ASSIGNMENT $2 \cdot 11$

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete arguments and justifications. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. They must be stapled, with your name and student number at the top of each page.

1. *In this problem we will see that not all related rates problems involve time.* You own a factory that is making a length of wire out of two metals: copper and iron. You have access to mines containing both iron and copper. The amount A you are able to produce is determined by the equation

$$A = 10000\sqrt{xy}$$

where x is the number of tonnes of copper mined and y is the number of tonnes of iron mined. However, you only have a limited number of mining resources. The amount of copper and iron you can mine is determined by the following relationship

$$x^2 + 4xy + 15y^2 = 144.$$

If we mine more copper then we are forced to mine less iron. In this way we can think of y as a function of x. The quantity dA/dx represents the change in the amount of wire we can produce as we change the amount of copper.

   (a) Find dA/dx as a function of x and y.
   (b) Find the value of dA/dx when the amount of copper mined is equal to the amount of iron mined. Is it positive or negative? Should you mine more or less copper than this amount?

2. Out in the wild west a man has been injured in a duel. He steadies himself against a light post that is three times as tall as he is. He summons his strength and begins to walk in a straight line away from the light post. He accelerates initially but soon slows. After making it 6 meters he stops to catch his breath. His speed v (measured in meters per second) throughout his journey is described by

$$v = -\frac{s^2}{4} + \frac{3s}{2}$$

where s is his distance from the light post in meters.

   (a) Find the rate of change of the length of his shadow when he has traveled 4 meters from the light post.
   (b) Show that his shadow was changing the fastest when he reached his top speed.

3. You stand 5 meters directly underneath a pulley. You hold in your hand a rope that loops up through the pulley and is connected to a heavy box of calculus exams which sits on the ground exactly 12 meters behind you. The rope is stretched tight at all times. You rush forward at 2 meters per second pulling the rope with you and the box slides along the ground without lifting up. How fast is the box moving after 2.5 seconds have elapsed? Include any assumptions you make to solve the problem.