Note: This assignment is optional. It does not need to be completed or submitted and will not impact grades. There is a corresponding webwork assignment that is also optional. The indention of this assignment is to practice asymptotes and curve sketching; topics that will be on the final exam. Feel free to submit the assignment if you would like feedback.

Questions:

1. Evaluate the following limits using any method covered in class. Conclude whether the functions have any asymptotes at the following points. *Hint: Consider rearranging the equation first.*

   (a)
   $$\lim_{x\to 0} \frac{2\sin(x) - \sin(2x)}{x - \sin(x)}.$$

   (b)
   $$\lim_{x\to 0} x\ln(x).$$

   (c)
   $$\lim_{x\to -\infty} \sqrt{x^2 - 2x} + x.$$

2. Hand-sketch the following functions using the steps covered in lectures (Be sure to highlight all important points/trends in your sketch).

   (a)
   $$f(x) = \frac{x^2+4}{x^2-4}.$$

   (b)
   $$g(x) = (x^2 - 4x)e^{-x}.$$

3. Sketch a graph of a function $f(x)$ with the following properties.
   - $f(x)$ is an odd function with a root at $x = 0$ and $\lim_{x\to\infty} f(x) = -1$,
   - $f(x)$ has the domain: $\mathbb{R} - \{-3, 3\}$,
   - $\lim_{x\to 3^-} f(x) = \infty$, $\lim_{x\to 3^+} f(x) = -\infty$,
   - $f(x)$ is negative on the interval $(0, 2) \cup (3, \infty)$ and positive on the interval $(2, 3)$,
   - $f(x)$ is differentiable on the entire domain with critical points at $x = 1, 5$,
   - $f'(x) > 0$ on the interval $(1, 3) \cup (3, 5)$ and $f'(x) < 0$ on $(0, 1) \cup (5, \infty)$,
   - The second derivative of $f(x)$ exists on the entire domain,
   - $f''(x) > 0$ on the interval $(0, 3) \cup (8, \infty)$ and $f''(x) < 0$ on the interval $(3, 5)$.

4. [Challenge] Find an algebraic equation satisfying the above conditions. *Hint: A piecewise function is probably most practical. Partition the domain in convenient places.*