ASSIGNMENT 2·2: Section 002

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete arguments and justifications. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. They must be stapled, with your name and student number at the top of each page.

1. For each of the following, construct a function that satisfies the given condition. That is, sketch a graph and provide the corresponding algebraic expression.

    (a) A continuous function on an open interval with a global max and no global min.

    (b) A discontinuous function on an open interval with a global max and global min.

    (c) A function on a close interval where the global max and global min occurs at points where the derivative is not defined.

    (d) A continuous function with both types of critical points (zero derivative and a derivative that does not exist).

    (e) A function with exactly two critical points, but no global max and no global min.

2. Let $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$.

    (a) Find all the global maxima of the function $f(x)$ on the interval $\left[\frac{-2}{\pi}, \frac{2}{\pi}\right]$. Justify your answer.

    (b) [Bonus] Is $f(x)$ continuous? Why or why not?

3. In the 1998 paper "Comparing Gompertz and Richards Functions to Estimate Freezing Injury in *Rhododendron* Using Electrolyte Leakage", published in the *Journal of the American Society for Horticultural Science*, the authors propose a function which models the effect of temperature on the "injury level" of rhododendron leaves. The function is
$$f(T) = 100e^{-be^{kT}},$$
where b and k are positive constants and T is temperature (in degrees Celcius).

    (a) The function $f'(T)$ has exactly one critical point. Find that point.

    (b) What might that critical point represent? Propose an explanation in a few sentences. Your answer should refer both to the function $f'(T)$ and the physical phenomenon being described. You may use a picture if you wish.