ASSIGNMENT 1·4 (Section 002) Solutions Part 2

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1. Please see Solutions Part 1

2. Please see Solutions Part 1
   An alternative answer to the bonus is: (taken from Wikipedia)
   If acceleration can be felt by a body as the force (hence pressure) exerted by the object bringing about the acceleration on the body, jerk can be felt as the change in this pressure. For example a passenger in an accelerating vehicle with zero jerk will feel a constant force from the seat on his or her body; whereas positive jerk will be felt as increasing force on the body, and negative jerk as decreasing force on the body

3. Consider the real valued function $f(x) = \sqrt{25 - x^2}$. Consider also a real valued function g whose domain is $(-\infty, 3)$.
   (a) Find the domain of f.
   (b) Find the domain of $g \circ f$.

   Solution
   Part (a). Since no real number is the square root of a negative number for function f we require,

$$25 - x^2 \geq 0.$$

To find the x that satisfy above we rearange,

$$\begin{aligned} 25 - x^2 &\geq 0 \\ 25 &\geq x^2 \end{aligned}$$

Taking both the positive and negative square root yields both,

$$x \geq -5, \quad x \leq 5.$$

Putting these together gives $-5 \leq x \leq 5$. So the domain of f is $[-5, 5]$ or $\{x \in \mathbb{R} : -5 \leq x \leq 5\}$.

Part (b). Before we get to the full solution let's do an example. We can ask: is 0 in the domain of $g \circ f$? First, we put 0 into f to get, $f(0) = 5$. Next, we put this output from f into g. We can't do this however since the domain of g is $(-\infty, 3)$ which does not include 5.
For the full solution we ask if a general x is in the domain of $g \circ f$. First, we put x into f to get,

$$f(x) = \sqrt{25 - x^2}.$$

Next, we ask if this output value from f is in the domain of g. In other words is,

$$\sqrt{25 - x^2} < 3.$$

Solving this inequality for x we see,

$$\begin{aligned} \sqrt{25 - x^2} &< 3 \\ 25 - x^2 &< 9 \\ 25 - 9 &< x^2 \\ 16 &< x^2 \\ 4 &< |x|. \end{aligned}$$

The above gives us both $x > 4$ and $x < -4$ or $x \in (-\infty, -4) \cup (4, \infty)$. To see this we can plot the graphs of both x^2 and 16 to see where the first is larger than the second. In addition to the condition that $x \in (-\infty, -4) \cup (4, \infty)$ we must also recall that x must be in the domain of f, which is $[-5, 5]$. Taking the intersection of these two sets gives the domain of $g \cdot f$ which is,

$$[-5, -4) \cup (4, 5].$$