There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete arguments and justifications. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. They must be stapled, with your name and student number at the top of each page.

Questions:

1. The *yawn factor* is a measure of the ability for an instructor to put their students to sleep during a lecture. The relationship between the yawn factor y of an instructor and the number of students who attends lectures s is given by:

$$\frac{1920}{y \cdot s} = \sqrt{y + s}$$

   (a) Validate that a yawn factor of 4 will result in an attendance of 60 students.

   (b) Use a linear approximation to estimate the attendance if the instructor works really hard to decrease their yawn factor to 3.

2. Consider the function:

$$g(x) = e^x \sin(x)$$

   (a) Find the linear approximation $L(x)$ around $a = 0$ and use it to approximate $g(1)$.

   (b) Find the second order approximation $T_2(x)$ around $a = 0$ and use it to approximate $g(1)$. That is let $T_2(x) = L(x) + bx^2$ and choose b so that $g''(0) = T_2''(0)$.

   (c) Find the third order approximation $T_3(x)$ around $a = 0$ and use it to approximate $g(1)$. That is let $T_3(x) = T_2(x) + cx^3$ and choose c so that $g'''(0) = T_3'''(0)$.

   (d) Compare these values to the calculator given value of $g(1)$.

3. For this question we are interested in the following function:

$$f(x) = \sin(x) - x^2$$

   (a) Show that the function has at least one critical point.

   (b) Use Newton's Method to find a critical point (using a calculator for the computations) accurate to 6 digits and justify how you know your answer is accurate to 6 digits.
   Note: Show your work for the first three iterations. After that, just provide the result of each iteration.