ASSIGNMENT 2·3: Section 002

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete arguments and justifications. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. They must be stapled, with your name and student number at the top of each page.

1. (a) Suppose two sprinters racing each other finish in a tie. Explain, using the Mean Value Theorem, why this means there must have been a moment in the race when the two sprinters were running at exactly the same speed.
   (b) Suppose three sprinters racing each other finish in a tie. Can we conclude that there must have been a moment in the race when the three sprinters were running at exactly the same speed? State your answer, and defend it carefully.

2. In the 2006 paper "Use of the gamma distribution to represent monthly rainfall in Africa for drought monitoring applications", published in the *International Journal of Climatology*, the authors refer to gamma probability distribution functions of the form

$$f(x) = Mx^a e^{-kx},$$

   where M, a and k are positive constants, and $f(x)$ is the probability that a particular location will have a monthly rainfall accumulation x. (This is a simplified form of the function labelled (2) in the paper.)

   (a) $f(x)$ (as given above) has exactly one critical point. Find that point.
   (b) Suppose the critical point in part (a) is denoted c. Is the graph of $f(x)$ increasing or decreasing on the interval (c, ∞)? Justify your answer.
   (c) Why does your answer in part (b) make sense in terms of the physical phenomenon being described? In other words, why does it make sense for the graph of the probability that a particular location will have a monthly rainfall accumulation x to be decreasing (or increasing) on (c, ∞)? Justify your answer in a few sentences.
   (d) What do you think the area under the graph of the function $f(x)$ (and above the x-axis) from $x = 5\,\text{cm}$ and $x = 50\,\text{cm}$ represents? [Bonus]

3. (a) Suppose two functions $f(x)$ and $g(x)$ have the same derivative. Show that f and g are *parallel* in the sense that $h(x) = f(x) - g(x)$ is constant.
   (b) Describe all functions with the derivative e^{2x}.
   (c) Find the function $f(x)$ such that $f'(x) = e^{2x}$ and $f(0) = 2$.