations exceed the world population?

4. **The power of Logs:** Find the equation of the tangent line at $x = 1$ of $g(x)$ defined by:

$$g(x) = \frac{x^7 e^{2x}}{3^{\ln(x)}(x^2 + 5x + 6)^4}$$