② April 4

- Curve Sketching
- Optimization
- Related Rates.

Curve Sketching

Sketch $f(x) = \dfrac{-(x-1)(x-3)}{x^2}$.

1) x-intercepts : $f(x) = 0$. when $x = 1, 3$.
y-int : $f(0)$ is not defined,
no y-int.

2) Domain: $\{x \in \mathbb{R} : x \neq 0\}$.

3) Local Max/min Intervals of Inc/Dec.

$f'(x) = \dfrac{6-4x}{x^3}$.

Critical points:
Can $f'(x) = 0$ when $x = 3/2$.

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not local min.

local. max

First derivative test:

	$(-\infty, 0)$	$(0, 3/2)$	$(3/2, \infty)$
$f(x)$	↘	↗	↘
$f'(x)$	<0	>0	<0

local max: $3/2$.

Concavity: $f''(x) = \dfrac{2(4x-9)}{x^4}$.

Possible inflection points: $x = 9/4$.

	$(-\infty, 0)$	$(0, 9/4)$	$(9/4, \infty)$
$f(x)$	C.D.	C.D	C.U.
$f''(x)$	<0	<0	>0

inflection point.

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Asymptotes! Horizontal.

$$\lim_{x \to \infty} \frac{-(x-1)(x-3)}{x^2} = \lim_{x \to \infty} \frac{-x^2+4x-3}{x^2}.$$

$$= \lim_{x \to \infty} \frac{-x^2/x^2 + 4x/x^2 - 3/x^2}{x^2/x^2}.$$

$$= \lim_{x \to \infty} \frac{-1 + 4/x - 3/x^2}{1}.$$

$$= \frac{-1 + 0 - 0}{1} = -1.$$

$\Rightarrow$ asymptote at $y = -1$.

$$\lim_{x \to -\infty} f(x) = \ldots = -1.$$

vertical asymptotes potential vertical asymptote at $x = 0$.

$$\lim_{x \to 0^+} \frac{-(x-1)(x-3)}{x^2} = \text{“}\frac{-1(-1)(-3)}{0^+}\text{”} = -\infty.$$

$$\lim_{x \to 0^-} \frac{-(x-1)(x-3)}{x^2} = -\infty.$$

⑤ April 4

L'Hôpital's Rule.

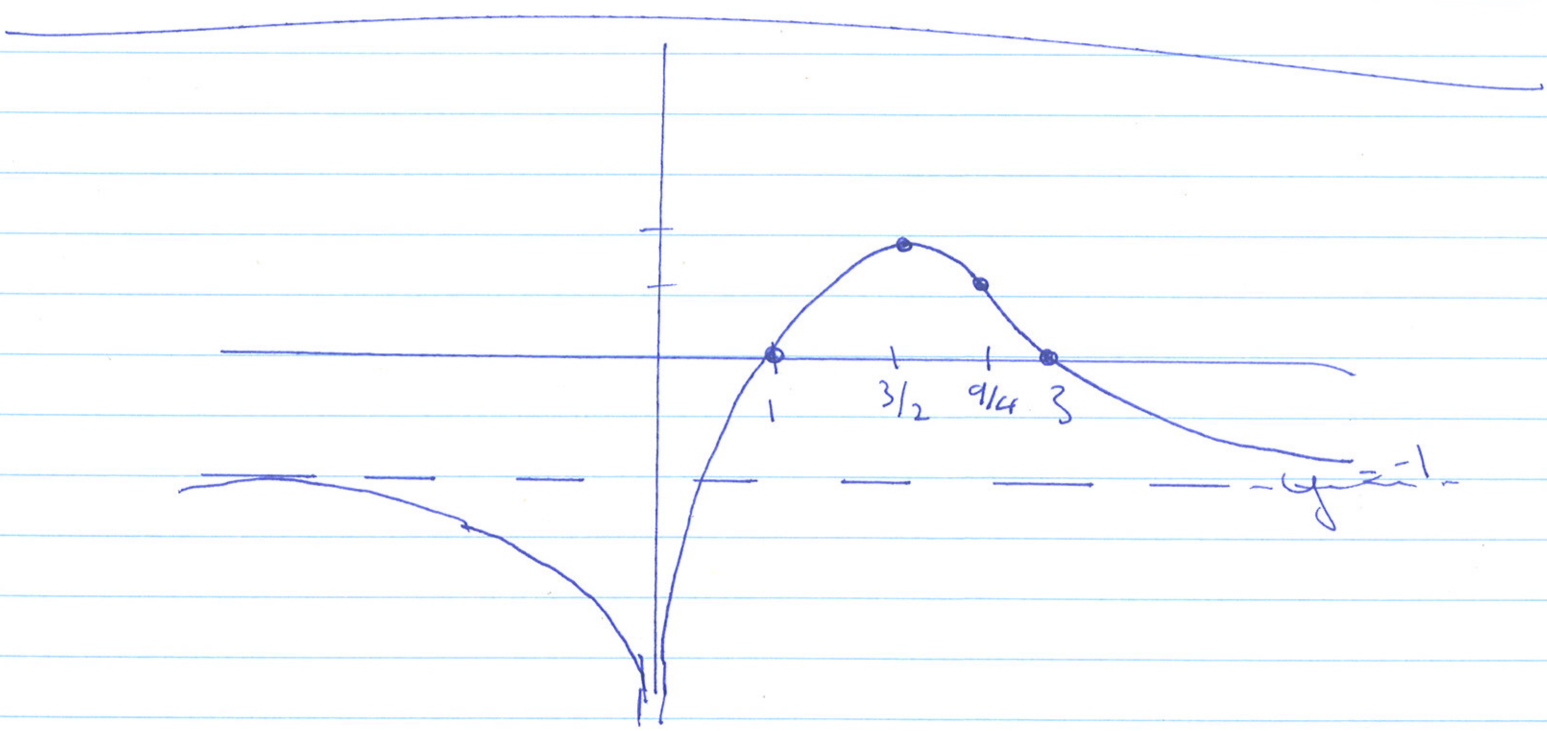
$$\lim_{x\to 0} \frac{\sin x}{x} \qquad \text{“}\frac{0}{0}\text{”}$$

$$\overset{LH}{=} \lim_{x\to 0} \frac{\cos x}{1}$$

$$= 1.$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} \overset{LH}{=} \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

of the form $\text{“}\frac{0}{0}\text{”}, \text{“}\frac{\infty}{\infty}\text{”}$.



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Problem Solving

Optimization

1. Understand. Picture
2. Notation.
   Identify desired quantity
3. Equation.
4. Eliminate to one variable
5. Calculus
   $\boxed{\text{(closed interval method}}$
   (first derivative test)

Related Rates

1. Understand. Picture.
2. Notation
   Rates: Have? Want?
3. Equation.
4. Chain Rule!
5. Plug / Solve

Plug in comes after derivative.

(3) (7) April 4 Post.

Optimization vs. Related Rates

1. A cylindical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

O.

2. A cylindrical oil tank with radius 5m is being filled at a rate of $3 m^3/min$. How fast is the height of the oil increasing.

R.R.

3. Two resistors with resistance R_1, R_2 are connected in parallel. The total resistance of the system will be

R.R.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

(R measured in ohms (Ω))

If R_1 and R_2 are increasing at rates of $0.3\ \Omega/s$ and $0.2\ \Omega/s$ respectively how fast is R changing when $R_1 = 80\Omega$ and $R_2 = 100\ \Omega$?

O.

4. If a resistor of R ohms is connected across a battery of E volts with internal resistance r ohms, then the power (in watts) in the external resistor is $P = \dfrac{E^2 R}{(R+r)^2}$.

If E and r are fixed but R varies, what is the max power?

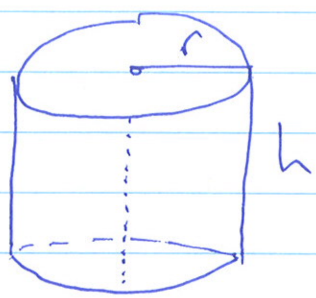
① April 7

- PASS today : BUCH B309
- Exam O.H. in LSK 303C
  10, 16, 17, 21, 23 : 11-1pm
- Review Session: 22nd BUCH A203
  12:30 - 2:30pm
- PASS Exam Sessions:
  19th, 4-6pm in ANGU 254
  22nd, 4-6pm in LSK 460.

Example: A cylindrical can is to be made to hold 1L of oil. Find the dimensions that will minimize the material required.

1. Picture, Understand.

2. Notation.



V - Volume

A - Surface area.

(2) April 7

3. Relationship(s)

1. $= V = \pi r^2 h$, $A = 2\pi r^2 + 2\pi r h$.

Area $=$ 2 $\cdot$ Area of circle + area rectangle.

4. Function of one variable. On some domain.

$\Rightarrow$ $h = \dfrac{1}{\pi r^2}$.

$$A(r) = 2\pi r^2 + 2\pi r\left(\frac{1}{\pi r^2}\right)$$
$$= 2\pi r^2 + \frac{2}{r}.$$

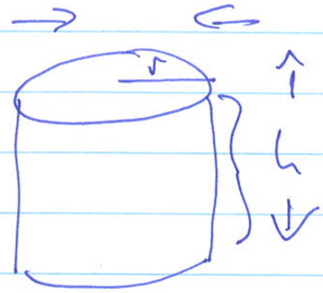
$r \in (0, \infty)$

5. Calculus. (first derivative test + uniqueness of critical point)

Example: The height of a cylinder is increasing at a rate of 7 m/s and the radius is decreasing at 3 m/s.
How fast is the volume changing when the cylinder is 5 m high and has a radius of 6 m?

③ April 7

1. Picture



2. Notation. V – volume. know: dh/dt, $\frac{dr}{dt}$.
Want: dV/dt.

3. Equation.

$$V = \pi r^2 h$$

4. Chain Rule

6

5

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}.$$

3 7

5. Solve / Substitute.