

Limit Definition of the Derivative

Find the equation of the line tangent to $y = \frac{1}{x}$ at the point $(-1, -1)$, using the limit definition of the derivative.

$$f(x) = \frac{1}{x}$$

$$\begin{aligned} f'(-1) &= \lim_{h \rightarrow 0} \frac{f(-1+h) - f(-1)}{h} \\ &= \lim_{h \rightarrow 0} \left(\frac{1}{h} \right) \left(\frac{1}{h-1} - -1 \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{1}{h} \right) \left(\frac{1}{h-1} + 1 \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{1}{h} \right) \left(\frac{1}{h-1} + 1 \left(\frac{h-1}{h-1} \right) \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{1}{h} \right) \left(\frac{1}{h-1} + \frac{h-1}{h-1} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{1}{h} \right) \left(\frac{1+h-1}{h-1} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{1}{h} \right) \left(\frac{h}{h-1} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{1}{h-1} \right) \\ &= \frac{1}{0-1} \\ &= -1 \end{aligned}$$

$$y + 1 = -(x + 1)$$

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$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

