

ASSIGNMENT 1.8 (Section 002) Due: Friday, October 25

There are two parts to this assignment. The first part is on WeBWork — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions (in full sentences) with complete arguments and justifications in a linear, coherent manner. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. Your work must be your own and must be self-contained. Assignments must be stapled, with your name and student number at the top of each page. The assignment is due at the beginning of class on the due date.

1. The height of a ball at time t is $x(t) = t(4 - t)$.
 - a) Use the definition of the derivative as a limit and interpretation of the velocity as the derivative of position with respect to time to find the velocity $v(t)$ of the ball at time t .
 - b) At what moment of time is the velocity equal to zero? At what moment of time is the height of the ball the largest? Is there any relation between these moments in time? If so, explain what it is and why it makes sense intuitively for such a relation to exist.

2. We define the **dderivative** (that's right: a double d!) of the function f at point x to be the limit

$$f^\nabla(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

- a) Let $f(x) = 2x + 3$. Find $f^\nabla(x)$.
- b) Let $f(x) = ax + b$ for some unspecified numbers a, b . What is the dderivative of f ?
- c) Let $f(x) = x^2$. Find $f^\nabla(x)$.
- d) Find the dderivative of the function $f(x) = x^3$ at points $x = 1$ and $x = -1$.
- e) The dderivative of a function f at point x being **positive** means that

$$\lim_{h \rightarrow 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} > 0.$$

This means that for really small h the value of $\frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$ is positive, which in turn means that

$$\frac{f(x+h) + f(x-h)}{2} > f(x)$$

(you are encouraged to do the algebra on a separate sheet of paper to verify that this is indeed correct). Similarly $f^\nabla(x) < 0$ means that for really small h , the opposite inequality

$$\frac{f(x+h) + f(x-h)}{2} < f(x)$$

holds.

Think about what these inequalities mean on a graph of the function f . Based on this interpretation of the sign of the dderivative at point x , sketch three graphs: one graph of a function which is increasing and has positive dderivative at any point, one graph of a function that is decreasing and has positive dderivative at any point, and one graph of a function that is increasing and has a negative dderivative at any point. Label your graphs with labels “ $f \nearrow, f^\nabla > 0$ ”, “ $f \searrow, f^\nabla > 0$ ” and “ $f \nearrow, f^\nabla < 0$ ” respectively.

3. Let

$$f(x) = \begin{cases} x & \text{if } x = \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \dots \\ 0 & \text{otherwise} \end{cases}$$

- a) Is f continuous at $x = 0$? Justify your answer.
- b) Does $f'(0)$ exist? Justify your answer. (Hint: use the definition of the derivative using limits).