

Midterm Test 2 Duration: 60 minutes*This test has 6 questions on 8 pages, for a total of 44 points.*

- Read all the questions carefully before starting to work.
- Q1 and Q2 are short-answer questions; put your answer in the boxes provided.
- All other questions are long-answer; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **None of the following are allowed:** documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

First Name: _____ Last Name: _____

Student-No: _____ Section: _____

Signature: _____

Question:	1	2	3	4	5	6	Total
Points:	6	12	4	7	7	8	44
Score:							

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (i) speaking or communicating with other examination candidates, unless otherwise authorized;
 - (ii) purposely exposing written papers to the view of other examination candidates or imaging devices;
 - (iii) purposely viewing the written papers of other examination candidates;
 - (iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Short-Answer Questions. Questions 1 and 2 are short-answer questions. Put your answer in the box provided. Full marks will be given for a correct answer placed in the box, unless otherwise specified. Show your work also, for part marks. Each part of Q1 is worth 2 marks and each part of Q2 is worth 3 marks. Not all parts are of equal difficulty. **Simplify your answers as much as possible in Questions 1 and 2.**

2 marks

1. (a) Let $f(x) = \ln\left(\frac{1}{x\sqrt{3x+1}}\right)$. Find $f'(x)$.

Answer:

2 marks

(b) Evaluate $\arcsin\left(\sin\left(\frac{7\pi}{6}\right)\right)$.

Answer:

2 marks

(c) Find the derivative of $(\sin x)^{\ln x}$. You can assume $x \in (0, \pi)$.

Answer:

3 marks

2. (a) Use a suitable linear approximation to estimate
- $\sqrt[3]{25}$
- .

Answer:

3 marks

- (b)
- [For this problem provide brief justification for your choice.]**

Under the right conditions the velocity of an object with air resistance can be described by the following differential equation:

$$\frac{dv}{dt} = v^2$$

Which of the following functions satisfies the above equation?

A: $v(t) = 2t$

B: $v(t) = \frac{1}{3}t^3$

C: $v(t) = \frac{1}{1-t}$

D: $v(t) = e^{2t}$

E: None of the above

Answer:

3 marks

- (c) Find the equation of the line tangent to the curve $e^{2y} - (y - 1)x = 4$ at the point $(3, 0)$.

Answer:

3 marks

- (d) Suppose that a particle travels according to the equation $x \cos y + y \ln x = 0$. If its x -coordinate is changing at a constant rate of $2/3$ units per second what is the rate of change of its y -coordinate when the particle is at $(1, \pi/2)$.

Answer:

Full-Solution Problems. In questions 3–6, justify your answers and **show all your work**. If a box is provided, write your final answer there. Unless otherwise indicated, **simplification of answers is not required in these questions**.

1 mark

3. (a) Use quotient rule to find the derivative of $\cot x$. Note that $\cot x = \frac{\cos x}{\sin x}$.

3 marks

(b) Show that

$$\frac{d}{dx}(\operatorname{arccot}(x)) = -\frac{1}{1+x^2}.$$

5 marks

4. (a) A glass of iced tea is taken from the refrigerator and placed on the counter where it is forgotten about. After an hour has passed it is $12^{\circ}C$. After two hours it is $17^{\circ}C$. The temperature of the air around the counter is $25^{\circ}C$. How cold is the refrigerator? You may assume the glass is warming according to Newton's law of cooling. Be sure to define all variables and units you use.

2 marks

- (b) If the glass is left on the counter indefinitely explain, using your model, what will happen to its temperature.

3 marks

5. (a) Find the second degree Taylor Polynomial for $f(x) = x^{1/4}$ around $x = 1$.

3 marks

(b) Your friend uses your polynomial to approximate $2^{1/4}$. Find a bound on the error in this approximation.

1 mark

(c) Show that the error is less than 0.1.

4 marks

6. (a) A woman walks in a straight line away from a light post that is three times as tall as she is. Her distance from the post is given by $s(t) = 4t - \frac{1}{2}t^2$ where s is in meters and t is in seconds. Find the rate of change of the length of her shadow after 3 seconds. Be sure to define all variables you use.

4 marks

- (b) What is the rate of change of the length of her shadow after she has walked a total of 10 meters?