There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete arguments and justifications. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. They must be stapled, with your name and student number at the top of each page.

1. Find the coordinates of all seven critical points on the curve

$$(x^2 + y^2)^2 = 24(x^2 - y^2).$$

(You are not required to classify those critical points.)

2. The demand function Q(p) for a commodity is used to describe the quantity of the commodity that can be sold at a price of p dollars. The price elasticity of demand, given by the equation

$$E(p) = \frac{\mathrm{d}Q}{\mathrm{d}p} \cdot \frac{p}{Q(p)},$$

is a measure used in economics to show responsiveness of the quantity demand to a change in its price p. More precisely, it provides the percentage change in the demand for a 1% change in price. For example, a value of E(p) > 1 would mean a greater than 1% change in the demand for a 1% increase in price.

(a) Find the elasticity of demand for unicorn sunglasses if its demand function is given by

$$Q(p) = e^{\left(\frac{p}{p^2 + 100}\right)}.$$

- (b) Find the price where the price elasticity of demand changes from increasing to decreasing.
- (c) Propose a real life example of a product whose price elasticity of demand changes from increasing to decreasing. Justify your answer.
- 3. Recent pogo stick research has shown that the displacement h(t) of a pogo stick spring under the weight of its user at time t is given by:

$$h(t) = -\cos\left(\frac{10\pi}{4 + e^{6-x}}\right)$$

(a) Find the domain and range of the inner function:

$$g(t) = \frac{10\pi}{4 + e^{6-x}}.$$

(b) Use the information above to find and classify all the critical points of h(t).