ASSIGNMENT 2·5: Section 002

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete arguments and justifications. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. They must be stapled, with your name and student number at the top of each page.

1. Given the function $y = f(x)$, do the following:
   i. Find the domain of f.
   ii. Find the x-intercepts and y-intercept of the graph of f.
   iii. Find where f is positive or negative.
   iv. Find the intervals where f is increasing ($\uparrow$), and the intervals where f is decreasing ($\downarrow$).
   v. Find all local maxima and all local minima of the function f. State how you know this, i.e. which test did you use?
   vi. Find all intervals where the graph of f is concave up ($\cup$) and all intervals where the graph of f is concave down ($\cap$).
   vii. Find all inflection points of the graph of f.
   viii. Plot each "important point" accurately and then connect the dots by using the information that you have gathered.

   Question 1.a) $y = \dfrac{\cos x}{2 + \sin x}$.

   Question 1.b) $y = \dfrac{x^2}{x^2+3}$.

2. The family of Normal (or Gaussian) curves play a very important roll in science, forestry, medicine, and business. You may have heard of the so-called Bell curve. These curves are defined by two parameters, the mean μ and the standard deviation $\sigma > 0$,

$$N(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}.$$

   (a) What is the range of $y = N(x)$?
   (b) Find and classify any extreme values of $y = N(x)$.
   (c) Explain why the critical number found in part (b) is actually a global maximum value, while there in no minimum value.
   (d) Find the intervals for concavity for $N(x)$ and explain what roll σ plays in shaping the graph of $N(x)$

3. Sketch the graph of a function defined on $[0, \infty)$ that satisfies all the given conditions.
   $f'(x) > 0$ if $0 < x < 2$, $f'(x) < 0$ if $x > 2$,
   $f'(2) = 0$, $\lim_{n \to \infty} f(x) = 1$,
   $f''(x) < 0$ if $0 < x < 3$, $f''(x) > 0$ if $x > 3$.

   Bonus: 2pts Extend your graph to all of $\mathbb{R}$ by using the following properties.
   i. The function is an _even_ function. That is, $f(-x) = f(x)$ for all x.
   ii. The function is an _odd_ function. That is, $f(-x) = -f(x)$ for all x.

   Bonus: 2pts When is $f(x)$ differentiable at $x = 0$ in the two extensions above?