① April 2.

Review Workshops Next Week.

Webwork #12 is up.

§ 4.8 Newton's Method.

Let's say we want to find the zeros of $f(x) = x^3 - 3x^2 + 1$.

We could use IVT to show a solution exists. (f is continuous).

$$\left.\begin{array}{l} f(0) = 1 > 0 \\ f(1) = -1 < 0 \end{array}\right\} \Rightarrow \text{there is a zero between } 0 \text{ and } 1.$$

We can compute

$f(0.5) = 0.375 > 0$.

$\Rightarrow$ there is a zero between 0.5 and 1.

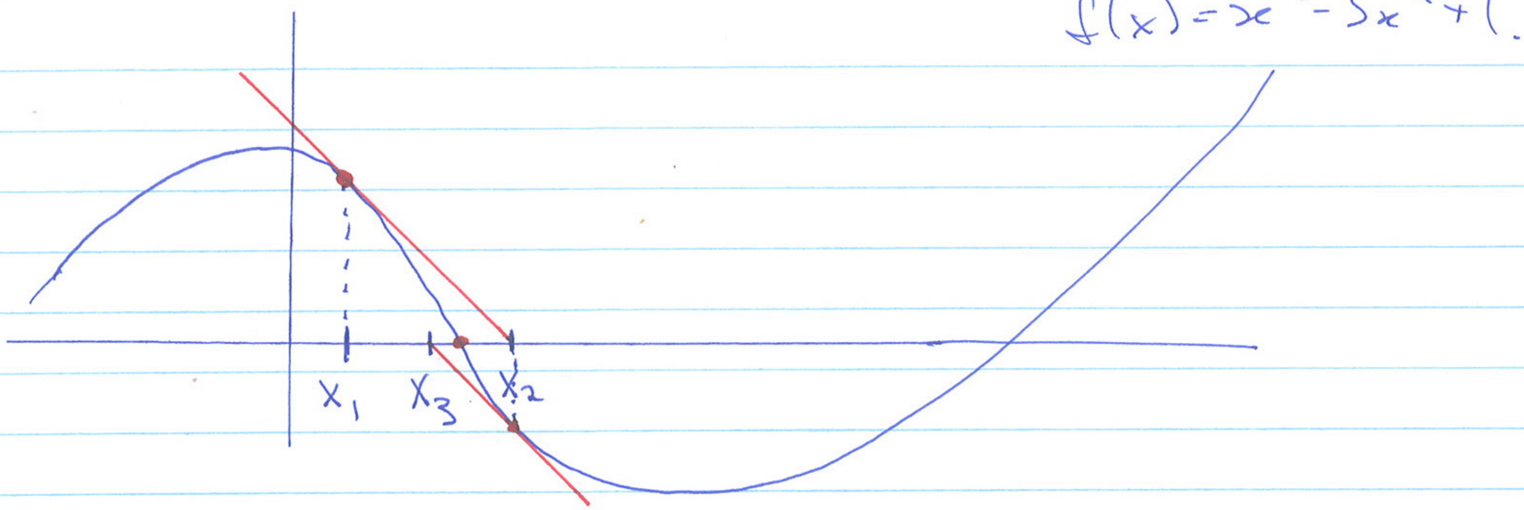
$f(0.75) = -0.26\ldots < 0$

$\Rightarrow$ there is a zero between 0.5 and 0.75.

This algorithm is called Bisection Method.

Instead we can use the tangent line to approximate the trajectory of the graph.

②



Let's say we start with a guess, x_1, to approximate the zero.

If we follow the tangent to the x-axis to find the point x_2.

The equation of the tangent line at X_1 is $y = f(x_1) + f'(x_1)(x - x_1)$.

So x_2 solves:

$$0 = f(x_1) + f'(x_1)(x_2 - x_1).$$

$$f'(x_1)(x_2 - x_1) = -f(x_1)$$

$$x_2 - x_1 = \frac{-f(x_1)}{f'(x_1)}$$

So,
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

③

There is no need to stop.
We can find

$$X_3 = x_2 - \frac{f(x_2)}{f'(X_2)}.$$

And in general,

$$\boxed{X_{n+1} = X_n - \frac{f(X_n)}{f'(x_n)}}$$

Example: $f(x) = x^3 - 3x^2 + 1.$
$f'(x) = 3x^2 - 6x.$

Start with an initial guess of $x_1 = 0.5$

$$X_2 = x_1 - \frac{f(x_1)}{f(x_1)} = 0.5 - \frac{f(0.5)}{f'(0.5)}.$$

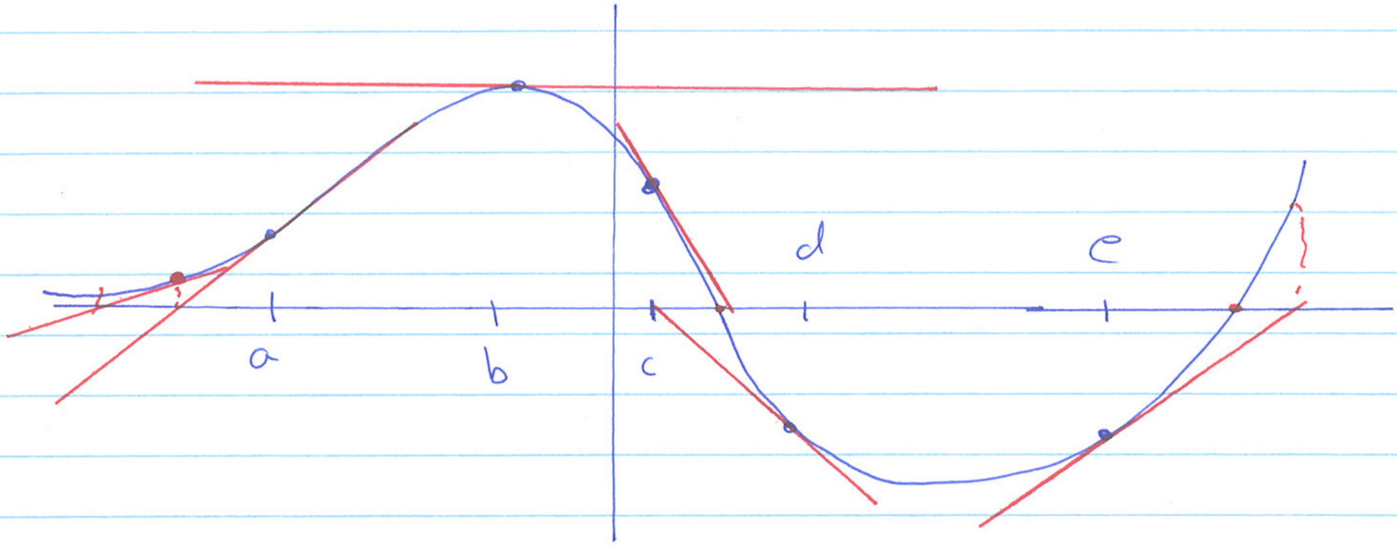
$$X_2 = 2/3.$$

$$x_3 = X_2 - \frac{f(x_2)}{f'(x_2)} = 0.65277\ldots$$

The true value is $0.65270\ldots$

4

Not all initial guesses will work.



Clicker Q: Is the point $x_1 = a$ a good initial guess?

A) Yes

B) No.

Is $x_1 = b$ good?

A) Yes

B) No

Note $f'(b) = 0$

Newton's method won't work if $f'(x_1) = 0$ or if $f'(x_1)$ D.N.E.

Points c, d and e are good choices

Example: Use $x_1 = \pi/6$ to approximate the critical point of

$$f(x) = \sin x - \frac{1}{2}x^2.$$

Find x_2. Try this later.

Critical point $f'(x) = 0$

$$g(x) = f'(x) = \cos x - x,$$

$$\boxed{X_2 = X_1 - \frac{g(x_1)}{g'(x_1)}} = \ldots = 0.7518\ldots$$

true ans. $0.739\ldots$

§ 4.9 Antiderivatives

Given a function $f(x)$ we may want to find $F(x)$ where

$$F'(x) = f(x).$$

$F(x)$ is ~~the~~ an antiderivative of $f(x)$.

Example: If $f(x) = x^2$
$F(x) = \frac{1}{3}x^3$.

Is $\frac{1}{3}x^3$ the only antiderivative?
No.

Also, $F(x) = \frac{1}{3}x^3 + C$

for any constant C will work.

Example: Find the general antiderivative of $f(x) = e^{2x}$.

Clicker!

A) $e^{2x} + C$

B) $2e^{2x} + C$

C) $\frac{1}{2}e^{2x} + C$.

Find a function $F(x)$ so that $F'(x) = e^{2x}$ and $F(0) = 2$.

$$F(x) = \frac{1}{2}e^{2x} + C$$

$$F(0) = 2 = \frac{1}{2}e^{0} + C$$

$$2 = \frac{1}{2} + C$$

$$C = 3/2.$$

So,

$$F(x) = \frac{1}{2}e^{2x} + 3/2.$$

Example: Find the general antiderivative of

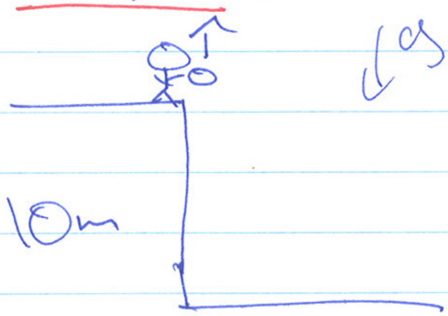
$$f(x) = \frac{2x + x^3}{\sqrt{x^2}}.$$

$$f(x) = \frac{2x}{x^{1/2}} + \frac{x^3}{x^{1/2}}$$

$$= 2x^{1/2} + x^{5/2}.$$

$$F(x) = 2 \cdot \frac{2}{3} x^{3/2} + \frac{2}{7} x^{7/2} + C.$$

Example:



Assume acceleration due to gravity is constant. The ball is thrown upwards with velocity 2m/s. Find an equation for $s(t)$ the distance of the ball from the ground.

$$a(t) = -g$$

$$v(t) = -gt + C$$

but $v(0) = 2$.

$$2 = v(0) = C \Rightarrow C = 2.$$

$$v(t) = -gt + 2.$$

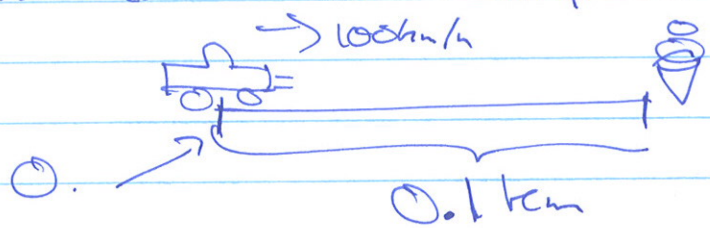
$$s(t) = -\frac{1}{2}gt^2 + 2t + C.$$

$$s(0) = 10 \Rightarrow C = 10.$$

$$s(t) = \frac{-1}{2}gt^2 + 2t + 10.$$

We can now easily find the highest point the ball reaches or the time when the ball hits the ground or the position of the ball at any time.

Example: You are driving at 100km/h. You see someone selling ice cream 100m ahead and slam on the breaks. What constant acceleration is needed to stop in time?



$$a(t) = -A$$

$$v(t) = -At + C$$

$$v(0) = 100$$

$$C = 100.$$

$$v(t) = -At + 100.$$

(9).

$$s(t) = -\frac{1}{2}At^2 + 100t + C.$$

$s(0) = 0$ so $C = 0$.

$$s(t) = -\frac{1}{2}At^2 + 100t.$$

We want when $v(t) = 0$ or

$$0 = -At + 100$$

$$t^* = \frac{100}{A}.$$

t^* is when the car stops.

We want $s(t^*) = 0.1$

$$0.1 = -\frac{1}{2}A(t^*)^2 + 100t^*.$$

$$0.1 = -\frac{1}{2}A\left(\frac{100}{A}\right)^2 + 100 \cdot \frac{100}{A}.$$

$$\frac{1}{10} = -50 \cdot \frac{100}{A} + 100 \cdot \frac{100}{A}.$$

$$\frac{A}{10} = -50 \cdot 100 + 100 \cdot 100 = 50 \cdot 100$$

$$A = 500 \cdot 100 = 50\,000 \text{ km/h}^2.$$

or $3.858\ldots$ m/s^2.

Need a constant acceleration of $-50\,000$ km/h^2.