ASSIGNMENT 2·8: Section 002

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete arguments and justifications. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. They must be stapled, with your name and student number at the top of each page.

In the following take great care when setting up the problems. Use the following steps as a guide.

1. Understand the problem. Read the problem carefully. Try to draw a picture or think about some examples. Get a feel for what the relevant quantities are, how the quantities can change and what constraints are present.

2. Introduce notation for the relevant quantities. Identify the quantity to be optimized.

3. Find relationships between the variables.

4. Find an equation for the quantity to be optimized as a function of a single variable on some suitable domain.

5. Do Calculus. Use techniques from the course to find the absolute maximum or minimum as required. For example, if you are working with a continuous function on a closed interval consider using the closed interval test.

6. Reflect. Take some time to think about your answer. Is it reasonable?

Question 1 Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.

Question 2 Your love has been lost at sea for months. Each day you stand on a cliff, at point $(0,0)$, looking out to sea wondering if they will return. One day you see a boat due north at point $(0,2)$. You notice the boat is traveling down the river along the curve $y = \sqrt{x+4}$ towards the harbour, which is west of you at point $(-4,0)$. When the boat reaches the point where it is closest to your position you recognize it as your love's boat. Find the coordinates of this point.

Question 3 You are selling subscriptions to an online magazine. Based on some market research you determine that if you set the price of the subscription to be p in dollars then $q(p)$ is the quantity of subscriptions you are able to sell (in thousands) where

$$q(p) = 6 - 2\sqrt{p}.$$

The revenue you will generate is give by

$$r(p) = \text{price} \cdot \text{quantity} = p \cdot q(p).$$

What price should you set in order to maximize revenue?