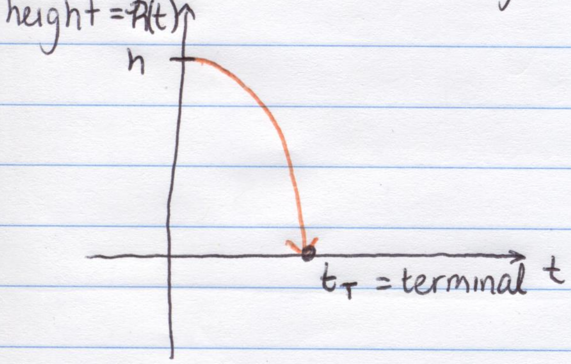
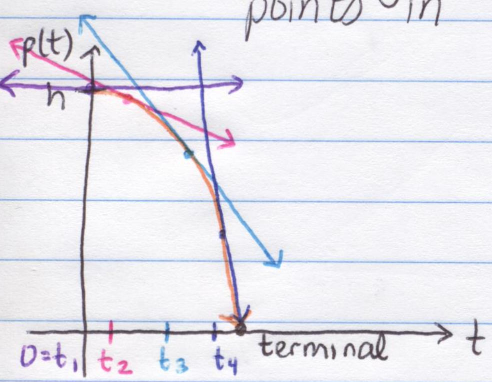
① Imagine you drop a stone from the Ladner clock tower. Sketch position, velocity, and acceleration graphs of the stone.

SOLUTION: Assuming the height of the tower is h, we have the following position graph for the stone

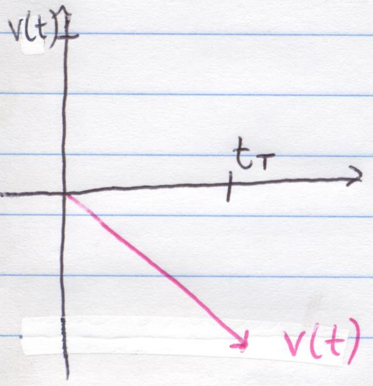


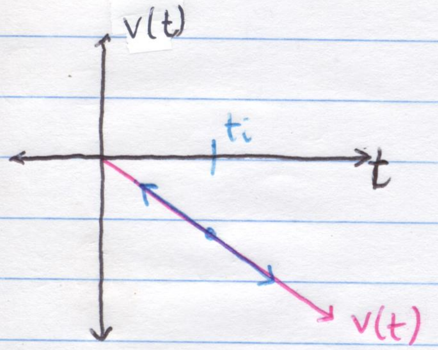
The graph is of this form since as we consider position with respect to time, at the very start of the drop, i.e $t=0$, position is not changing as quickly as it does as time increases.

Using the notion of "change in position", what we know as velocity, we can sketch the graph of velocity. We should look at tangent lines at certain points in time, say t_1, t_2, t_3, t_4

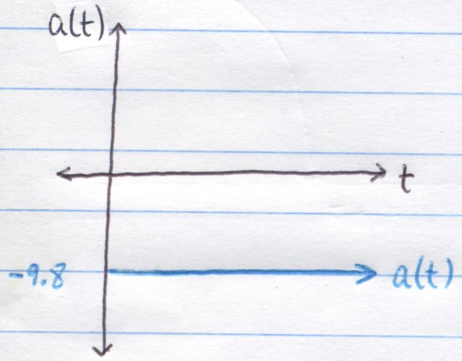


- We observe that as time increases the slope of the tangent line is getting more and more negative, consistently.
- There is no point t_i in time where our change in velocity increases.
- As mentioned in class, since $p(t)$ is decreasing, we must have $v(t) < 0$ for $t > 0$ and $v(t) = 0$ at $t = 0$ since we are at a peak
- So, velocity is depicted by figure ③
  - Every possible rate of change < 0 is observed





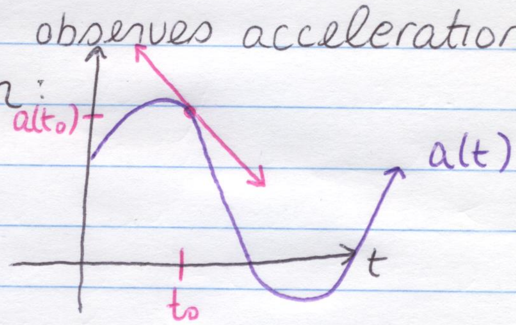
- In the same manner as before, we see that since $v(t)$ is always decreasing, $a(t)$ must be < 0.
- What if we try and look at tangent lines again?
- If I choose a point $t = t_i$, I see that the tangent line is equivalent to $v(t)$
- So, the change in velocity, i.e acceleration, is never changing, thus constant.
- We can also realize that acceleration is due to the force of gravity, which is $-9.8\,m/s^2$. So, the slope of $v(t)$ is -9.8.



#2 In physics, jerk is defined to be the change in acceleration with time.

(a) Describe, using a graph/picture, how to find the instantaneous jerk of an object, given the graph of acceleration. If acceleration is measured in m/s^2, what are the units of jerk?

<u>SOLUTION</u>: Suppose some object observes acceleration given by the following graph:



As jerk = rate of change of $a(t)$, we need to look at tangent lines!

The slope of the tangent line at $(t_0, a(t_0))$ gives the instantaneous jerk, since tangent lines correspond to instantaneous rates of change.

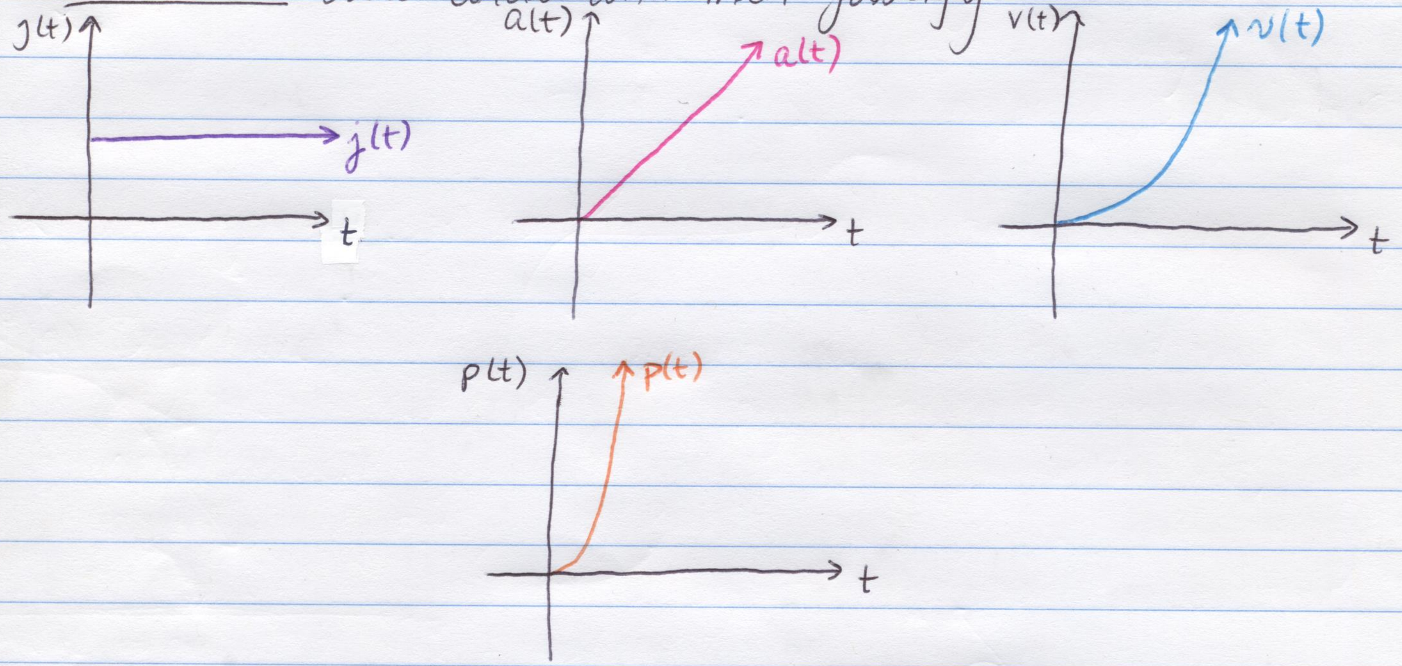
#2(a) cont...

As jerk means the change of acceleration over time, this means, mathematically: $\frac{\Delta \text{acceleration}}{\Delta \text{time}} = \frac{\Delta(a(t))}{\Delta t}$

Since units of $a(t)$ are m/s^2 and units of time are seconds, $\frac{\Delta a(t)}{\Delta t}$ is measured by $\frac{m/s^2}{s} = m/s^3$

(b) If an object has constant, positive jerk, what does the graph of its position wrt time look like?

SOLUTION: Lets draw and then justify



Since $j(t)$, i.e $\Delta a(t)$ is constant, we know that the rate of change of $a(t)$ must be the same as time increases. The type of graph with this property is a line, so $a(t)$ is linear. Similarly, since $a(t) = \Delta v(t)$, we know that the tangent lines of $v(t)$ must have positive slope, and increase in slope with time. $v(t)$ in the diagram satisfies these conditions with the slope of the tangent line at $t=0$ being 0. Finally, $p(t) = \Delta v(t)$ must have ....

#2(b) CONT...

... tangent lines with (always) positive slope and increasing with time. Unlike $a(t) = \Delta v(t)$, $v(t) = \Delta p(t)$ increases at a slower rate at first, but then increases faster at some $t = t_0$ than $a(t)$. So, for $p(t)$, the graph should not change much at the beginning, and then begin to shoot off after $t = t_0$.

(c) <u>BONUS</u>: Explain what constant jerk on the human body feels like.

<u>SOLUTION</u>: In order to understand what <u>constant</u> jerk feels like, let's try and think about what a general change in acceleration feels like.
If all of the sudden, one accelerates very quickly you would feel a "jerking" motion backwards. This is due to a change in the G-force. If jerk were constant, then acceleration would be linear as in part (a). So, even though we are accelerating (or decelerating) the push back (or forward) would be constant.