ASSIGNMENT 1·6 (Section 002) Due: Friday, October 11

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions (in full sentences) with complete arguments and justifications in a linear, coherent manner. You will be graded on the correctness, clarity and elegance of your solutions. Your answers must be typed or very neatly written. Your work must be your own and must be self-contained. Assignments must be stapled, with your name and student number at the top of each page. The assignment is due at the beginning of class on the due date.

1. Let $f(x)$ be the function defined by:

$$f(x) = \begin{cases} 2x + 5 & x < -2 \\ 2ax^2 + b & -2 \leq x < 5 \\ 7 - 3x & x \geq 5 \end{cases}$$

   Find the values of a and b that make this function continuous.

2. Consider the function

$$f(x) = x^3 - 2x^2 - 2x + 2.$$

   (a) Show that $f(x)$ has at least 3 roots (zeros).
   (b) Pick one of the roots and using the Bisection Algorithm find an interval of size 0.1 or smaller containing the root. Include the results of each iteration (feel free to use a calculator).
   (c) **Bonus** Compute, using the Bisection Algorithm one of the zeros to three decimal places. There will be a prize for the person who computes the most decimal places of a zero (if you use a computer it must be programmed by yourself).

3. The other weekend, I decided to go for a hike. At the bottom of the mountain around 10 : 00am, I started my hike up the mountain. Along the way, I made stops to eat, relax, and even had to go backwards along the trail for no adequately explored reason. I finally reached the summit at 3 : 00pm. I decided to stay the night at the summit and leave to go back down at 10 : 00am the next day. Taking the same trail down with stops and backtracking at various times, I made it back down to the point I started from the day before at 3 : 00pm.

   Prove that there existed a time where I was at the exact same location on the mountain at the exact same time on *both days*. You should provide a precise mathematical argument as well as a picture depicting the situation.

   HINT: Consider the difference in height at time t where t is the time after 10am.